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Research Article

Solution of Fuzzy Fractional Integro-Differential Equations Using A Domian Decomposition Method

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Abstract. In this paper, we present Adomian decomposition method to solve linear fuzzy fractional integro-differential equation with fuzzy initial conditions. Results are compared with the results obtained using Fuzzy Laplace transform method.

Keywords. Adomian decomposition method; Fractional integro-differential equation; Fuzzy differential equations

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1. Introduction

Fractional differential equations have been used to model many physical phenomenain acoustics, electromagnetics, viscoelasticity and hydrology etc. [4, 9]. Podlubny [9] have presented a survey on the applications of fractional calculus in various fields of science.

The topics of *fuzzy integral equations* (FIE) have been rapidly growing in recent years in particular to relation in fuzzy control. Allahviranloo *et al*. [1] have applied a novel method for solving fuzzy integro-differential equation under generalized differentiability. Solution of Fuzzy Integro-Differential Equations with compactness type conditions was discussed by Donchev *et al*. [5].

Armand *et al*. [2] investigated the existence and uniqueness solutions for fuzzy fractional integro-differential equations under generalized Caputo differentiability. Mittal *et al*. [8] have used Adomian decomposition method for solving the fractional integro-differential equations. Priyadharsini *et al*. [10] applied Fuzzy Laplace Transform method to solve fuzzy fractional integro-differential equations. Jameel [6] applied Adomian decomposition method for nonlinear two point fuzzy boundary value problem.

In this article, we have solved the fuzzy fractional integro-differential equation of the form

$$^{C}D^{\alpha}y(t) = a y(t) + \int_0^t K(s-t)y(s)ds \tag{1.1}$$

with fuzzy initial conditionsby Adomain decomposition method, where $^{C}D^{\alpha}$ is a Caputo fractional derivative.

In this article, Section 2 provides basic definitions of fractional calculus. In Section 3, Adomian decomposition procedure is provided to solve the fractional integro-differential equations. In Section 4, solution obtained using Adomian decomposition method for fuzzy fractional integro-differential equation is elucidated in detail through illustration.

2. Basic Definitions of Fractional Calculus

Several definitions of fractional calculus are available in literature [3, 7]. Caputo and Riemann-Liouville definitions are primarily used by several researchers in many areas. Definitions and properties of these two types of fractional derivatives are provided in this section.

2.1 Abel-Riemann Fractional Integral and Derivatives

Abel-Riemann fractional integral of any order $\alpha > 0$ of $f(x)$ is given as

$$J^{\alpha}f(x) = \begin{cases} \dfrac{1}{\Gamma(\alpha)}\displaystyle\int_0^x (x-t)^{\alpha-1}f(t)dt, & \alpha > 0, \\ f(x), & \alpha = 0. \end{cases} \tag{2.1}$$

2.2 Caputo Fractional Derivatives

$$D^{\alpha}f(x)=\begin{cases}\dfrac{1}{\Gamma(m-\alpha)}\displaystyle\int_0^x \dfrac{f^m(t)}{(x-t)^{\alpha+1-m}}dt, & 0\le m-1<\alpha\le m,\\ \dfrac{d^m f(x)}{dx^m}, & \alpha=m.\end{cases} \tag{2.2}$$

Properties of the operators J^{α} and $D^{\alpha}$are

(i) $J^{\alpha}J^{\beta}f(x)=J^{\alpha+\beta}f(x)=J^{\beta}J^{\alpha}f(x)$,

(ii) $J^{\alpha}t^{\gamma}=\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+1+\alpha)}t^{\gamma+\alpha}$,

(iii) $D^{\alpha}t^{\gamma}=\dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma+1-\alpha)}t^{\gamma-\alpha}$,

(iv) $D^{\alpha}[J^{\alpha}f(x)]=f(x)$,

(v) $J^{\alpha}[D^{\alpha}f(x)]=f(x)-\displaystyle\sum_{k=0}^{m-1}f^k(0^+)\dfrac{x^k}{k!}$.

3. Adomian Decomposition Method

Adomian decomposition method have been applied to solve ordinary differential equations and partial differential equations arise in science and engineering. We provide the procedure for Adomian decomposition method to solve the fuzzy fractional integro-differential equation of form

$${}^{C}D^{\alpha}y(t)=ay(t)+\int_0^t K(s-t)y(s)ds \tag{3.1}$$

with the fuzzy initial condition $y(0)=(\underline{y}_0,\overline{y}_0)$, where $0<\alpha<1$.

Operating with J^{α} on both sides of the above fuzzy fractional integro-differential equation (3.1) implies that

$$\underline{y}(t;r)=\underline{y}_0+J^{\alpha}\left(a\underline{y}(t;r)+\int_0^t K(s-t)\underline{y}(s;r)ds\right), \tag{3.2}$$

$$\overline{y}(t;r)=\overline{y}_0+J^{\alpha}\left(a\overline{y}(t;r)+\int_0^t K(s-t)\overline{y}(s;r)ds\right), \tag{3.3}$$

where $0\le r\le 1$.

Adomian decomposition method defines the solutions $\underline{y}(t;r)$ and $\overline{y}(t;r)$ are sums $\underline{y}(t;r)=\sum_{n=0}^{\infty}\underline{y}_n(t;r)$ and $\overline{y}(t;r)=\sum_{n=0}^{\infty}\overline{y}_n(t;r)$.

The components $\underline{y}_1,\underline{y}_2,\dots$ and $\overline{y}_1,\overline{y}_2,\dots$ are determined recursively as follows

$$\underline{y}_{k+1}(t,r)=J^{\alpha}\left(a\underline{y}_k(t;r)+\int_0^t K(s-t)\underline{y}_k(s;r)ds\right) \tag{3.4}$$

and

$$\overline{y}_{k+1}(t,r) = J^{\alpha}\left(a\overline{y}_k(t;r) + \int_0^t K(s-t)\overline{y}_k(s;r)ds\right), \tag{3.5}$$

where $k = 0,1,2,3,\dots,\infty$.

From (3.4) and (3.5), we can compute the components of $\underline{y}_n(t;r)$ and $\overline{y}_n(t;r)$ hence the series solutions $\underline{y}(t;r)$ and $\overline{y}(t;r)$ can be immediately obtained.

4. Numerical Example

We consider the linear fuzzy fractional integro-differential equation of the form

$$^{C}D^{\alpha}y(t) + \int_0^t y(s)ds = 0, \tag{4.1}$$

where $0 < \alpha < 1$ and with the initial condition $y(0) = (r-1, 1-r)$.

Exact solution of (4.1) is

$$\underline{y}(t;r) = (r-1)E_{\alpha+1}(-t^{\alpha+1}), \quad \overline{y}(t;r) = (1-r)E_{\alpha+1}(-t^{\alpha+1}),$$

where $E_{\alpha}(t)$ is Mittag-Leffler function and $0 \le r \le 1$.

According to Adomian decomposition procedure presented in Section 3, equation (4.1) can be written as follows

$$\underline{y}(t;r) = \underline{y}_0 - J^{\alpha}\int_0^t \underline{y}(s;r)ds \quad \text{and} \quad \overline{y}(t;r) = \overline{y}_0 - J^{\alpha}\int_0^t \overline{y}(s;r)ds.$$

Adomian's method defines the solutions $\underline{y}(t;r)$ and $\overline{y}(t;r)$ by the series

$$\underline{y}(t;r) = \sum_{n=0}^{\infty}\underline{y}_n(t;r) \quad \text{and} \quad \overline{y}(t;r) = \sum_{n=0}^{\infty}\overline{y}_n(t;r).$$

The terms $\underline{y}_n(t;r)$ can be obtained by recursive relation

$$\underline{y}_0(t;r) = r-1,$$

$$\underline{y}_{k+1}(t;r) = -J^{\alpha}\int_0^t \underline{y}_k(s;r)ds,$$

for $k = 0,1,2,3,\dots$, we get

$$\underline{y}_1(t;r) = -J^{\alpha}\int_0^t \underline{y}_0(s;r)ds = -(r-1)\frac{t^{\alpha+1}}{\Gamma(\alpha+2)},$$

$$\underline{y}_2(t;r) = -J^{\alpha}\int_0^t \underline{y}_1(s;r)ds = (r-1)\frac{t^{2\alpha+2}}{\Gamma(2\alpha+3)},$$

$$\underline{y}_3(t;r) = -J^{\alpha}\int_0^t \underline{y}_2(s;r)ds = -(r-1)\frac{t^{3\alpha+3}}{\Gamma(3\alpha+4)}.$$

Similarly we can obtain the $\underline{y}_k(t;r)$ as follows

$$\underline{y}_k(t;r) = -J^{\alpha}\int_0^t \underline{y}_{k-1}(s;r)ds = (-1)^k(r-1)\frac{t^{k\alpha+k}}{\Gamma(k\alpha+k+1)}.$$

Hence

$$\underline{y}(t;r) = \sum_{n=0}^{\infty} \underline{y}_n(t;r) = (r-1)E_{\alpha+1}(-t^{\alpha+1}).$$

Similarly, we can obtain $\overline{y}(t;r)$ as follows

$$\overline{y}(t;r) = \sum_{n=0}^{\infty} \overline{y}_n(t;r) = (1-r)E_{\alpha+1}(-t^{\alpha+1}).$$

Figure 1 and Figure 2 shows the graphs of the solutions for different values of α and r.

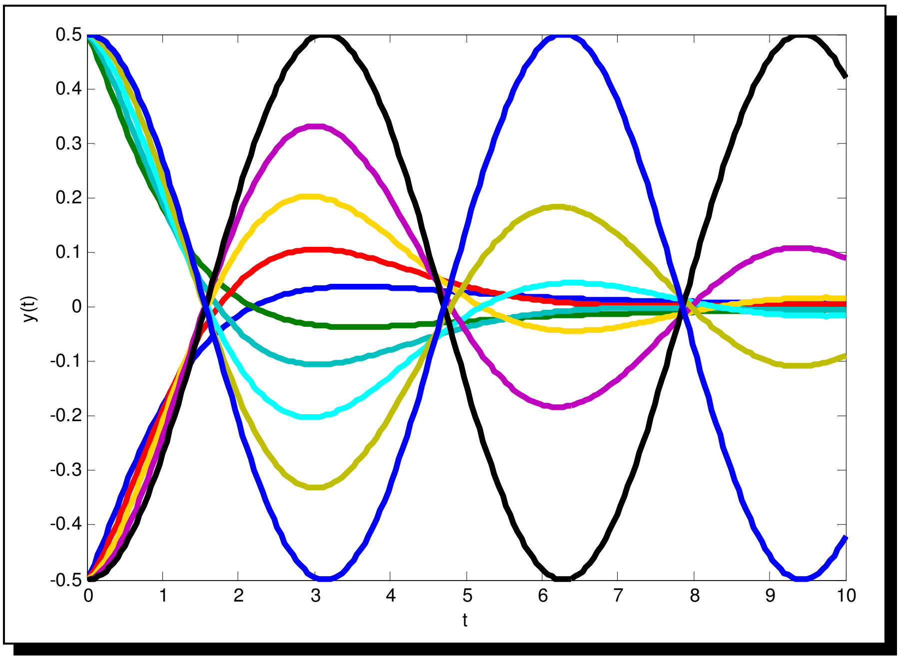

Figure 1. $\alpha = 0.2, 0.4, 0.6, 0.8, 1$, $r = 0.5$

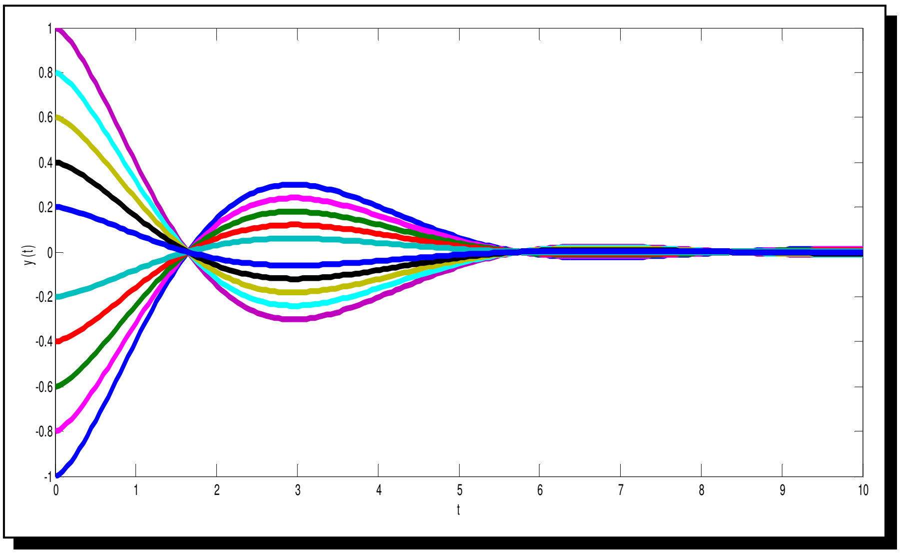

Figure 2. $r = 0, 0.2, 0.4, 0.6, 0.8$, $\alpha = 0.5$

Figure 3 shows comparison between integer order and a fractional order.

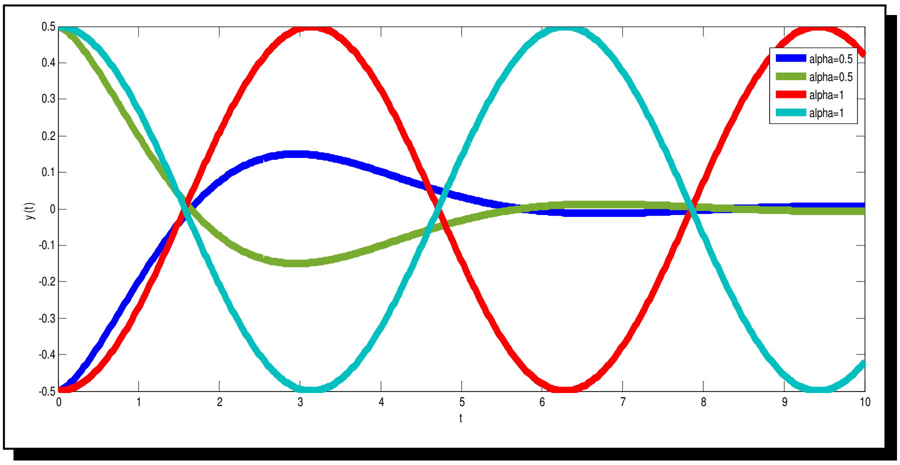

Figure 3. Comparison between integer order and fractional order

The above results almost match with the results obtained using Laplace transform method [10] for the equation (4.1).

5. Conclusion

In this paper, we applied the *Adomian Decomposition Method* (ADM) to obtain an analytical approximate solution for fuzzy fractional integro-differential equation and the results are compared with the exact solution obtained using Laplace transform. This method is so powerful and efficient, also one can apply this method for nonlinear case.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] T. Allahviranloo, S. Abbasbandy, O. Sedaghatfar and P. Darabi, A new method for solving fuzzy integro-differential equation under generalized differentiability, *Neural Computing and Applications* **21** (1967), 191–196.

[2] A. Armand and Z. Gouyandeh, Fuzzy fractional integro-differential equations under generalized Caputo differentiability, *Annals of Fuzzy Mathematics and Informatics* **10** (2015), 789–798.

[3] M. Caputo, Linear models of dissipation whose Q is almost frequency independent-II, *Geophysical Journal of the Royal Astronomical Society* **13** (1967), 529–539.

[4] K. Diethelm and N.J. Ford, *Analysis of Fractional Differential Equations*, Springer, New York (2004).

[5] T. Donchev, A. Nosheen and V. Lupulescu, Fuzzy integro-differential equations with compactness type conditions, *Hacettepe Journal of Mathematics and Statistics* **43** (2014), 259–267.

[6] A.F. Jameel, Semi analytical-solution of nonlinear two points fuzzy boundary value problems by adomian decomposition method, *Journal of Applied Computational Mathematics* **4** (2015) doi:10.4172/2168-9679.1000199.

[7] A. Loverro, *Fractional Calculus: History, Definitions and Applications for the Engineer*, U.S.A. (2004).

[8] R.C. Mittal and R. Nigam, Solution of fractional integro-differential equations by Adomian decomposition method, *International Journal of Application, Mathematics and Mechanics* **4** (2008), 87–94.

[9] I. Podlubny, *Fractional Differential Equation*, Academic Press, New York (1999).

[10] S. Priyadharsini, V. Parthiban and A. Manivannan, Solution of fractional integro-differential system with fuzzy initial condition, *International Journal of Pure and Applied Mathematics* **106** (2016), 107–112.