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Research Article

Local Closure Functions in Hereditary Generalized Topological Spaces

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Abstract. In this paper, we introduce and study the notions of local closure functions and also we introduce the operator Ψ_Γ in hereditary generalized topological spaces.

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1. Introduction

In 2002, Csaszar [1] introduced the notions of generalized topology and generalized continuity. A nonempty family $\mathscr{H}$ of subsets of X is said to be *hereditary class* [2], if $A \in \mathscr{H}$ and $B \subset A$, then $B \in \mathscr{H}$. Given a generalized topological space (X,μ) with a hereditary class $\mathscr{H}$, for each $A \subseteq X$, $A^*(\mathscr{H},\mu) = \{x \in X : A \cap V \notin \mathscr{H}$ for every $V \in \mu$ such that $x \in V\}$ [2]. If $c^*_\mu(A) = A \cup A^*(\mathscr{H},\mu)$ for every subset A of X, then $\mu^* = \{A \subset X : X - A = c^*_\mu(X - A)\}$ is a GT, μ^* is finer than μ ([2, Theorem 3.6]). In [3], $c_\theta(A) = \{x \in X : c_\mu(U) \cap A \neq \emptyset$ for every $U \in \mu\}$ and a set A is θ-closed if and only if $A = c_\theta(A)$ [3]. The generalized topological space (X,μ) is μ-regular [7] if and only if $\mu = \mu_\theta$. In this paper, $(X,\mu,\mathscr{H})$ denotes a hereditary generalized topological space, we define a operator $\Gamma(A)(\mathscr{H},\mu)$ called the local closure function of A with respect to $\mathscr{H}$ and μ as follows: $\Gamma(A)(\mathscr{H},\mu) = \{x \in X : A \cap c_\mu(U) \notin \mathscr{H}$ for every $U \in \mu(x)\}$. Moreover, by using $\Gamma(A)(\mathscr{H},\mu)$, we introduce a operator $\Psi_\Gamma : \wp(X) \to \mu$ satisfying $\Psi_\Gamma(A) = X - \Gamma(X-A)$ for each $A \in \wp(X)$. We set $\sigma = \{A \subseteq X : A \subseteq \Psi_\Gamma(A)\}$ and $\sigma_0 = \{A \subseteq X : A \subseteq i_\mu(c_\mu(\Psi_\Gamma(A)))\}$ and show that $\mu_\theta \subseteq \sigma \subseteq \sigma_0$.

In Section 2, we introduce and study the notion of local closure function in HGTS. In Section 3, we introduce and study the properties of Ψ_Γ-operator in HGTS.

2. Local Closure Function in HGTS

Definition 2.1. Let $(X,\mu,\mathscr{H})$ be a hereditary generalized topological space. For a subset A of X, we define the following operator: $\Gamma(A)(\mathscr{H},\mu) = \{x \in X : A \cap c_\mu(U) \notin \mathscr{H}\}$, for every $U \in \mu(x)$, where $\mu(x) = \{U \in \mu : x \in U\}$. In case, $\Gamma(A)(\mathscr{H},\mu)$ is briefly denoted by $\Gamma(A)$ and is called the local closure function of A with respect to $\mathscr{H}$ and μ.

Lemma 2.2. *Let $(X,\mu,\mathscr{H})$ be a hereditary generalized topological space. Then $A^*(\mathscr{H},\mu) \subseteq \Gamma(A)(\mathscr{H},\mu)$ for every subset A of X.*

Proof. Let $x \in A^*(\mathscr{H},\mu)$. Then, $A \cap U \notin \mathscr{H}$ for every μ-open set U containing x. Since, $A \cap U \subseteq A \cap c_\mu(U)$, we have $A \cap c_\mu(U) \notin \mathscr{H}$ and hence $x \in \Gamma(A)(\mathscr{H},\mu)$. □

Lemma 2.3. *Let (X,μ) be a generalized topological space and A be a subset of X. Then*

(a) *If A is μ-open, then $c_\mu(A) = c_\theta(A)$.*

(b) *If A is μ-closed, then $i_\mu(A) = i_\theta(A)$.*

Theorem 2.4. *Let (X,μ) be a generalized topological space, $\mathscr{H}$ and $\mathscr{J}$ be two hereditarys on X, and let A and B be subsets of X. Then the following properties hold:*

(i) *If $A \subseteq B$, then $\Gamma(A) \subseteq \Gamma(B)$.*

(ii) *If $\mathscr{H} \subseteq \mathscr{J}$, then $\Gamma(A)(\mathscr{H}) \supseteq \Gamma(A)(\mathscr{J})$.*

(iii) *$\Gamma(A) = c_\mu(\Gamma(A)) \subseteq c_\theta(A)$ and $\Gamma(A)$ is μ-closed.*

(iv) *If $A \subseteq \Gamma(A)$ and $\Gamma(A)$ is μ-open, then $\Gamma(A) = c_\theta(A)$.*

(v) *If $A \in \mathscr{I}$, then $\Gamma(A) = \emptyset$*

Proof. (i) Suppose that $x \notin \Gamma(B)$. Then there exists $U \in \mu(x)$ such that $B \cap c_\mu(U) \in \mathscr{H}$. Since $A \cap c_\mu(U) \subseteq B \cap c_\mu(U), A \cap c_\mu(U) \in \mathscr{H}$. Hence, $x \notin \Gamma(A)$. Thus $\Gamma(A) \subseteq \Gamma(B)$.

(ii) Suppose that $x \notin \Gamma(A)(\mathscr{H})$. Then there exists $U \in \mu(x)$ such that $A \cap c_\mu(U) \in \mathscr{H}$. Since $\mathscr{H} \subseteq \mathscr{J}$, $A \cap c_\mu(U) \in \mathscr{J}$ and $x \notin \Gamma(A)(\mathscr{J})$. Therefore, $\Gamma(A)(\mathscr{J}) \subseteq \Gamma(A)(\mathscr{H})$.

(iii) We have $\Gamma(A) \subseteq c_\mu(\Gamma(A))$ in general. Let $x \in c_\mu(\Gamma(A))$. Then $\Gamma(A) \cap U \neq \emptyset$ for every $U \in \mu(x)$. Therefore, there exists some $y \in \Gamma(A) \cap U$ and $U \in \mu(y)$. Since $y \in \Gamma(A), A \cap c_\mu(U) \notin \mathscr{H}$ and hence $x \in \Gamma(A)$. Hence we have $c_\mu(\Gamma(A)) \subseteq \Gamma(A)$ and hence $c_\mu(\Gamma(A)) = \Gamma(A)$. Again, let $x \in c_\mu(\Gamma(A)) = \Gamma(A)$, then $A \cap c_\mu(U) \notin \mathscr{H}$ for every $U \in \mu(x)$. This implies $A \cap c_\mu(U) \neq \emptyset$ for every $U \in \mu(x)$. Therefore, $x \in c_\theta(A)$. This shows that $\Gamma(A)(\mathscr{H}) = c_\mu(\Gamma(A)) \subseteq c_\theta(A)$.

(iv) For any subset A of X, by (3) we have $\Gamma(A) = c_\mu(\Gamma(A)) \subseteq c_\theta(A)$. Since $A \subseteq \Gamma(A)$ and $\Gamma(A)$ is μ-open, by Lemma 2.3, $c_\theta(A) \subseteq c_\theta(\Gamma(A)) = c_\mu(\Gamma(A)) = \Gamma(A) \subseteq c_\theta(A)$ and hence $\Gamma(A) = c_\theta(A)$.

(v) Suppose that $x \in \Gamma(A)$. Then for any $U \in \mu(x), A \cap c_\mu(U) \notin \mathscr{H}$. Since $A \in \mathscr{H}, A \cap c_\mu(U) \in \mathscr{H}$ for every $U \in \mu(x)$. This is a contradiction. Hence $\Gamma(A) = \emptyset$. □

Theorem 2.5. *Let $(X,\mu,\mathscr{H})$ be an hereditary generalized topological space. If $U \in \mu_\theta$, then $U \cap \Gamma(A) = U \cap \Gamma(U \cap A) \subseteq \Gamma(U \cap A)$ for any subset A of X.*

Proof. Suppose that $U \in \mu_\theta$ and $x \in U \cap \Gamma(A)$. Then $x \in U$ and $x \in \Gamma(A)$. Since $U \in \mu_\theta$, then there exists $W \in \mu$ such that $x \in W \subseteq c_\mu(W) \subseteq U$. Let V be any μ-open set containing x. Then $V \cap W \in \mu(x)$ and $c_\mu(V \cap W) \cap A \notin \mathscr{H}$ and hence $c_\mu(V) \cap (U \cap A) \notin \mathscr{H}$. This shows that $x \in \Gamma(U \cap A)$ and hence we obtain $U \cap \Gamma(A) \subseteq \Gamma(U \cap A)$. Moreover, $U \cap \Gamma(A) \supseteq U \cap \Gamma(U \cap A)$ and by Theorem 2.4 $\Gamma(U \cap A) \subseteq \Gamma(A)$ and $U \cap \Gamma(A) \subseteq U \cap \Gamma(U \cap A)$. Therefore, $U \cap \Gamma(A) = U \cap \Gamma(U \cap A)$. □

Theorem 2.6. *Let $(X,\mu,\mathscr{H})$ be an hereditary generalized topological space and A, B be any subsets of X. Then the following properties hold:*

(i) $\Gamma(\emptyset) = \emptyset$.

(ii) $\Gamma(A) \cup \Gamma(B) = \Gamma(A \cup B)$.

Theorem 2.7. *Let $(X,\mu,\mathscr{H})$ be an hereditary generalized topological space and A, B be any subsets of X. Then $\Gamma(A) - \Gamma(B) = \Gamma(A - B) - \Gamma(B)$.*

Proof. We have by Theorem 2.6 $\Gamma(A) = \Gamma[(A - B) \cup (A \cup B)] = \Gamma(A - B) \cup \Gamma(A \cap B) \subseteq \Gamma(A - B) \cup \Gamma(B)$. Thus $\Gamma(A) - \Gamma(B) \subseteq \Gamma(A - B) - \Gamma(B)$. By Theorem 2.4, $\Gamma(A - B) \subseteq \Gamma(A)$ and hence $\Gamma(A) - \Gamma(B) \supseteq \Gamma(A - B) - \Gamma(B)$. Hence, $\Gamma(A) - \Gamma(B) = \Gamma(A - B) - \Gamma(B)$. □

Corollary 2.8. *Let $(X,\mu,\mathscr{H})$ be an hereditary generalized topological space and A,B be any subsets of X with $B \in \mathscr{H}$. Then $\Gamma(A \cup B) = \Gamma(A) = \Gamma(A - B)$.*

Proof. Since $B \in \mathscr{H}$, by Theorem 2.4 $\Gamma(B) = \emptyset$. By Theorem 2.7, $\Gamma(A) = \Gamma(A - B)$ and by Theorem 2.6 $\Gamma(A) \cup \Gamma(B) = \Gamma(A \cup B) = \Gamma(A)$. □

3. Ψ_Γ-Operator in HGTS

Definition 3.1. Let $(X,\mu,\mathscr{H})$ be an hereditary generalized topological space. An operator $\Psi_\Gamma : \wp(X) \to \mu$ is defined as follows: for every $A \in X$, $\Psi_\Gamma(A) = \{x \in X :$ there exists $U \in \mu(x)$ such that $c_\mu(U) - A \in \mathscr{H}\}$ and observe that $\Psi_\Gamma(A) = X - \Gamma(X - A)$.

Theorem 3.2. *In hereditary generalized topological space $(X,\mu,\mathscr{H})$, the following holds:*

(i) *If $A \subseteq X$, then $\Psi_\Gamma(A)$ is μ-open.*

(ii) *If $A \subseteq B$, then $\Psi_\Gamma(A) \subseteq \Psi_\Gamma(A)$.*

(iii) *If $A,B \in \wp(X)$, then $\Psi_\Gamma(A \cap B) = \Psi_\Gamma(A) \cap \Psi_\Gamma(B)$.*

(iv) *If* $A \subseteq X$*, then* $\Psi_\Gamma(A) = \Psi_\Gamma(\Psi_\Gamma(A))$ *if and only if* $\Gamma(X - A) = \Gamma(\Gamma(X - A))$.

(v) *If* $A \in \mathscr{H}$*, then* $\Psi_\Gamma(A) = X - \Gamma(X)$.

(vi) *If* $A \subseteq X, H \in \mathscr{H}$*, then* $\Psi_\Gamma(A - H) = \Psi_\Gamma(A)$.

(vii) *If* $A \subseteq X, H \in \mathscr{H}$*, then* $\Psi_\Gamma(A \cup H) = \Psi_\Gamma(A)$.

(viii) *If* $(A - B) \cup (B - A) \in \mathscr{H}$*, then* $\Psi_\Gamma(A) = \Psi_\Gamma(B)$.

Proof. (i) This follows from Theorem 2.4(iii).

(ii) This follows from Theorem 2.4(i).

(iii)
$$\begin{aligned}\Psi_\Gamma(A \cap B) &= X - \Gamma(X - (A \cap B))\\ &= X - \Gamma[(X - A) \cup (X - B)]\\ &= X - [\Gamma(X - A) \cup \Gamma(X - B)]\\ &= [X - \Gamma(X - A)] \cap [X - \Gamma(X - B)]\\ &= \Psi_\Gamma(A) \cap \Psi_\Gamma(A).\end{aligned}$$

(iv) This follows from the facts:

(a) $\Psi_\Gamma(A) = X - \Gamma(X - A)$.

(b) $\Psi_\Gamma(\Psi_\Gamma(A)) = X - \Gamma[X - (X - \Gamma(X - A))] = X - \Gamma(\Gamma(X - A))$.

(v) By Corollary 2.8 we obtain that $\Gamma(X - A) = \Gamma(X)$ if $A \in \mathscr{H}$.

(vi) This follows from Corollary 2.8 and $\Psi_\Gamma(A - I) = X - \Gamma[X - (A - I)] = X - \Gamma[(X - A) \cup I] = X - \Gamma(X - A) = \Psi_\Gamma(A)$.

(vii) This follows from Corollary 2.8 and $\Psi_\Gamma(A \cup I) = X - \Gamma[X - (A \cup I)] = X - \Gamma[(X - A) - I] = X - \Gamma(X - A) = \Psi_\Gamma(A)$.

(viii) Assume $(A - B) \cup (B - A) \in \mathscr{H}$. Let $A - B = I$ and $B - A = J$. Observe that $I, J \in \mathscr{H}$. Also observe that $B = (A - I) \cup J$. Thus $\Psi_\Gamma(A) = \Psi_\Gamma(A - I) = \Psi_\Gamma[(A - I) \cup J] = \Psi_\Gamma(B)$ by (vi) and (vii). □

Corollary 3.3. *Let* $(X, \mu, \mathscr{H})$ *be an hereditary generalized topological space. Then* $U \subseteq \Psi_\Gamma(U)$ *for every* θ*-open set* $U \subseteq X$.

Proof. We know that $\Psi_\Gamma(U) = X - \Gamma(X - U)$. Now $\Gamma(X - U) \subseteq c_\theta(X - U) = X - U$, since $X - U$ is θ-closed. Therefore, $U = X - (X - U) \subseteq X - \Gamma(X - U) = \Psi_\Gamma(U)$.

Now we give an example of a set A which is not θ-open but satisfies $A \subseteq \Psi_\Gamma(A)$. □

Example 3.4. Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a, c\}, \{d\}, \{a, c, d\}\}$ and $\mathscr{H} = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$. Let $A = \{a\}$. Then $\Psi_\Gamma(\{a\}) = X - \Gamma(X - \{a\}) = X - \Gamma(\{b, c, d\}) = X - \{b, d\} = \{a, c\}$. Therefore. $A \subseteq \Psi_\Gamma(A)$, but A is not θ-open.

Theorem 3.5. *Let* $(X, \mu, \mathscr{H})$ *be an hereditary generalized topological space and* $A \subseteq X$*. Then the following properties holds:*

(i) $\Psi_\Gamma(A) = \cup\{U \in \mu : c_\mu(U) - A \in \mathscr{H}\}$.

(ii) $\Psi_\Gamma(A) \supseteq \cup\{U \in \mu : (c_\mu(U) - A) \cup (A - c_\mu(U)) \in \mathscr{H}\}$

Proof. (i) This follows immediately from the definition of Ψ_Γ-operator.

(ii) Since $\mathscr{H}$ is hereditary, it is obvious that $\cup\{U \in \mu : (c_\mu(U) - A) \cup (A - c_\mu(U)) \in \mathscr{H}\} \subseteq \cup\{U \in \mu : c_\mu(U) - A \in \mathscr{H}\} = \Psi_\Gamma(A)$ for every $A \subseteq X$. □

Theorem 3.6. *Let $(X, \mu, \mathscr{H})$ be an hereditary generalized topological space. If $\sigma = \{A \subseteq X : A \subseteq \Psi_\Gamma(A)\}$. Then σ is a generalized topology for X.*

Proof. Let $\sigma = \{A \subseteq X : A \subseteq \Psi_\Gamma(A)\}$. Since $\emptyset \in \mathscr{H}$, by Theorem 2.4(v) $\Gamma(\emptyset) = \emptyset$ and $\Psi_\Gamma(X) = X - \Gamma(X - X) = X - \Gamma(\emptyset) = X$. Moreover, $\Psi_\Gamma(\emptyset) = X - \Gamma(X - \emptyset) = X - X = \emptyset$. Therefore we obtain that $\emptyset \subseteq \Psi_\Gamma(\emptyset)$ and $X \subseteq \Psi_\Gamma(X) = X$ and thus $\emptyset$ and $X \in \sigma$. Now if $A, B \in \sigma$, then by Theorem 3.2 $A \cap B \subseteq \Psi_\Gamma(A) \cap \Psi_\Gamma(B) = \Psi_\Gamma(A \cap B)$ which implies that $A \cap B \in \sigma$. If $\{A_\alpha : \alpha \in \Delta\} \subseteq \sigma$, then $A_\alpha \subseteq \Psi_\Gamma(A_\alpha) \subseteq \Psi_\Gamma(\cup A_\alpha)$ for every α and hence $\cup A_\alpha \subseteq \Psi_\Gamma(\cup A_\alpha)$. This shows that σ is a generalized topology. □

Lemma 3.7. *If either $A \in \mu$ or $B \in \mu$, then $i_\mu(c_\mu(A \cap B)) = i_\mu\left(c_\mu(A)\right) \cap i_\mu(c_\mu(B))$.*

Theorem 3.8. *Let $\sigma_0 = \{A \subseteq X : A \subseteq i_\mu(c_\mu(\Psi_\Gamma(A)))\}$, then σ_0 is a generalized topology for X.*

Proof. By Theorem 3.2, for any subset A of $X, \Psi_\Gamma(A)$ is μ-open and $\sigma \subset \sigma_0$. Therefore, $\emptyset, X \in \sigma_0$. Let $A, B \in \sigma_0$. Then by Lemma 3.7 and Theorem 3.2, we have $A \cap B \subset i_\mu(c_\mu(\Psi_\Gamma(A))) \cap i_\mu(c_\mu(\Psi_\Gamma(B))) = i_\mu(c_\mu(\Psi_\Gamma(A) \cap \Psi_\Gamma(B))) = i_\mu(c_\mu(\Psi_\Gamma(A \cap B)))$. Therefore, $A \cap B \in \sigma_0$. Let $A_\alpha \in \sigma_0$ for each $\alpha \in \Delta$. By Theorem 3.2, for each $\alpha \in \Delta, A_\alpha \subseteq i_\mu(c_\mu(\Psi_\Gamma(A_\alpha))) \subseteq i_\mu(c_\mu(\Psi_\Gamma(\cup A_\alpha)))$ and hence $\cup A_\alpha \subset i_\mu(c_\mu(\Psi_\Gamma(\cup A_\alpha)))$. Hence $\cup A_\alpha \in \sigma_0$. This shows that σ_0 is a generalized topology for X. □

Remark 3.9. (i) In Example 3.4, A is σ-open but it is not μ-open. Therefore every σ_0-open set is not μ-open.

(ii) Let $X = \{a, b, c\}$ with $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\mathscr{H} = \{\emptyset, \{a\}\}$ be an hereditary on X. We observe that $\{a\}$ is μ-open but it is not σ_0-open. Since $\Psi_\Gamma(\{a\}) = X - \Gamma(\{b, c\}) = X - X = \emptyset$. Also, $\{c\}$ is not μ-open but it is σ-open set, since $\Psi_\Gamma(\{c\}) = X - \Gamma(\{a, b\}) = X - \{b\} = \{a, c\}$.

Definition 3.10. Let $(X, \mu, \mathscr{H})$ be an hereditary generalized topological space. We say μ is closure compatible with hereditary $\mathscr{H}$, denoted $\mu \sim_\Gamma \mathscr{H}$, if the following holds for every $A \subseteq X$, if for every $x \in A$ there exists $U \in \mu(x)$ such that $c_\mu(U) \cap A \in \mathscr{H}$, then $A \in \mathscr{H}$.

Theorem 3.11. *Let $(X, \mu, \mathscr{H})$ be an hereditary generalized topological space. Then $\mu \sim_\Gamma \mathscr{H}$ if and only if $\Psi_\Gamma(A) - A \in \mathscr{H}$ for every $A \subseteq X$.*

Proof. *Necessity:* Assume $\mu \sim_\Gamma \mathscr{H}$ and let $A \subseteq X$. Observe that $x \in \Psi_\Gamma(A) - A$ if and only if $x \notin A$ and $x \notin \Gamma(X - A)$ if and only if $x \notin A$ and there exists $U_x \in \mu(x)$ such that $c_\mu(U_x) - A \in \mathscr{H}$ if and only if there exists $U_x \in \mu(x)$ such that $x \in c_\mu(U_x) - A \in \mathscr{H}$. Now, for each $x \in \Psi_\Gamma(A) - A$ and

$U_x \in \mu(x), c_\mu(U_x) \cap (\Psi_\Gamma(A) - A) \in \mathscr{H}$ by hereditary and hence $\Psi_\Gamma(A) - A \in \mathscr{H}$ by assumption that $\mu \sim_\Gamma \mathscr{H}$.

Sufficiency: Let $A \subseteq X$ and assume that for each $x \in A$ there exists $U_x \in \mu(x)$ such that $c_\mu(U_x) \cap A \in \mathscr{H}$. Observe that $\Psi_\Gamma(X - A) - (X - A) = A - \Gamma(A) = \{x :$ there exists $U_x \in \mu(x)$ such that $x \in c_\mu(U_x) \cap A \in \mathscr{H}\}$. Thus we have $A \subseteq \Psi_\Gamma(X - A) - (X - A) \in \mathscr{H}$ and hence $A \in \mathscr{H}$ by hereditary of $\mathscr{H}$. □

Theorem 3.12. *Let $(X, \mu, \mathscr{H})$ be an hereditary generalized topological space with $\mu \sim_\Gamma \mathscr{H}$, $A \subseteq X$. If N is a nonempty μ-open subset of $\Gamma(A) \cap \Psi_\Gamma(A)$, then $N - A \in \mathscr{H}$ and $c_\mu(N) \cap A \notin \mathscr{H}$.*

Proof. If $N \subseteq \Gamma(A) \cap \Psi_\Gamma(A)$, then $N - A \subseteq \Psi_\Gamma(A) - A \in \mathscr{H}$, by Theorem 3.11 and hence $N - A \in \mathscr{H}$ by hereditary. Since $N \in \mu - \{\emptyset\}$ and $N \subseteq \Gamma(A)$, we have $c_\mu(N) \cap A \notin \mathscr{H}$ by the definition of $\Gamma(A)$. □

Theorem 3.13. *Let $(X, \mu, \mathscr{H})$ be an hereditary generalized topological space with $\mu \sim_\Gamma \mathscr{H}$, where $c_\mu(\mu) \cap \mathscr{H} = \emptyset$. Then for $A \subseteq X, \Psi_\Gamma(A) \subseteq \Gamma(A)$.*

Proof. Suppose $x \in \Psi_\Gamma(A)$ and $x \notin \Gamma(A)$. Then there exists a nonempty neighborhood $U_x \in \mu(x)$ such that $c_\mu(U_x) \cap A \in \mathscr{H}$. Since $x \in \Psi_\Gamma(A)$, by Theorem 3.5 $\in \cup \{U \in \mu : c_\mu(U) - A \in \mathscr{H}\}$ and there exists $V \in \mu(x)$ and $c_\mu(V) - A \in \mathscr{H}$. Now we have $U_x \cap V \in \mu(x), c_\mu(U_x \cap V) \cap A \in \mathscr{H}$ and $c_\mu(U_x \cap V) - A \in \mathscr{H}$ by hereditary. Hence by finite additivity we have $c_\mu((U_x \cap V) \cap A) \cup \left(c_\mu(U_x \cap V) - A\right) = c_\mu(U_x \cap V) \in \mathscr{H}$. Since $(U_x \cap V) \in \mu(x)$, this is contrary to $c_\mu(\mu) \cap \mathscr{H} = \emptyset$. Therefore, $x \in \Gamma(A)$. This implies that $\Psi_\Gamma(A) \subseteq \Gamma(A)$. □

4. Conclusion

Local closure function play vital role in topological spaces. Many authors contributed ([4], [5], [6]) in the field to improve the results in topological spaces. We have defined and discussed the properties of local closure function in hereditary generalized topological spaces.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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