Journal of Informatics and Mathematical Sciences
Vol. 10, Nos. 1 & 2, pp. 247–259, 2018
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications
http://dx.doi.org/10.26713/jims.v10i1-2.679

http://www.rgnpublications.com

Research Article

A Dual Method to Study Motion of A Robot End-Effector

Burak Şahiner[1,*], Mustafa Kazaz[1] and Hasan Hüseyin Uğurlu[2]

[1] *Department of Mathematics, Manisa Celal Bayar University, Manisa, Turkey*
[2] *Department of Mathematics Teaching, Gazi University, Ankara, Turkey*
***Corresponding author:** burak.sahiner@cbu.edu.tr

Abstract. This paper presents a dual method to study motion of a robot end-effector by using the curvature theory of a dual curve which corresponds to a ruled surface generated by a line fixed in the end-effector. By using dual method, translational and angular differential properties of motion such as velocity and acceleration are determined without redundant parameters. These properties are important information in robot trajectory planning. As a practical example, motion of a robot end-effector in which a line fixed in the end-effector generates a surface of hyperbolic paraboloid is investigated.

Keywords. Curvature theory; Dual Darboux frame; Dual tool frame; Robot end-effector; Robot trajectory planning; Ruled surface

MSC. 53A05; 53A17; 53A25

Received: September 7, 2017 **Accepted:** March 31, 2018

1. Introduction

Recently, since robot end-effectors have a wide range of usage from surgical operations to bomb disposal, accurate robot trajectory planning becomes an important research area of robotics. Describing path of a robot end-effector, representing orientation and position, and determining linear and angular differential properties are some important problems in this area. Traditional methods such as (4×4) homogeneous transformation, Quaternions and Euler angle representation in [3, 10, 11] are based on matrix representation that is required intense computation. Moreover, these methods have not been adequate for a smooth and differentiable trajectory which robot end-effector follows. Ryuh and Pennock introduced a method based on the curvature theory of a ruled surface generated by a line fixed in the end-effector [12, 13, 14]. They represented motion of a robot end-effector by using the ruled surface and an additional parameter called spin angle. Thus, they found an efficient relationship between the kinematics of a robot end-effector and the differential geometry of a ruled surface. Their papers were first attempts to use the curvature theory which investigates the intrinsic geometric properties of trajectories for robot trajectory planning.

The research area of motion of a robot end-effector is also interesting for the authors who study in Lorentzian space. For example, Ekici *et al*. [5] study motion of a robot end-effector in Lorentzian space by using the curvature theory of timelike ruled surface with timelike ruling.

On the other hand, by the aid of the E. Study mapping or transference principle which can be stated as: "there exists one-to-one correspondence between the directed lines in line space and dual unit vectors in dual space" [9, 16], a relationship can also be found between a ruled surface in real space and a dual spherical curve in dual space. Using this correspondence, several authors have applied dual quantities to their research concerning kinematics, analysis and synthesis of spatial mechanisms and many other areas.

In this paper, we use the relationship between dual space and kinematics. In this way, we study differential properties of motion of a robot end-effector by using the curvature theory of dual spherical curves. First, we discuss three reference frames related to a robot end-effector and a ruled surface generated by a line fixed in the end-effector in real space. From E. Study mapping, a dual curve corresponds to the ruled surface. Then, the dual Darboux frame of the dual curve is given briefly. Afterwards, we define a dual frame called dual tool frame on the robot end-effector. By relating between dual Darboux frame and dual tool frame and by using the dual instantaneous rotation vector of the dual tool frame, the translational and angular differential properties of motion of a robot end-effector such as velocity and acceleration which are important information in robot trajectory planning are determined.

The paper is organized as follows: In Section 2, we give some basic concepts about dual space. In Section 3, we mention mathematical background of the motion of a robot end-effector in real space. In Section 4, we propose a dual method to examine the motion of robot end-effector. In Section 5, an example is given. Finally, the conclusions of this paper are presented in Section 6.

2. Preliminaries

In this section, dual space and its basic concepts will be given briefly.

A dual number, as introduced by W. Clifford, can be defined as an ordered pair $\bar{a} = (a, a^*)$ of real numbers, where a and a^* are called real part and dual part of the dual number, respectively. Dual numbers can also be expressed as $\bar{a} = a + \varepsilon a^*$, where $\varepsilon^2 = 0$ and $\varepsilon = (0,1)$ is called dual unit [17]. The set of all dual numbers is denoted by $\mathbb{D}$. Two inner operations and equality in $\mathbb{D}$ are defined as follows [1, 7]:

Addition: $(a, a^*) + (b, b^*) = (a + b, a^* + b^*)$,

Multiplication: $(a, a^*)(b, b^*) = (ab, ab^* + a^* b)$, Equality: $(a, a^*) = (b, b^*) \Leftrightarrow a = b, a^* = b^*$.

The set $\mathbb{D}$ with the above operations is a commutative ring, not a field. The function of a dual number $f(\bar{a})$ can be expanded in a Maclaurin series as

$$f(\bar{a}) = f(a + \varepsilon a^*) = f(a) + \varepsilon a^* f'(a),$$

where $f'(a)$ is derivative of $f(a)$ with respect to a [2].

A dual vector can also be defined as an ordered pair (a, a^*) of two real vectors and can be expressed as $\widetilde{a} = a + \varepsilon a^*$, where $a, a^* \in \mathbb{R}^3$ and $\varepsilon^2 = 0$. The set of all dual vectors is a module over the ring $\mathbb{D}$ and is called dual space or $\mathbb{D}$-module, denoted by $\mathbb{D}^3$ [15]. Let $\widetilde{a} = a + \varepsilon a^*$ and $\widetilde{b} = b + \varepsilon b^*$ be two dual vectors in $\mathbb{D}^3$. Then the dual inner product and the dual vector product can be defined as

$$\langle \widetilde{a}, \widetilde{b} \rangle = \langle a, b \rangle + \varepsilon(\langle a, b^* \rangle + \langle a^*, b \rangle)$$

and

$$\widetilde{a} \times \widetilde{b} = a \times b + \varepsilon(a \times b^* + a^* \times b),$$

respectively [17]. The norm of a dual vector $\widetilde{a}$ can be given by [7, 17]

$$\|\widetilde{a}\| = \|a\| + \varepsilon \frac{\langle a, a^* \rangle}{\|a\|}, \quad (a \neq 0).$$

A dual vector $\widetilde{a}$ is called dual unit vector if and only if $\|\widetilde{a}\| = 1$. An oriented straight line in three dimensional Euclidean space $\mathbb{R}^3$ can be represented by a dual unit vector in the dual space $\mathbb{D}^3$ as

$$\widetilde{a} = a + \varepsilon a^* \quad (\langle a, a \rangle = 1, \langle a, a^* \rangle = 0)$$

where a is a unit vector along the straight line, a^* is the moment of a about the origin and ε is the dual unit with the property $\varepsilon^2 = 0$. The set of all directed straight lines in $\mathbb{R}^3$ are in one-to-one correspondence with the set of all points of dual unit sphere $\langle \widetilde{a}, \widetilde{a} \rangle = 1$ in $\mathbb{D}^3$ [9, 16]. This correspondence is known as E. Study mapping or transference principle.

A dual angle between two oriented lines in real space can be defined as $\bar{\theta} = \theta + \varepsilon \theta^*$, where θ and θ^* are the real angle and the shortest distance between these lines, respectively [1].

3. Mathematical Background of Motion of A Robot End-Effector in Real Space

In this section, we mention three reference frames described by Ryuh and Pennock [13] in detail. These frames are used in the study of motion of a robot end-effector in three dimensional real space $\mathbb{R}^3$. These frames are: a tool frame, a surface frame and a generator trihedron. We need to emphasize that the generator trihedron has been defined as the Frenet frame of the directing cone of a ruled surface by Karger and Novak [8] before the papers [11, 12, 13]. The goal of this section is to better understand the geometrical meaning of the dual procedure to be used in Section 4.

The tool frame consists of three orthonormal vectors which are the orientation vector O, the approach vector A, and the normal vector N (see Figure 1). The origin of the tool frame is called tool center point and denoted by TCP [12].

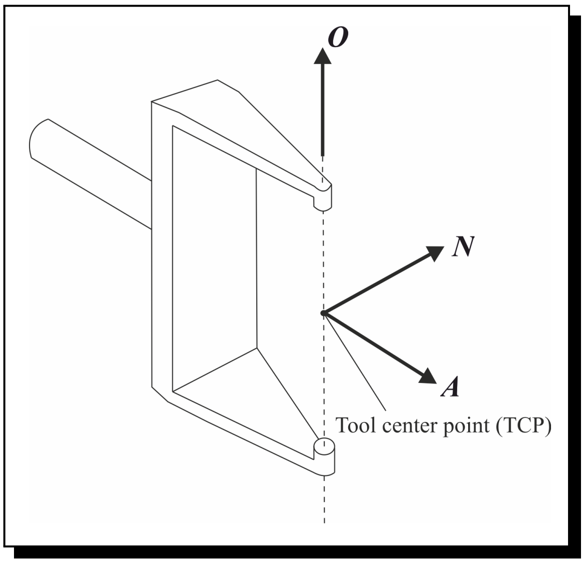

Figure 1. A robot end-effector and its tool frame.

As a robot end-effector moves on a specified trajectory, a line fixed in the end-effector called tool line which passes through TCP and whose direction vector is parallel to the orientation vector O generates a ruled surface [12]. This ruled surface can be expressed as

$$X(t,v) = \alpha(t) + vR(t),$$

where $\alpha(t)$ is the directrix of the ruled surface and is also the specified trajectory of the robot end-effector, $R(t)$ is a vector of constant magnitude called ruling parallel to the orientation vector O, and t is the parameter of time. In order to simplify the formulations, we use a normalized parameter s which is the arc-length parameter of the spherical image curve of R instead of the time parameter t and it can be found as [8]

$$s(t) = \int_0^t \left\| \frac{dR}{dt} \right\| dt.$$

During motion of the robot end-effector, the approach vector A may not be always perpendicular to the ruled surface. There may be an angle between the approach vector A and the surface normal vector which may be denoted by S_n (see Figure 2). This angle is called spin angle and denoted by η [13].

The surface frame consists of three orthonormal vectors: the orientation vector O, the surface normal vector S_n, and the surface binormal vector S_b [12]. The surface frame can be used to describe the orientation of the tool frame relative to the ruled surface. The surface normal vector S_n can be given as

$$S_n = \left. \frac{X_v \times X_s}{\|X_v \times X_s\|} \right|_{v=0},$$

where X_v and X_s are derivatives of X with respect to v and s, respectively. The surface binormal vector which is perpendicular to both the orientation vector and the surface normal vector, can also be given as

$$S_b = O \times S_n.$$

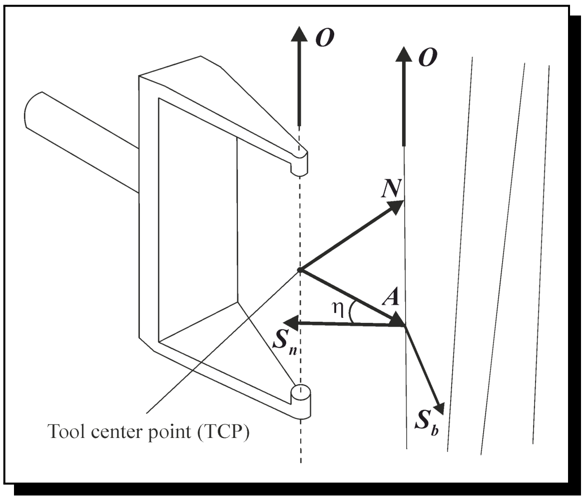

Figure 2. The spin angle which is the angle between approach vector and the surface normal vector.

The line of striction of the ruled surface can be denoted by c such that $\langle c', R' \rangle = 0$ and it can be given as $c(s) = \alpha(s) - \mu(s)R(s)$, where $\mu = \langle \alpha', R' \rangle$ [4]. The distance from the line of striction to the directrix along the ruling is μR, where R denotes the magnitude of the ruling, i.e., $R = \|R\|$.

The generator trihedron (can also be known as Frenet frame of directing cone of the ruled surface) defined on the line of striction of the ruled surface consists of three orthonormal vectors: the generator vector e which is the same as the orientation vector O, the central normal vector t, and the central tangent vector g. These vectors can be given as, respectively,

$$e = \frac{R}{R}, \quad t = R', \quad g = e \times t,$$

where the prime indicates differentiation with respect to s [8].

The relation between the tool frame and the generator trihedron can be obtained as [12]

$$\begin{bmatrix} O \\ A \\ N \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} e \\ t \\ g \end{bmatrix},$$

where $\varphi = \eta + \sigma$, such that σ is the angle between S_n and t. Figure 3 shows the relationships between the tool frame, the surface frame and the generator trihedron.

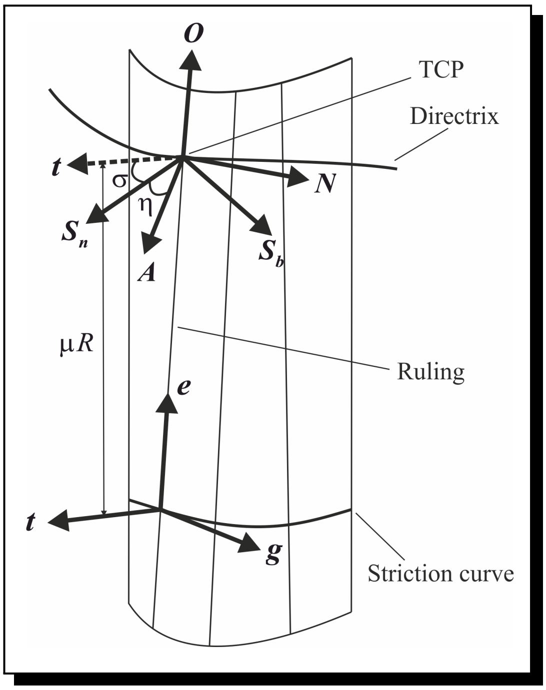

Figure 3. Relations between three reference frames as a robot end-effector moves on a specified trajectory.

4. Dual Method

From E. Study mapping (or transference principle), a dual curve lying fully on the dual unit sphere corresponds to a ruled surface in real space. In this section, we examine the study of motion of a robot end-effector by using the curvature theory of a dual curve corresponds to the ruled surface generated by a line fixed in the robot end-effector. For this purpose, we first give the dual Darboux frame of the corresponding dual curve which is well-known and used in the dual curvature theory of dual curves. Then, we define a dual frame called dual tool frame on the robot end-effector. Finally, we find the dual instantaneous rotation vector of the dual tool frame in terms of the elements of dual Darboux frame. This dual vector can be considered as the dual velocity vector of robot end-effector and it plays leading role to determine both translational and angular differential properties of motion of a robot end-effector.

Let the ruled surface generated by a line fixed in the robot end-effector be given by the equation

$$X(s,v) = \alpha(s) + vR(s),$$

where $\alpha(s)$ is trajectory of the robot end-effector, $R(s)$ is the ruling of the ruled surface which is a vector of constant magnitude and s is the normalized parameter discussed in Section 3. Let a dual curve which corresponds to the ruled surface be represented by $\widetilde{e}(s) = e(s) + \varepsilon e^*(s)$, where $e(s)$ is the generator vector of the ruled surface as defined in Section 3 and e^* is the moment vector of e about the origin which can be obtained as $e^*(s) = c(s) \times e(s)$, where $c(s)$ is the line of striction of the ruled surface [17]. From now on, we consider the case without $e(s) = \text{constant}$ which means the ruled surface is a cylinder and $e^*(s) = 0$ which means the ruled surface is a cone.

The dual Darboux frame (or dual geodesic frame) of a dual curve which was described by Veldkamp [17] in detail consists of three orthonormal dual unit vectors. The first dual unit vector is the dual curve itself, i.e., $\widetilde{e}(s)$. The dual arc-length of the dual curve $\widetilde{e}$ can be given by [17]

$$\bar{s} = \int_0^s \|\widetilde{e}'(u)\| du = \int_0^s (1+\varepsilon\Delta) du = s + \varepsilon \int_0^s \Delta du,$$

where $\Delta = \langle c' \times e, t \rangle$. The second dual unit vector is the dual tangent vector of the dual curve $\widetilde{e}(s)$, and it can be given by [17]

$$\widetilde{t} = \frac{d\widetilde{e}}{d\bar{s}} = \frac{\widetilde{e}'}{\bar{s}'} = \frac{\widetilde{e}'}{1+\varepsilon\Delta} = t + \varepsilon(c \times t),$$

where t is the central normal vector of the ruled surface. The third dual unit vector can also be given by [17]

$$\widetilde{g} = \widetilde{e} \times \widetilde{t} = g + \varepsilon c \times g,$$

where $g = e \times t$ is the central tangent vector of the ruled surface. The derivation formulae of the dual Darboux frame can be expressed in matrix form as

$$\frac{d}{d\bar{s}} \begin{bmatrix} \widetilde{e} \\ \widetilde{t} \\ \widetilde{g} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & \bar{\gamma} \\ 0 & -\bar{\gamma} & 0 \end{bmatrix} \begin{bmatrix} \widetilde{e} \\ \widetilde{t} \\ \widetilde{g} \end{bmatrix}, \tag{4.1}$$

where $\bar{\gamma}$ is called dual geodesic curvature [17].

Now, we define a dual frame called dual tool frame which will be used to determine differential properties of motion of a robot end-effector by using relationships with dual Darboux frame. The dual tool frame can be defined by three orthonormal dual unit vectors which correspond to three oriented straight lines, the orientation line, the approach line and the normal line, all pass through the tool center point of the robot end-effector, and their direction vectors are the orientation vector O, the approach vector A and the normal vector N, respectively (see Figure 4). These dual unit vectors can be denoted by $\widetilde{O}$, $\widetilde{A}$ and $\widetilde{N}$, and may be called dual orientation vector, dual approach vector and dual normal vector, respectively.

The dual tool frame and the dual Darboux frame have a common dual vector $\widetilde{O}$ (or $\widetilde{e}$) which corresponds to the ruling of the ruled surface. Let $\bar{\varphi} = \varphi + \varepsilon\varphi^*$ be a dual angle between the dual unit vectors $\widetilde{A}$ and $\widetilde{t}$, where φ is the real angle and φ^* is the shortest distance between the lines which correspond to the dual unit vectors $\widetilde{A}$ and $\widetilde{t}$ (see Figure 5). Also from Section 3, we know that φ is the real angle between the approach vector A and the central normal vector t, and φ^* is the shortest distance from the line of striction to the directrix, i.e., μR.

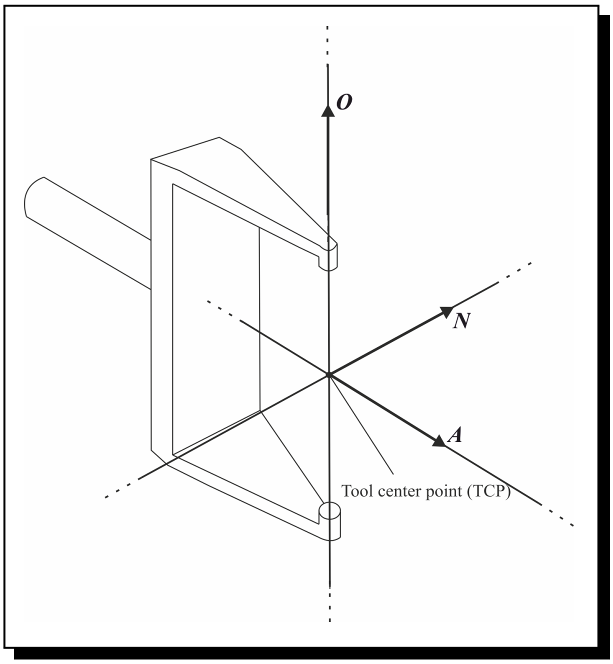

Figure 4. Dual tool frame of a robot end-effector.

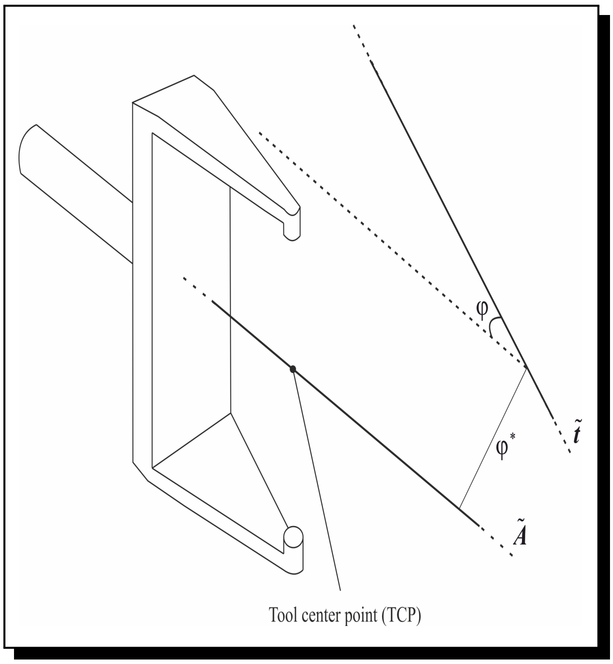

Figure 5. The dual angle between the dual unit vectors $\widetilde{A}$ and $\widetilde{t}$.

Thus, the relations between the dual Darboux frame and dual tool frame can be given in matrix form as

$$\begin{bmatrix} \widetilde{O} \\ \widetilde{A} \\ \widetilde{N} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\bar{\varphi} & \sin\bar{\varphi} \\ 0 & -\sin\bar{\varphi} & \cos\bar{\varphi} \end{bmatrix} \begin{bmatrix} \widetilde{e} \\ \widetilde{t} \\ \widetilde{g} \end{bmatrix}. \tag{4.2}$$

By differentiating equation (4.2) and substituting equation (4.1) into the result, we have

$$\begin{bmatrix}\widetilde{O}'\\ \widetilde{A}'\\ \widetilde{N}'\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ -\cos\bar{\varphi} & -\bar{\delta}\sin\bar{\varphi} & \bar{\delta}\cos\bar{\varphi}\\ \sin\bar{\varphi} & -\bar{\delta}\cos\bar{\varphi} & -\bar{\delta}\sin\bar{\varphi}\end{bmatrix}\begin{bmatrix}\widetilde{e}\\ \widetilde{t}\\ \widetilde{g}\end{bmatrix},$$

where $\bar{\delta}=\bar{\varphi}'+\bar{\gamma}$ and the prime denotes derivation with respect to the dual arc-length parameter $\bar{s}$. By using equation (4.2), the derivation formulas of the dual tool frame can be obtained by itself in matrix form as

$$\begin{bmatrix}\widetilde{O}'\\ \widetilde{A}'\\ \widetilde{N}'\end{bmatrix}=\begin{bmatrix}0 & \cos\bar{\varphi} & -\sin\bar{\varphi}\\ -\cos\bar{\varphi} & 0 & \bar{\delta}\\ \sin\bar{\varphi} & -\bar{\delta} & 0\end{bmatrix}\begin{bmatrix}\widetilde{O}\\ \widetilde{A}\\ \widetilde{N}\end{bmatrix}.$$

The dual instantaneous rotation vector of the dual tool frame which plays an important role to determine the differential properties of motion of a robot end-effector can be obtained as

$$\widetilde{w}_O=\bar{\delta}\widetilde{O}+\sin\bar{\varphi}\widetilde{A}+\cos\bar{\varphi}\widetilde{N}.$$

Note that, the following equalities hold for the dual vector $\widetilde{w}_O$,

$$\widetilde{O}'=\widetilde{w}_O\times\widetilde{O},\quad \widetilde{A}'=\widetilde{w}_O\times\widetilde{A},\quad \widetilde{N}'=\widetilde{w}_O\times\widetilde{N},$$

where the prime indicates the differentiation with respect to the dual arc-length parameter $\bar{s}$. By using equation (4.2), the dual instantaneous rotation vector of the dual tool frame can be expressed in terms of the dual Darboux frame as

$$\widetilde{w}_O=\bar{\delta}\widetilde{e}+\widetilde{g}. \tag{4.3}$$

The dual vector $\widetilde{w}_O=w_O+\varepsilon w_O^*$ plays the same role with dual Pfaff vector which we encounter in dual spherical motion [6], so this dual vector can be considered as dual velocity vector of the robot end-effector. The dual tool frame attached to the robot end-effector moves along the unit direction $\frac{\widetilde{w}_O}{\|\widetilde{w}_O\|}$ with the dual angle $\|\widetilde{w}_O\|$. This dual motion contains both rotational and translational motion in real space. The real and dual parts of the dual instantaneous rotation vector, w_O and w_O^*, correspond to the instantaneous angular velocity vector and the instantaneous translational velocity vector, respectively. By separating equation (4.3) into the real and dual parts, these vectors can be obtained as

$$w_O=\delta e+g \tag{4.4}$$

and

$$w_O^*=\delta e^*+\delta^* e+g^* \tag{4.5}$$

respectively. By differentiating equation (4.3) and using equation (4.1), the derivative of the dual instantaneous rotation vector of the dual tool frame which may be called dual acceleration vector can be obtained as

$$\widetilde{w}'_O=\bar{\delta}'\widetilde{e}+\bar{\varphi}'\widetilde{t}. \tag{4.6}$$

By separating equation (4.6) into the real and dual parts, the instantaneous angular acceleration vector and the instantaneous translational acceleration vector can be found as

$$w'_O=\delta' e+\varphi' t \tag{4.7}$$

and

$$w_O^{*\prime} = \delta' e^* + \delta^{*\prime} e + \varphi' t^* + \varphi^{*\prime} t. \tag{4.8}$$

The vectors given in (4.4), (4.5), (4.7) and (4.8) are obtained in terms of s which is the arc-length parameter of spherical image curve of R. These vectors should be associated with t which is the parameter of time to determine the time dependent differential properties of motion of a robot end-effector. We can give the following corollaries concerning the translational and angular time dependent differential properties of motion of a robot end-effector.

Corollary 4.1. *Let a motion of a robot end-effector be represented by a ruled surface $X(t,v) = \alpha(t) + vR(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector and R is the ruling of the ruled surface parallel to the orientation vector O. Then the angular and translational velocities of the robot end-effector can be given, respectively, as*

$$v_A = w_O \dot{s} \tag{4.9}$$

and

$$v_T = w_O^* \dot{s} \tag{4.10}$$

where w_O and w_O^* are given by equations (4.4) *and* (4.5)*, respectively, and the dot indicates differentiation with respect to time, i.e., $\dot{s} = \frac{ds}{dt}$.*

Corollary 4.2. *Let a motion of a robot end-effector be represented by a ruled surface $X(t,v) = \alpha(t) + vR(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector and R is the ruling of the ruled surface parallel to the orientation vector O. Then the angular and translational accelerations of the robot end-effector can be given, respectively, as*

$$a_A = w_O \ddot{s} + w_O' \dot{s}^2 \tag{4.11}$$

and

$$a_T = w_O^* \ddot{s} + w_O^{*\prime} \dot{s}^2 \tag{4.12}$$

where w_O' and $w_O^{*\prime}$ are given by equations (4.7) *and* (4.8)*, respectively.*

If a robot end-effector moves on a specified trajectory such that it is always perpendicular to the ruled surface, then the spin angle η is equal to zero. More general, the spin angle η may be constant during the motion. For this case, we can give the following corollaries.

Corollary 4.3. *Let a motion of a robot end-effector be represented by a ruled surface $X(t,v) = \alpha(t) + vR(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector and R is the ruling of the ruled surface parallel to the orientation vector O. If the spin angle η is constant, then the angular and translational velocities of the robot end-effector can be expressed as*

$$v_A = ((\sigma' + \gamma)e + g)\dot{s}$$

and

$$v_T = ((\sigma' + \gamma)e^* + \delta^* e + g^*)\dot{s}$$

respectively.

Corollary 4.4. *Let a motion of a robot end-effector be represented by a ruled surface $X(t,v) = \alpha(t)+vR(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector and R is the ruling of the ruled surface parallel to the orientation vector O. If the spin angle η is constant, then the angular and translational accelerations of the robot end-effector can be expressed as*

$$a_A = ((\sigma'+\gamma)e+g)\ddot{s}+((\sigma''+\gamma')e+\varphi' t)\dot{s}^2$$

and

$$a_T = ((\sigma'+\gamma)e^*+\delta^* e+g^*)\ddot{s}+((\sigma''+\gamma')e^*+\delta^{*\prime}e+\sigma' t^*+\varphi^{*\prime} t)\dot{s}^2$$

respectively.

A specified trajectory which robot end-effector follows can be the line of striction of the ruled surface, in other words, the directrix of the ruled surface can also be the line of the striction of the ruled surface. Now, we can give the following corollaries for this case.

Corollary 4.5. *Let a motion of a robot end-effector be represented by a ruled surface $X(t,v) = \alpha(t)+vR(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector and R is the ruling of the ruled surface parallel to the orientation vector O. If the specified trajectory is also the line of striction of the ruled surface on which robot end-effector moves, then the angular and translational velocities of the robot end-effector can be expressed as*

$$v_A = ((\eta'+\gamma)e+g)\dot{s}$$

and

$$v_T = ((\eta'+\gamma)e^*+\gamma^* e+g^*)\dot{s}$$

respectively.

Corollary 4.6. *Let a motion of a robot end-effector be represented by a ruled surface $X(t,v) = \alpha(t)+vR(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector and R is the ruling of the ruled surface parallel to the orientation vector O. If the specified trajectory is also the line of striction of the ruled surface on which robot end-effector moves, then the angular and translational accelerations of the robot end-effector can be expressed as*

$$a_A = ((\eta'+\gamma)e+g)\ddot{s}+((\eta''+\gamma')e+\eta' t)\dot{s}^2$$

and

$$a_T = ((\eta'+\gamma)e^*+\gamma^* e+g^*)\ddot{s}+((\eta''+\gamma')e^*+\gamma^{*\prime}e+\eta' t^*)\dot{s}^2$$

respectively.

5. Example

Let a motion of a robot end-effector be represented by a hyperbolic paraboloid given by the equation $X(t,v) = (t,v,tv)$ and a spin angle η (see Figure 6), where t is the parameter of time and v is an arbitrary parameter. The directrix and the ruling of the hyperbolic paraboloid are $\alpha(t) = (t,0,0)$ and $R(t) = (0,1,t)$, respectively. Since $\mu = \langle \alpha', R' \rangle = 0$, it is seen that the directrix and the line of striction are the same curve, i.e., $c = \alpha$.

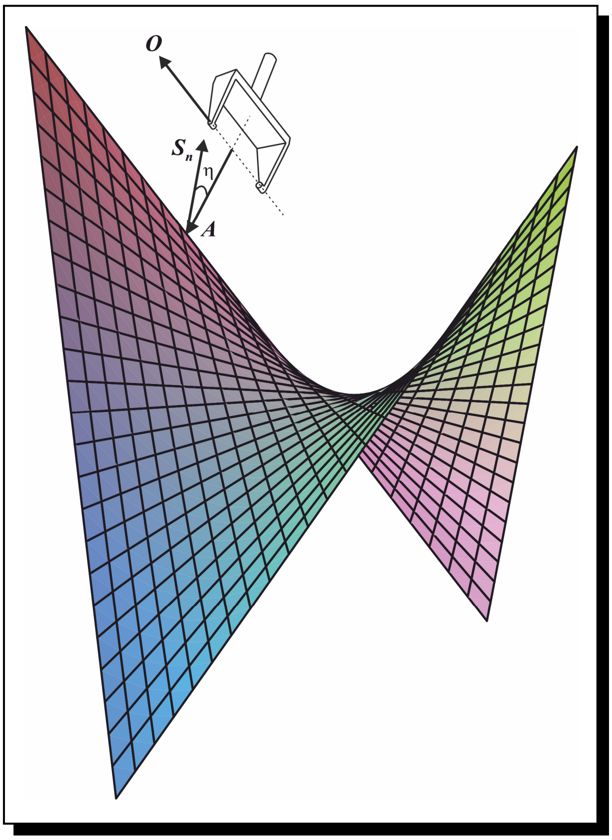

Figure 6. A robot end-effector which moves on the surface of a hyperbolic paraboloid.

The hyperbolic paraboloid corresponds to a dual curve which can be expressed as

$$\tilde{e}(s) = \frac{1}{\sqrt{1+\tan^2 s}}[(0,1,\tan s) + \varepsilon(0,-\tan^2 s,\tan s)],$$

where s is the arc-length parameter of the spherical image curve of R. The first element of the dual Darboux frame is the dual curve $\tilde{e}(s)$ itself. The second and third elements of the dual Darboux frame can be found as

$$\tilde{t}(s) = \frac{1}{\sqrt{1+\tan^2 s}}[(0,-\tan s,1) + \varepsilon(0,-\tan s,-\tan^2 s)]$$

and

$$\tilde{g}(s) = (1,0,0),$$

respectively. By using equation (4.1), the dual geodesic curvature of the hyperbolic paraboloid can be found as $\bar{\gamma} = 0$. Let $\bar{\varphi} = \varphi + \varepsilon\varphi^*$ be a dual angle between the dual unit vectors $\tilde{A}$ and $\tilde{t}$, where φ and φ^* are the real angle and the shortest distance between the lines correspond to the dual vectors $\tilde{A}$ and $\tilde{t}$, respectively. Since the directrix is also the line of striction, the distance between these curves equals to zero, i.e., $\varphi^* = 0$, and the normal vector S_n and central normal vector t are the same vectors, i.e., $\sigma = 0$. Thus, we have $\bar{\varphi} = \eta$. The dual instantaneous rotation vector of the dual tool frame can be found as

$$\tilde{w}_O = w_O + \varepsilon w_O^* = \left[\frac{\eta'}{\sqrt{1+\tan^2 s}}(0,1,\tan s) + (1,0,0)\right] + \varepsilon\frac{\eta'}{\sqrt{1+\tan^2 s}}(0,-\tan^2 s,\tan s).$$

The angular and translational velocities of the robot end-effector can be obtained by substituting w_O and w_O^* into equations (4.9) and (4.10), respectively. By differentiating the dual instantaneous rotation vector, we get

$$\begin{aligned}\widetilde{w}_O' &= w_O' + \varepsilon w_O^{*\prime}\\ &= \frac{1}{\sqrt{1+\tan^2 s}}\left[\eta''(0,1,\tan s)+\eta'(0,-\tan s,1)\right]\\ &\quad + \varepsilon\frac{1}{\sqrt{1+\tan^2 s}}\left[\eta''(0,-\tan^2 s,\tan s)+\eta'(0,-\tan s,-\tan^2 s)\right].\end{aligned}$$

The angular and translational accelerations of the robot end-effector can be obtained by substituting w_O' and $w_O^{*\prime}$ into equations (4.11) and (4.12), respectively.

6. Conclusions

In this paper, a dual method based on the curvature theory of a dual unit spherical curve which corresponds to a ruled surface generated by a line fixed in the robot end-effector is proposed. By using this dual method, translational and angular differential properties, such as velocity and acceleration, of motion of a robot end-effector which are important information in robot trajectory planning are determined. The dual curvature theory used in this paper is much simpler in expression than the curvature theory in real space and it reduces parameters in formulations. It is believed that this method may reduce computation time in computer programming and contribute to the research area of robotics.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] W. Blaschke, *Vorlesungen Über Differentialgeometrie*, Bd. 1, Dover Publications, New York (1945).

[2] O. Bottema and B. Roth, *Theoretical Kinematics*, North-Holland Publ. Co., Amsterdam, p. 556 (1979).

[3] M. Brady, J.M. Hollerbach, T.L. Johnson, T. Lozano-Perez and M.T. Mason, *Robot Motion: Planning and Control*, The MIT Press, Cambridge, Massachusetts, p. 585 (1982).

[4] M.P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Vol. 2, Prentice-hall, Englewood Cliffs (1976).

[5] C. Ekici, Y. Ünlütürk, M. Dede and B.S. Ryuh, On motion of robot end-effector using the curvature theory of timelike ruled surfaces with timelike ruling, *Mathematical Problems in Engineering* **2008** (2008), Article ID 362783.

[6] H.H. Hacısalihoğlu, On the pitch of a closed ruled surface, *Mechanism and Machine Theory* **7** (1972), 291 – 305.

[7] H.H. Hacısalihoğlu, *Hareket Geometrisi ve Kuaterniyonlar Teorisi*, Gazi University Faculty of Sciences and Arts (1983).

[8] A. Karger and J. Novak, *Space Kinematics and Lie Groups*, STNL Publishers of Technical Lit., Prague, Czechoslovakia (1978).

[9] A.P. Kotelnikov, *Screw Calculus and Some Applications to Geometry and Mechanics*, Annals of Imperial University of Kazan (1895).

[10] F.L. Litvin and X.C. Gao, Analytical representation of trajectory of manipulators, trends and developments in mechanisms, machines, and robotics, *The ASME Design Technology Conferences, the 20th Biennial Mechanisms Conference*, Vol. 15-3, pp. 481 – 485, Kissimmee, Florida, USA, September 25-28 (1988).

[11] R.P. Paul, Manipulator cartesian path control, *IEEE Trans. Systems, Man., Cybernetics* **SMC-9** (11) (1979), 702 – 711.

[12] B.S. Ryuh, *Robot Trajectory Planning Using the Curvature Theory of Ruled Surfaces*, Doctoral dissertation, Purdue University, West Lafayette, Ind., USA (1989).

[13] B.S. Ryuh and G.R. Pennock, Accurate motion of a robot end-effector using the curvature theory of ruled surfaces, *Journal of Mechanisms, Transmissions, and Automation in Design* **110** (4) (1988), 383 – 388.

[14] B.S. Ryuh and G.R. Pennock, Trajectory planning using the Ferguson curve model and curvature theory of a ruled surface, *Journal of Mechanical Design* **112** (1990), 377 – 383.

[15] J.A. Schaaf, *Curvature Theory of Line Trajectories in Spatial Kinematics*, Doctoral dissertation, University of California, Davis, 111 p. (1988).

[16] E. Study, *Geometrie der Dynamen*, Leipzig (1903).

[17] G.R. Veldkamp, On the use of dual numbers, vectors and matrices in instantaneous spatial kinematics, *Mechanism and Machine Theory* **II** (1976), 141 – 156.