



Surface Waves in Fluid-saturated Porous Layer Bounded by A Liquid Layer and An Orthotropic Elastic Half Space

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Abstract. The dispersion equation of surface waves is studied in a fluid saturated porous layer occupying an orthotropic elastic half space and lying under the uniform liquid layer. The frequency equation is obtained as determinant of order ten. Some special cases are derived. Numerical results are obtained for two different cases and are shown by graphs.

1. Introduction

Liquid saturated porous material are often present on and below the surface of the earth in the form of sandstone, limestone and other sediments permeated by groundwater or oil. Porous media theories play an important role in many branches of engineering including material science, petroleum industry, chemical engineering, biomechanics, soil mechanics and water table determination etc.

The propagation of waves in a porous elastic solid saturated by a single-phase compressible viscous fluid was first analyzed by Biot's in several interested research papers. The most prominent results of Biot's theory of wave propagation in porous media is that there are in contrast with the conventional elastic theory three bulk waves: two compressional and one shear wave. The study of propagation of waves in a layered heterogeneous medium is in particular of great importance. The propagation of SH-wave in homogeneous layer over a porous medium has been studied the many author. Deresiewicz and Rice [1] have discussed the effect of boundaries on wave propagation in liquid-Filled porous solid. Wadhwa [16] has studied the disturbance due to a homogeneous liquid layer over a heterogeneous liquid half-space. Sharma et al. [6, 7] have discussed the surface wave propagation in a liquid saturated porous solid and isotropic half-space. Rajapakse and Senjuntichai [11] have studied the Dynamic response of a multi-layered poroelastic medium. Kumar and Deswal [15] have

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discussed the Surface wave propagation in liquid-saturated porous layer over a liquid-saturated porous half-space with loosely bonded interface. Destrade [8] has discussed the surface waves in orthotropic incompressible material. Recently Kumar and Hundal [12, 13, 14] have studied the problem of wave propagation in fluid-saturated incompressible porous media. Pal [10] has studied on shear wave propagation in a multilayered medium including a fluid saturated porous solid stratum. Pal and Sen [9] have discussed the Propagation of SH-type waves in three layered medium including a fluid-saturated porous solid stratum. Rangelov et al. [17] have discussed the wave propagation in a restricted class of orthotropic inhomogeneous half-planes.

In this paper we investigate the theory of propagation of elastic waves in a system composed of a fluid-saturated porous solid, homogeneous fluid and orthotropic solid. Dispersion equation is obtained and numerical results are shown graphically.

2.

2.1. Formulation of problem and solution

Let h_1 and h_2 are the thickness of the homogeneous liquid layer and fluid saturated porous solid respectively. X axis is taken as the wave propagation and Z axis is taken as vertically downwards. $z = -h_1$ is the free surface of the liquid layer, intermediate layer makes contact with the fluid saturated porous solid at $z = 0$ and lower layer makes contact with the orthotropic solid half space at $z = h_2$.

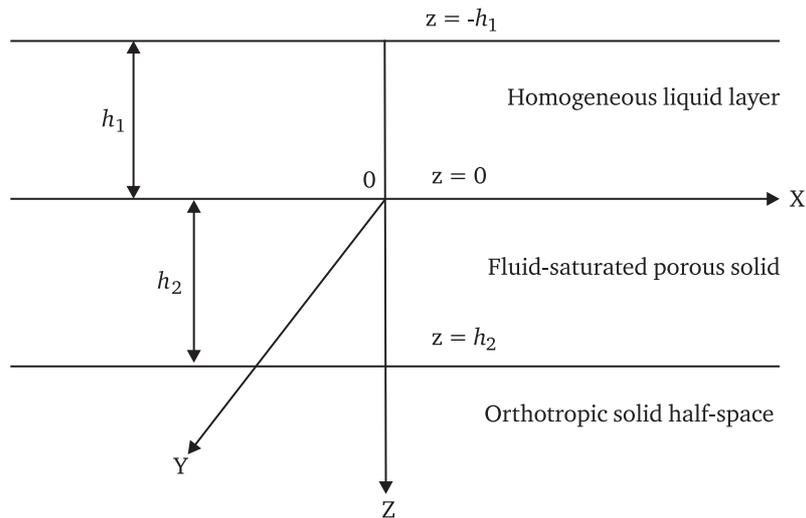


Figure 1. Geometry of problem

Equation of motion in homogeneous liquid layer

$$\frac{\partial^2 \varphi_0}{\partial x^2} + \frac{\partial^2 \varphi_0}{\partial z^2} = \frac{1}{\alpha^2} \frac{\partial^2 \varphi_0}{\partial t^2}, \quad (1)$$

where $\alpha = \frac{\lambda_0}{\rho_0}$.

Let us define the displacement

$$\varphi_0 = \bar{\varphi}_0(z) e^{ik(x-ct)}. \quad (2)$$

We get the homogeneous equation

$$\frac{d^2 \bar{\varphi}_0}{dz^2} - k^2 s^2 \bar{\varphi}_0 = 0, \quad (3)$$

where $s^2 = 1 - \frac{c^2}{\alpha^2}$.

Solution of equation (3), and put the equation (2), we get

$$\varphi_0 = (A_1 e^{ksz} + A_2 e^{-ksz}) e^{ik(x-ct)}. \quad (4)$$

From equation (4), we get

$$u_0 = \frac{\partial \varphi_0}{\partial x} = ik(A_1 e^{ksz} + A_2 e^{-ksz}) e^{ik(x-ct)}, \quad (5)$$

$$w_0 = \frac{\partial \varphi_0}{\partial z} = ks(A_1 e^{ksz} - A_2 e^{-ksz}) e^{ik(x-ct)}, \quad (6)$$

$$\sigma_{zz} = \lambda_0 \frac{\partial \omega_0}{\partial z} = k^2 s^2 \lambda_0 (A_1 e^{ksz} - A_2 e^{-ksz}) e^{ik(x-ct)}. \quad (7)$$

According to Biot's [5] formulation of the equations of motion governing the displacement u , of the solid matrix and U , of the interstitial liquid, we have

$$\left. \begin{aligned} N \nabla^2 \bar{u} + \text{grad}[(D + N) \text{div} \bar{u} + Q \text{div} \bar{U}] &= \frac{\partial^2}{\partial t^2} (\rho_{11} \bar{u} + \rho_{12} \bar{U}) \\ \text{grad}(Q \text{div} \bar{u} + R \text{div} \bar{U}) &= \frac{\partial^2}{\partial t^2} (\rho_{12} \bar{u} + \rho_{22} \bar{U}) \end{aligned} \right\}, \quad (8)$$

where D , N , Q and R , all nonnegative, are elastic Moduli; ρ_{11} , ρ_{12} and ρ_{22} dynamical coefficients.

The Moduli enter into the relations between the stress in the solid, σ_{ij} and that in the liquid, σ and the strain in the solid,

$$\varepsilon_{ij} = \frac{1}{2}(u_{ij} + u_{ji}) \quad (9)$$

and the dilatations,

$$\left. \begin{aligned} e &= \text{div} \bar{u} \quad \text{and} \quad \varepsilon = \text{div} \bar{U} \\ \sigma_{ij} &= (De + Q\varepsilon) \delta_{ij} + 2N \varepsilon_{ij} \\ \sigma &= Qe + R\varepsilon \end{aligned} \right\} \quad (10)$$

where, δ_{ij} is kronecker delta.

The dynamical coefficients $\rho_{11} > 0$, $\rho_{12} \leq 0$ and $\rho_{22} \geq 0$ are related to the mass of solid, $\rho(s)$ and of the fluid, $\rho(f)$

$$\left. \begin{aligned} \rho(s) &= \rho_{11} + \rho_{12} = (1-\beta)\rho_s \\ \rho(f) &= \rho_{11} + \rho_{22} = \beta\rho_f \end{aligned} \right\}, \quad (11)$$

where ρ_s and ρ_f denote the mass densities of the solid and fluid, respectively moreover, the elastic and dynamical constants have been shown (Biot, [5]) to satisfy the inequalities.

$$PR - Q^2 > 0, \quad \rho_{11}P + \rho_{22}R - 2\rho_{12}Q > 0,$$

where $P = D + 2N$.

We now consider a Helmholtz resolution of each of two displacement vectors, of the form

$$\left. \begin{aligned} \bar{u} &= \text{grad } \varphi + \text{curl } H \\ \bar{U} &= \text{grad } \psi + \text{curl } G \end{aligned} \right\}. \quad (12)$$

Intersection of (12) in (8), yields a pair of equations which are satisfied identically by setting

$$\left. \begin{aligned} P\nabla^2\varphi + Q\nabla^2\psi &= \frac{\partial^2}{\partial t^2}(\rho_{11}\varphi + \rho_{12}\psi) \\ Q\nabla^2\varphi + R\nabla^2\psi &= \frac{\partial^2}{\partial t^2}(\rho_{12}\varphi + \rho_{22}\psi) \end{aligned} \right\} \quad (13)$$

and

$$\left. \begin{aligned} N\nabla^2H &= \frac{\partial^2}{\partial t^2}(\rho_{11}H + \rho_{12}G) \\ 0 &= \frac{\partial^2}{\partial t^2}(\rho_{12}H + \rho_{22}G) \end{aligned} \right\}. \quad (14)$$

Let

$$\left. \begin{aligned} \varphi &= \bar{\varphi}(z)e^{ik(x-ct)} \\ \psi &= \bar{\psi}(z)e^{ik(x-ct)} \end{aligned} \right\} \quad (15)$$

be the potentials of the solid and relative fluid displacement.

Enable us to eliminate ψ from (13) the resulting equation governing $\bar{\varphi}$ reducible to

$$p\nabla^4\bar{\varphi} + k^2(q - 2p)\nabla^2\bar{\varphi} + k^4(p - q + r)\bar{\varphi} = 0, \quad (16)$$

where

$$\left. \begin{aligned} p &= PR - Q^2 \\ q &= c^2(\rho_{11}R - 2\rho_{12}Q + \rho_{22}P) \\ r &= c^4(\rho_{11}\rho_{22} - \rho_{12}^2) \end{aligned} \right\}. \quad (17)$$

The solution of (16) may be written in the form

$$\bar{\varphi} = \bar{\varphi}_1 + \bar{\varphi}_2, \quad (18)$$

where

$$(\nabla^2 - k^2 \alpha_j) \bar{\varphi}_j = 0, \quad j = 1, 2, \quad (19)$$

$$\alpha_j = \alpha_{1,2} = \frac{(2p - q) \pm \sqrt{(q^2 - 4pr)}}{2p} = m_1 \quad (\text{say}). \quad (20)$$

Solution of equation (18) and (19) is

$$\bar{\varphi} = 2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_2}}. \quad (21)$$

Using the relation (21) in (15), we get

$$\varphi = (2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_2}}) e^{ik(x-ct)}. \quad (22)$$

With the aid of (18) and (19), the remaining scalar potential is then found to be

$$\bar{\psi} = \mu_1 \bar{\varphi}_1 + \mu_2 \bar{\varphi}_2, \quad (23)$$

where

$$\mu_j = \frac{c^2(\rho_{11}R - \rho_{12}Q) - p(1 - m_1)}{c^2(\rho_{22}Q - \rho_{12}R)} = n_1 \quad (\text{say}), \quad j = 1, 2. \quad (24)$$

Similarly we put

$$\left. \begin{aligned} H &= \bar{H}(z) e^{ik(x-ct)} \\ G &= \bar{G}(z) e^{ik(x-ct)} \end{aligned} \right\}. \quad (25)$$

When inserted in (14), we get

$$\bar{G} = -\frac{\rho_{12}}{\rho_{22}} \bar{H}, \quad (26)$$

$$(\nabla^2 - k^2 m_2) \bar{H} = 0, \quad (27)$$

where

$$m_2 = 1 - \frac{c^2(\rho_{11}\rho_{22} - \rho_{12}^2)}{N\rho_{22}}. \quad (28)$$

$\psi_1 = (-\bar{H})_y$, and is given by

$$\psi_1 = (A_5 e^{zk\sqrt{m_2}} + A_6 e^{-zk\sqrt{m_2}}) e^{ik(x-ct)}. \quad (29)$$

For two dimensional motion x-z plane the displacements $\bar{u} = (u, 0, w)$ in the solid and $\bar{U} = (U, 0, W)$ in liquid will be given as follows

$$\left. \begin{aligned} u &= \frac{\partial \bar{\varphi}_1}{\partial x} + \frac{\partial \bar{\varphi}_2}{\partial x} + \frac{\partial \psi_1}{\partial z} \\ w &= \frac{\partial \bar{\varphi}_1}{\partial z} + \frac{\partial \bar{\varphi}_2}{\partial z} - \frac{\partial \psi_1}{\partial x} \\ U &= \mu_1 \frac{\partial \bar{\varphi}_1}{\partial x} + \mu_2 \frac{\partial \bar{\varphi}_2}{\partial x} + n_2 \frac{\partial \psi_1}{\partial z} \\ W &= \mu_1 \frac{\partial \bar{\varphi}_1}{\partial z} + \mu_2 \frac{\partial \bar{\varphi}_2}{\partial z} - n_2 \frac{\partial \psi_1}{\partial x} \end{aligned} \right\} \quad (30)$$

where

$$n_2 = \frac{\rho_{12}}{\rho_{22}}. \quad (31)$$

Using the relation (22), (24), (29) and (31) in (30), we get

$$\left. \begin{aligned} u &= k \left\{ i(2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_1}}) + \sqrt{m_2}(A_5 e^{zk\sqrt{m_2}} - A_6 e^{-zk\sqrt{m_2}}) \right\} e^{ik(x-ct)} \\ w &= k \left\{ \sqrt{m_1}(2A_3 e^{zk\sqrt{m_1}} - 2A_4 e^{-zk\sqrt{m_1}}) - i(A_5 e^{zk\sqrt{m_2}} + A_6 e^{-zk\sqrt{m_2}}) \right\} e^{ik(x-ct)} \\ U &= k \left\{ in_1(2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_1}}) + n_2 \sqrt{m_2}(A_5 e^{zk\sqrt{m_2}} - A_6 e^{-zk\sqrt{m_2}}) \right\} e^{ik(x-ct)} \\ W &= k \left\{ n_1 \sqrt{m_1}(2A_3 e^{zk\sqrt{m_1}} - 2A_4 e^{-zk\sqrt{m_1}}) - in_2(A_5 e^{zk\sqrt{m_2}} + A_6 e^{-zk\sqrt{m_2}}) \right\} e^{ik(x-ct)} \end{aligned} \right\} \quad (32)$$

From equation (10), we obtain

$$\left. \begin{aligned} \sigma_{zz} &= k^2(m_1 - 1)(D + Qn_1)(2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_1}}) e^{ik(x-ct)} \\ &\quad + 2Nk^2 m_1 (2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_1}}) e^{ik(x-ct)} \\ &\quad - i2Nk^2 \sqrt{m_2} (A_5 e^{zk\sqrt{m_2}} - A_6 e^{-zk\sqrt{m_2}}) e^{ik(x-ct)} \end{aligned} \right\}, \quad (33)$$

$$\sigma_{zx} = \left\{ 2ik^2 N \sqrt{m_1} (2A_3 e^{zk\sqrt{m_1}} - 2A_4 e^{-zk\sqrt{m_1}}) + k^2 N (m_2 + 1) (A_5 e^{zk\sqrt{m_2}} + A_6 e^{-zk\sqrt{m_2}}) \right\} e^{ik(x-ct)}, \quad (34)$$

$$\sigma = k^2(m_1 - 1)(Q + Rn_1)(2A_3 e^{zk\sqrt{m_1}} + 2A_4 e^{-zk\sqrt{m_1}}) e^{ik(x-ct)}. \quad (35)$$

Equation of motion in orthotropic material

$$\left. \begin{aligned} c_{11} \frac{\partial^2 u^*}{\partial x^2} + c_{66} \frac{\partial^2 u^*}{\partial z^2} + (c_{66} + c_{12}) \frac{\partial^2 u^*}{\partial x \partial z} &= \rho^* \frac{\partial^2 u^*}{\partial t^2} \\ \text{and } c_{12} \frac{\partial^2 w^*}{\partial x^2} + c_{66} \frac{\partial^2 w^*}{\partial z^2} + (c_{66} + c_{12}) \frac{\partial^2 w^*}{\partial x \partial z} &= \rho^* \frac{\partial^2 w^*}{\partial t^2} \end{aligned} \right\}. \quad (36)$$

We seek harmonic solution and so we put

$$\left. \begin{aligned} u^* &= U(z) e^{ik(x-ct)} \\ w^* &= W(z) e^{ik(x-ct)} \end{aligned} \right\}. \quad (37)$$

In (36), we get the homogeneous equation

$$\left. \begin{aligned} \frac{d^2U}{dz^2} + ik\alpha_1 \frac{dU}{dz} - k^2\alpha_2 U &= 0 \\ \frac{d^2W}{dz^2} + ik\beta_1 \frac{dW}{dz} - k^2\beta_2 W &= 0 \end{aligned} \right\}, \quad (38)$$

where

$$\alpha_1 = \frac{c_{66} + c_{12}}{c_{12}}, \quad \alpha_2 = \frac{c_{11} - \rho^* c^2}{c_{12}}, \quad \beta_1 = \frac{c_{66} + c_{12}}{c_{66}}, \quad \beta_2 = \frac{c_{12} - \rho^* c^2}{c_{66}}.$$

Solution (38) and use in (37), we get

$$\left. \begin{aligned} u^* &= (A_7 e^{ks_1 z} + A_8 e^{-ks_2 z}) e^{ik(x-ct)} \\ w^* &= (A_9 e^{ks_3 z} + A_{10} e^{-ks_4 z}) e^{ik(x-ct)} \end{aligned} \right\}, \quad (39)$$

where

$$\begin{aligned} s_1 &= \frac{-i\alpha_1 + \sqrt{(-\alpha_1^2 + 4\alpha_2)}}{2}, & s_2 &= \frac{i\alpha_1 + \sqrt{(-\alpha_1^2 + 4\alpha_2)}}{2}, \\ s_3 &= \frac{-i\beta_1 + \sqrt{(-\beta_1^2 + 4\beta_2)}}{2}, & s_4 &= \frac{i\beta_1 + \sqrt{(-\beta_1^2 + 4\beta_2)}}{2}, \end{aligned}$$

$$\begin{aligned} \sigma_{zz}^* &= c_{11} \frac{\partial u^*}{\partial x} + c_{12} \frac{\partial w^*}{\partial z} \\ &= k \left\{ ic_{11}(A_7 e^{ks_1 z} + A_8 e^{-ks_2 z}) + c_{12}(s_3 A_9 e^{ks_3 z} - s_4 A_{10} e^{-ks_4 z}) \right\} e^{ik(x-ct)}, \end{aligned} \quad (40)$$

$$\begin{aligned} \sigma_{zx}^* &= c_{44} \left(\frac{\partial w^*}{\partial x} + \frac{\partial u^*}{\partial z} \right) \\ &= c_{44} k (iA_9 e^{ks_3 z} + iA_{10} e^{-ks_4 z} + s_1 A_7 e^{ks_1 z} - s_2 A_8 e^{-ks_2 z}) e^{ik(x-ct)}. \end{aligned} \quad (41)$$

2.2. Boundary conditions

At $z = h_2$ i.e. at the interface of fluid saturated porous solid and orthotropic elastic solid half-space.

$$\left. \begin{aligned} (\sigma_{zz} + \sigma)_{II} &= (\sigma_{zz}^*)_{III} \\ (\sigma_{zx})_{II} &= (\sigma_{zx}^*)_{III} \\ (w)_{II} &= (w^*)_{III} \\ (u)_{II} &= (u^*)_{III} \\ (\dot{w} - \dot{W})_{II} &= 0 \end{aligned} \right\} \quad (42)$$

At $z = 0$ i.e. at the interface of homogenous liquid layer and fluid saturated porous solid.

$$\left. \begin{aligned} (\sigma_{zz} + \sigma)_{II} &= (\sigma_{zz})_I \\ (\sigma_{zz})_{II} &= 0 \\ (w)_{II} &= (w_0)_I \\ (u)_{II} &= (u_0)_I \end{aligned} \right\} \quad (43)$$

At $z = -h_1$ i.e. liquid layer surface is stress free.

$$(\sigma_{zz})_1 = 0 \quad (44)$$

Using the relations (5), (6), (7), (32), (33), (34), (35), (39), (40), (41) in boundary conditions (42), (43) and (44), we get

$$\begin{aligned} k \{ (m_1 - 1)(D + Qn_1 + Q + Rn_1) + 2Nm_1 \} (2A_3 e^{h_2 k \sqrt{m_1}} + 2A_4 e^{-h_2 k \sqrt{m_1}}) \\ - ik2N \sqrt{m_2} (A_5 e^{h_2 k \sqrt{m_2}} - A_6 e^{-h_2 k \sqrt{m_2}}) - ic_{11} (A_7 e^{h_2 ks_1} + A_8 e^{-h_2 ks_2}) \\ - c_{12} (s_3 A_9 e^{h_2 ks_3} - s_4 A_{10} e^{-h_2 ks_4}) = 0, \end{aligned} \quad (45)$$

$$\begin{aligned} 2ikN \sqrt{m_1} (2A_3 e^{h_2 k \sqrt{m_1}} - 2A_4 e^{-h_2 k \sqrt{m_1}}) - kN(m_2 + 1) (A_5 e^{h_2 k \sqrt{m_2}} + A_6 e^{-h_2 k \sqrt{m_2}}) \\ - c_{44} \{ (s_1 A_7 e^{h_2 ks_1} - s_2 A_8 e^{-h_2 ks_2}) + i(A_9 e^{h_2 ks_3} + A_{10} e^{-h_2 ks_4}) \} = 0, \end{aligned} \quad (46)$$

$$\begin{aligned} 2k \sqrt{m_1} (A_3 e^{h_2 k \sqrt{m_1}} - A_4 e^{-h_2 k \sqrt{m_1}}) - ik (A_5 e^{h_2 k \sqrt{m_2}} + A_6 e^{-h_2 k \sqrt{m_2}}) \\ - (A_9 e^{h_2 ks_3} + A_{10} e^{-h_2 ks_4}) = 0, \end{aligned} \quad (47)$$

$$\begin{aligned} 2ik (A_3 e^{h_2 k \sqrt{m_1}} + A_4 e^{-h_2 k \sqrt{m_1}}) + k \sqrt{m_2} (A_5 e^{h_2 k \sqrt{m_2}} - A_6 e^{-h_2 k \sqrt{m_2}}) \\ - (A_7 e^{h_2 ks_1} + A_8 e^{-h_2 ks_2}) = 0, \end{aligned} \quad (48)$$

$$\begin{aligned} 2(1 - n_1) \sqrt{m_1} (A_3 e^{h_2 k \sqrt{m_1}} - A_4 e^{-h_2 k \sqrt{m_1}}) \\ + (n_2 - 1) \sqrt{m_2} (A_5 e^{h_2 k \sqrt{m_2}} + A_6 e^{-h_2 k \sqrt{m_2}}) = 0, \end{aligned} \quad (49)$$

$$\begin{aligned} \frac{\lambda_0 s^2}{2} (A_1 + A_2) - \{ (m_1 - 1)(D + Qn_1 + Q + Rn_1) + 2Nm_1 \} \\ \times (A_3 + A_4) + iN \sqrt{m_2} (A_5 - A_6) = 0, \end{aligned} \quad (50)$$

$$\{ (m_1 - 1)(D + Qn_1) + 2Nm_1 \} (A_3 + A_4) - iN \sqrt{m_2} (A_5 - A_6) = 0, \quad (51)$$

$$s(A_1 - A_2) - 2\sqrt{m_1} (A_3 - A_4) + i(A_5 + A_6) = 0, \quad (52)$$

$$(A_1 + A_2) - 2(A_3 + A_4) + i\sqrt{m_2} (A_5 - A_6) = 0, \quad (53)$$

$$A_1 e^{-h_1 ks} + A_2 e^{h_1 ks} = 0. \quad (54)$$

From the equation (45) to (54), we eliminate the coefficient $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$ and A_{10} , we get dispersion equation

$$(n_1 - 1) \cosh(h_2 k \sqrt{m_1}) = (n_2 - 1) \cosh(h_2 k \sqrt{m_2}). \quad (55)$$

3. Special cases

Case 1: If $m_1=0$, we get the dispersion equation (55) as

$$\tanh(h_2 k \sqrt{m_2}) = \pm \frac{\sqrt{(n_1 - n_2)(n_1 + n_2 - 2)}}{(n_1 - 1)}.$$

Case 2: If $m_2=0$, we get the dispersion equation (55) as

$$\tanh(h_2 k \sqrt{m_1}) = \pm \frac{\sqrt{(n_1 - n_2)(2 - n_1 - n_2)}}{(n_2 - 1)}.$$

Further if $kh_2 \rightarrow \infty$, then the liquid layer become liquid half-space and the dispersion equation become the equation of surface wave propagation in liquid-saturated porous layer bounded between liquid and orthotropic elastic half-space.

4. Numerical calculations and discussions

The numerical calculations are performed by considering the second layer as water saturated sand stone. The data's are taken form Santos *et al.* [2]. We consider $\rho_{11} = 2.02$, $\rho_{12} = 0.20$, $\rho_{22} = 1.62$ as gm/cm³. $P = 3.9$, $Q = 2.2$, $R = 1.37$, $N = 4.6$ as GPA.

For both cases 1 and 2, the variations of phase velocity within frequency are considered. It is interesting to note that in case 1, the phase velocity decreases as frequency increases i.e. they are inverse in nature while in case 2, the variation of phase velocity and frequency are proportional. These short of remarkable change is due to the presence of fluid layer and an orthotropic solid half-space.

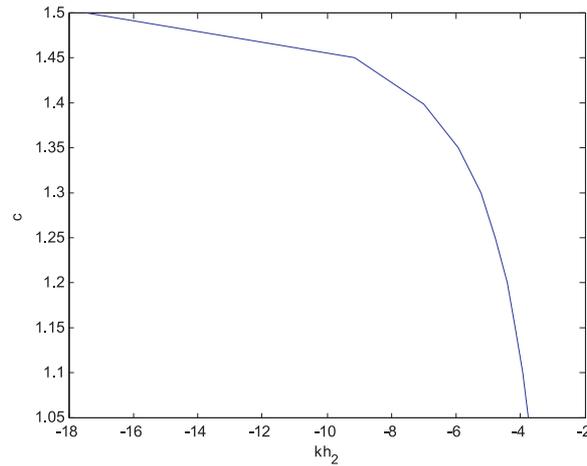


Figure 2. Variation of frequency and phase velocity (Case 1)

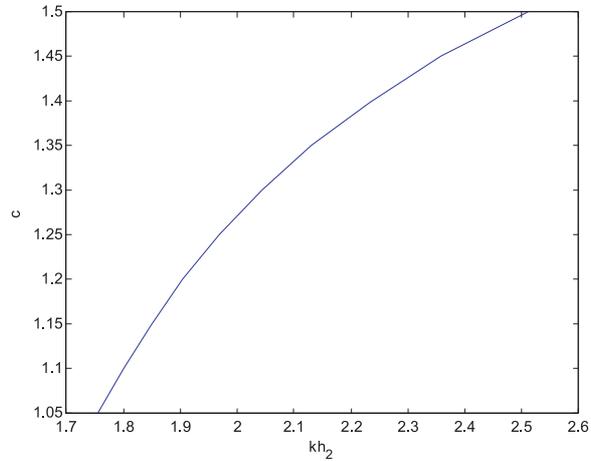


Figure 3. Variation of frequency and phase velocity (Case 2)

5. Acknowledgement

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