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Numerical Analysis of Natural Gas Delivery Discrepancy

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Abstract. The article describes algorithm for optimization of discrepancies in natural gas supply to consumers. Numerical monitoring makes it possible to obtain computational estimates of actual gas deliveries over given time spans and to estimate their difference from corresponding values reported by gas consumers. Mathematical analysis of the discrepancy is based on a statement and numerical solution of identification problem of a physically proved gas dynamics mode of natural gas transmission through specified gas distribution networks. The identified mode parameters should have a minimum discrepancy with field measurements of gas transport at specified reference points of the simulated pipeline network.

1. **Problem Statement**

Numerical monitoring of the discrepancy is based on a statement (for a specified time gap) and numerical solution of identification problem of a physically proved quasi-steady and transient gas dynamics mode of natural gas transmission through specified gas distribution networks. In large communities, natural gas is supplied to the consumers using medium or low pressure ring mains, being several dozen kilometres long. Gas from the supplier is transmitted to such mains through a *gas transmission networks* (GTN) after its pressure is reduced by means of a system of gas reducers installed at inlet *gas distribution stations* (GDSs). Major parameters of gas supplied by the gas transportation company to the seller are also measured at the GDS outlets. Here, major parameters of natural gas include its flow rate, pressure and temperature, varying with time.

Gas from inlet GDSs is delivered to the ring main via the *connecting gas pipelines* (CGP) network of the gas seller. Consumers receive gas from the ring mains through outlet CGPs leading from the ring main to the consumer. In the first approximation, each consumer is considered independent and provided with gas through one CGP, which is completely associated with the consumer (called "associated CGP" as the text goes). Consumer independence means that

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the consumer's gas cannot be delivered to other consumers. Thus, the *gas distribution network* (GDN) under consideration comprises inlet CGPs from inlet GDSs, a ring main and associated CGPs.

If the GDN operates properly, the seller seeks to sell the whole amount of gas received from the supplier. An exception in this case is natural gas forcedly accumulated in the GDN. For settlement of accounts, consumers submit reports to the seller, in which they indicate estimated, varying with time, volumes of received gas. These reports are usually generated either by processing the consumers' field flow meter readings or by simplified calculations based on the rates formally established for the given category of consumers.

Verification of data provided by the consumers consists in the comparison of their estimates with data obtained by processing the seller's flow meter readings in compliance with current guidelines. The central difficulty in such verification is that the amount of field measurements of supplied gas that can be used as a reliable basis is rather limited in the present-day gas industry. Such a situation results in occasional discrepancies (especially during the heating season) in analyzing the volume of natural gas supplied to the consumers. The total discrepancy over a given time period is determined as a difference between two estimates of the gas volume, varying with time. The first estimate represents the total gas volume actually received during the time period in question as reported by all consumers, and the second estimate, the total volume of natural gas delivered by the supplier to the seller less the gas volume accumulated in the GDN.

One of the most promising ways to resolve the above problem is to use high-accuracy *computational fluid dynamics* (CFD) simulators of modes of gas mixture transmission through long, branched pipeline systems (CFD-simulator) [1]. Such CFD-simulators form the basis of computational kernel for Gas Distribution Discrepancy *Computer Analytical System* (CAS). Their implementation is aimed at getting high accurate estimates of spatial-temporal distributions of natural gas flow in pipeline network under examination.

Input data: layout chart of the GDN; sensor locations in the GDN, where gas parameters are measured; given time interval of GDN operation; results of field measurements of gas parameters in the GDN in the given time interval; actual errors of instruments used to measure gas parameters; data on received gas volumes as reported by each consumer for the given time interval.

Target data: (1) physically based gas flow parameters in the GDN in the given time interval having a minimum discrepancy compared to respective field measurement data at identification points (IP) and providing the closest possible agreement between calculated flow rate values at the outlet of each associated CGP and corresponding reported values (further as the text goes, this mode will be called "the identified gas flow" (IGF)); (2) associated CGPs with underreported gas volumes as against the identified gas flow; (3) calculated estimates of discrepancies between gas volumes delivered in the given time interval through each associated CGP as an arithmetic difference between the calculated gas volume corresponding to the identified gas flow and the reported value; (4) calculated estimates of discrepancies between gas volumes delivered in the given time interval through each inlet GDS as an arithmetic difference between the calculated gas volume corresponding to the identified gas flow and the reported value.

2. **Solution Method**

Correct simulation of item 1 in the problem statement makes it possible to obtain credible information on physically consistent space-time distributions of flow rates, pressures and temperatures for the gas flow, which is most reasonable for the given time interval with the given field measurement data. It follows from the above problem statement that numerical monitoring of gas distribution discrepancy under items 2-4 in the list of target values in essence consists in performing straightforward arithmetic operations with output data of item 1.

Therefore, special attention below will be paid to the algorithm of this calculation. This algorithm was proposed by [2].

In order to calculate non-isothermal transient gas flow parameters in the GDN under consideration, the following boundary conditions of "Type I" need to be specified: laws of variation with time for pressure, temperature and composition are defined at the outlet of each inlet GDS; laws of variation with time for mass flow rate and gas temperature are defined at the outlet of each associated CGP.

Using the CFD-simulator with the given boundary conditions and fixed GDN characteristics, one can unambiguously determine physically based spatial-temporal distributions of calculated estimates of transient GDN operation parameters [2], [3]. Spatial-temporal distributions of parameters here mean their distributions along the pipelines.

A diagram of identification locations is generated on the given layout of sensor locations in the GDN. The preferred location of each IP should correspond to the key requirement: a considerable change in the fluid dynamics conditions of GDN operation should be accompanied by considerable changes in the gas parameters actually measured at this point.

The distribution of IPs over the GDN diagram should be as uniform as possible. An IP can be located both inside the GDN and at its boundaries. At each IP, different combinations of major gas flow parameters can be measured.

The process of finding the identified gas flow comes to the statement and solution of the problem of conditional optimization for target function varying with time (equivalent for dynamic control problem):

$$\int_{\Delta T} \|\vec{\mathbf{p}}_{\text{calc}}[\vec{\mathbf{X}}(t)] - \vec{\mathbf{p}}_{\text{meas}}^{const}(t)\|_L \, dt \rightarrow \min \quad \text{subject to } \vec{\mathbf{X}}(t) \in \widetilde{\Omega} \subset R^n, \tag{1}$$

where $\widetilde{\Omega}=\left\{\vec{\mathbf{X}}(t)\in R^n: \vec{\mathbf{a}}(t)\leq\vec{\mathbf{X}}(t)\leq\vec{\mathbf{b}}(t); \int_{\Delta T} \|\vec{\mathbf{q}}^{\text{GDS}}_{\text{calc}}[\vec{\mathbf{X}}(t)] - \vec{\mathbf{q}}^{const}_{\text{meas\_GDS}}(t)\|_D dt \leq \varepsilon\right\}$; $\|\cdots\|_L$ is the vector norm, the type of which is defined by specifying the parameter L, $(L = 1, 2)$; $\|\cdots\|_D$ is the vector norm, the type of which is specified by specifying the parameter D, $(D = 1, 2)$; $\mathbf{p}_{\text{calc}}[\mathbf{X}(t)]$, $\mathbf{p}_{\text{calc}} \in R^m$, is the vector function of calculated gas pressure estimates at the IP in the m-dimensional Euclidean space R^m (these values are calculated using the CFD-simulator); $\mathbf{p}^{const}_{meas} \in R^m$ is a specified vector of measured gas pressure values at the IP; $m = M_{IP}$ is the number of specified correctly functioning IPs in the pipeline network diagram; $\mathbf{X}(t) \in \widetilde{\Omega} \subset R^n$ is the vector of independent controlled variables in the n-dimensional Euclidean space R^n; $\mathbf{a}(t) \in R^n$ and $\mathbf{b}(t) \in R^n$ are specified vectors defining the boundaries in simple constraints on the range of admissible variations of the vector of independent controlled variables, $\mathbf{0} < \mathbf{a}(t) < \mathbf{b}(t)$, $\mathbf{0}$ is the zero vector; n is the number of independent controlled variables; $D = 1$ is the parameter determining the type of norm; $\mathbf{q}^{\text{GDS}}_{\text{calc}}[\mathbf{X}(t)]$, $\mathbf{q}^{\text{GDS}}_{\text{calc}} \in R^l$, is the vector function of calculated gas mass flow rates at GDS outlets in the l-dimensional Euclidean space R^l (these values are calculated using the CFD simulator); l is the number of GDSs; $\mathbf{q}^{const}_{\text{meas\_GDS}} \in R^l$ is a specified vector of measured gas mass flow rates at GDS outlets.

The latter constraint in the form of the one-sided unstrict inequality in (1) formalizes the predetermined assumption that the seller trusts the supplier. Fulfillment of this inequality results in the automatic fulfillment of the condition

$$\|\vec{\mathbf{q}}^{\text{GDS}}_{\text{calc}}[\vec{\mathbf{X}}(t)] - \vec{\mathbf{q}}^{const}_{\text{meas\_GDS}}(t)\|_0 \leq \tau^{\text{GDS}}_{\text{flow\_rate}}, \tag{2}$$

where $\tau^{\text{GDS}}_{\text{flow\_rate}} = \text{const}$ is a specified upper estimate of the actual (rated) absolute error of flow meters installed at GDS outlets, $\tau^{\text{GDS}}_{\text{flow\_rate}} > 0$.

Components $x_i(t)$ of independent controlled variables here mean some boundary conditions (BC) of the first kind specified for calculating gas dynamic modes using the CFD-simulator. Practice has shown [2] that good results in solving problem (1) can be obtained if independent controlled variables include a combined set of mass flow values at outlet boundaries of associated branches $(x_i(t),\ i = 1, \ldots, k)$ and gas pressures at GDS outlets $(x_i(t),\ i = k + 1, \ldots, n,$ $n = k + l)$, where k is the number of associated branches in the pipeline network of interest. Thus, here we speak about specifying natural BC. It should be emphasized that the set of variables proposed above is not the only possible choice. For example, as components of the vector of variables one can use a combined set of parameters $Z(t) \in R^n$, namely: pressure time histories at u $(u \leq l)$ GDS outlets and outlet boundaries s $(s \leq k)$ of associated branches, i.e. $(z_i(t),\ i = 1, \ldots, u + s)$; time histories of flow rates at $(l - u)$ GDS outlet and outlet boundaries of $(k - s)$ associated branches, i.e. $(z_i(t),\ i = u + s + 1, \ldots, n,\ n = k + l)$.

As noted above, the time interval of interest in numerical simulations of problem (1) is divided into time steps, $0, \ldots, N_t$. In this case, through straightforward transformations provided that $(L = D = 2)$, problem (1) converts to the equivalent discrete form:

$$\begin{cases} \sum_{m=0}^{N_t} \sqrt{\sum_{i=1}^{M_{\rm IP}} |p_{\rm calc}[\vec{\mathbf{X}}(t_m)] - p_{\rm meas}^{const}(t_m)|_i^2} \to \min \text{ subject to} \\ \vec{\mathbf{X}}(t) \in \Omega^* = \Bigg\{ \vec{\mathbf{X}}(t_m) \in R^n : \vec{\mathbf{a}}(t_m) \le \vec{\mathbf{X}}(t_m) \le \vec{\mathbf{b}}(t_m),\ m = \overline{0, N_t}; \\ \sum_{m=0}^{N_t} \sqrt{\sum_{j=1}^{l} |q_{\rm calc}^{\rm GDS}[\vec{\mathbf{X}}(t_m)] - q_{\rm meas\_GDS}^{const}(t_m)|_j^2} \le \varepsilon' \Bigg\}, \end{cases} \tag{3}$$

where $\varepsilon' = \varepsilon / \Delta t$, $\varepsilon' \sim \varepsilon$.

It does not seem possible to solve problems (3) in such a statement using computing facilities available to a wide range of gas industry specialists. It is therefore reasonable to extend the list of transient IGF setting criteria by introducing an additional physically and mathematically consistent requirement that, when defining the IGF, one should try to ensure that the discrepancy between computational and measured pressure time histories should be as small as possible at each time step (time slice). With computing facilities currently available to specialists of gas distribution companies, this requirement can be satisfied if the assumption that gas flow processes are quasi-steady-state is supposed to hold true within one time step, into which the specified time interval ΔT is divided.

Thus, provided that the above requirement is satisfied and the assumption that gas dynamic processes are quasi-steady-state within one time step is true, the mathematically consistent search for the local minimum in problem (3) reduces to a successive search of quasi-steady-state IGFs at each time slice by solving conditional optimization problems (the first computational scenario "Transient IGF Definition Subject to Corporate Trust in GDSs"):

$$\begin{cases} \sqrt{\sum_{i=1}^{M_{\rm IP}} |p_{\rm calc}(\vec{\mathbf{X}}) - p_{\rm meas}^{const}|_i^2} \to \min \text{ subject to} \\ \vec{\mathbf{X}} \in \Omega = \Bigg\{ \vec{\mathbf{X}} \in R^n : \vec{\mathbf{a}} \le \vec{\mathbf{X}} \le \vec{\mathbf{b}}; \sqrt{\sum_{j=1}^{l} |q_{\rm calc}^{\rm GDS}(\vec{\mathbf{X}}) - q_{\rm meas\_GDS}^{const}|_j^2} - \varepsilon \le 0 \Bigg\}, \end{cases} \tag{4}$$

for example, using the modified Lagrange function method [4].

The second computational scenario "Transient IGF Definition Subject to Corporate Trust in Customers" is similar to the first scenario in the underlying reasoning. In this case, however, instead of conditional optimization problem (4), the following sub-problems will be solved at each time slice:

$$\begin{cases} \sqrt{\sum_{i=1}^{M_{\rm IP}} |p_{\rm calc}(\vec{\mathbf{Y}}) - p_{\rm meas}^{const}|_i^2} \to \min \text{ subject to} \\ \vec{\mathbf{Y}} \in \Upsilon = \Bigg\{ \vec{\mathbf{Y}} \in R^n : \vec{\mathbf{c}} \le \vec{\mathbf{Y}} \le \vec{\mathbf{d}}; \sqrt{\sum_{j=1}^{k} |q_{\rm calc}^{\rm Consumer}(\vec{\mathbf{Y}}) - q_{\rm meas\_Consumer}^{const}|_j^2} - \varepsilon \le 0 \Bigg\}, \end{cases} \tag{5}$$

where $\mathbf{c} \in R^n$ and $\mathbf{d} \in R^n$ are specified vectors establishing the boundaries in simple constraints on the range of admissible variations of the vector of independent controlled variables, $\mathbf{0} < \mathbf{c} < \mathbf{d}$; $\mathbf{Y} \in R^n$ is the vector of controlled variables; $[q_{\text{calc}}^{\text{Consumer}}(Y)]_j$, $q_{\text{calc}}^{\text{Consumer}} \in R^k$, is the calculated gas mass flow rate at the outlet of the j-th associated branch, $j = 1, \ldots, k$ (these values are calculated using the CFD simulator); $[q_{\text{meas\_Consumer}}^{const}]_j$, $q_{\text{meas\_Consumer}}^{const} \in R^k$ is the reported gas mass flow rate at the outlet of the j-th associated branch, $j = 1, \ldots, k$.

The third computational scenario "No-Trust Transient IGF Definition" is the last scenario in the set. In its structure, it is similar to the first and second scenario, and is largely their combination. Accordingly, in this scenario, instead of conditional optimization problems (4) and (5), the following sub-problems will be solved at each time slice,

$$\sum_{i=1}^{l}[q_{\text{meas\_GDS}}^{const}]_i - \varepsilon \geq \sum_{j=1}^{k}[q_{\text{meas\_Consumer}}^{const}]_j + \varepsilon \ :$$

$$\begin{cases} \sqrt{\sum\limits_{i=1}^{M_{\text{IP}}} |p_{\text{calc}}(\vec{\mathbf{Z}}) - p_{\text{meas}}^{const}|_i^2} \to \min \ \text{subject to} \\ \vec{\mathbf{Z}} \in \Lambda = \Big\{ \vec{\mathbf{Z}} \in R^n : \vec{\mathbf{g}} \leq \vec{\mathbf{Z}} \leq \vec{\mathbf{f}}; \sum\limits_{j=1}^{k} [q_{\text{meas\_Consumer}}^{const}]_j - \sum\limits_{i=1}^{u} [q_{\text{calc}}^{\text{GDS}}(\vec{\mathbf{Z}})]_i - \\ - \sum\limits_{i=u+1}^{l} [q_{\text{meas\_GDS}}^{const}]_i + \varepsilon < 0; \sum\limits_{i=1}^{u} [q_{\text{calc}}^{\text{GDS}}(\vec{\mathbf{Z}}) - q_{\text{meas\_GDS}}^{const}]_i + \varepsilon < 0 \Big\}, \end{cases} \tag{6}$$

$$\sum_{i=1}^{l}[q_{\text{meas\_GDS}}^{const}]_i - \varepsilon \geq \sum_{j=1}^{k}[q_{\text{meas\_Consumer}}^{const}]_j + \varepsilon \ :$$

$$\text{as} \begin{cases} \sqrt{\sum\limits_{i=1}^{M_{\text{IP}}} |p_{\text{calc}}(\vec{\mathbf{Z}}) - p_{\text{meas}}^{const}|_i^2} \to \min \ \text{subject to} \\ \vec{\mathbf{Z}} \in \Lambda = \Big\{ \vec{\mathbf{Z}} \in R^n : \vec{\mathbf{g}} \leq \vec{\mathbf{Z}} \leq \vec{\mathbf{f}}; \sum\limits_{j=1}^{k} [q_{\text{meas\_Consumer}}^{const}]_j - \sum\limits_{i=1}^{u} [q_{\text{calc}}^{\text{GDS}}(\vec{\mathbf{Z}})]_i - \\ - \sum\limits_{i=u+1}^{l} [q_{\text{meas\_GDS}}^{const}]_i + \varepsilon < 0; \sum\limits_{i=1}^{u} [q_{\text{calc}}^{\text{GDS}}(\vec{\mathbf{Z}}) - q_{\text{meas\_GDS}}^{const}]_i + \varepsilon < 0 \Big\}, \end{cases} \tag{7}$$

where $\mathbf{g} \in R^n$ and $\mathbf{f} \in R^n$ are specified vectors establishing the boundaries in simple constraints on the range of admissible variations of the vector of independent controlled variables.

One should emphasize that the set of three computational scenarios presented above in fact covers almost all practically significant IGF search problem statements.

In this case, the *Basic Identified Gas Flow* (BIGF) is chosen among the three resulting IGFs obtained as a result of running each of the scenarios.

3. **Evaluation of Identification**

As a criterion in setting up a BIGF for a given time interval within the technique of transient analysis of gas supply discrepancies we use the achievement by the index of gas flow parameter identification level in the whole pipeline network of interest *P_Ident* of its highest possible value. In addition, this index should grow, as corresponding calculated and measured time histories of physical gas flow parameters in the pipeline system of interest get closer.

In order to establish the values of the index *P_Ident* at each IP, the proposed technique prescribes numerical analysis of closeness between corresponding calculated and measured time histories. Closeness is considered here in three senses, namely: closeness of the quality of two functional relationships; closeness of two functional relationships in time-weighted average metrics defined using the octahedral (L_1) or Euclidean (L_2) norm; closeness of two functional relationships within the framework of their uniform deviation (i.e. closeness to the metrics based on the cubic norm (L_0)). In the first sense, closeness between two (measured and calculated) functional relationships of gas flow parameters vs. time is evaluated based on the fraction of sign coincidences of their partial time derivatives during the time interval of interest.

4. **Accounting for Gas Accumulation**

Transition to solving the problem at each time slice in the quasi-steady-state setup requires developing a procedure to account for gas accumulation in the pipelines of the gas distribution system under control of the gas seller. Let us consider one of the possible ways to account for such effects using field measurement data.

At present, the seller usually receives measured pressure time histories in the common collector. In addition, gas temperature data are supplied from a number of GDSs. If corresponding measured values of gas pressure and temperature in the collector are averaged for each time interval under consideration, one can approximately estimate the quality (volume or mass) of gas $q_{\text{accumulation}}(t)$ accumulated in the pipelines of the pipeline system of interest based on the known conditions of heat exchange with the environment. This allows us to plot computational and experimental estimates of the rate of gas accumulation change.

Let us denote the rate of gas accumulation change by

$$\vartheta_{q\_\text{accumulation}}(t) = \partial q_{\text{accumulation}}(t)/\partial t\,, \tag{8}$$

where $\vartheta_{q\_\text{accumulation}}(t) > 0$ means that gas is accumulated in pipelines of the gas distribution system. Then, subject to complete trust in the supplier, one can plot the so-called function of actual gas consumption from the gas distribution system that will correspond to the difference between the total gas supply from all GDSs and the function of gas accumulation change in gas distribution pipelines:

$$\Theta_{\text{RC}}(t) = \sum_{i=1}^{M_{\text{GDS}}} [Q_{\text{meas}}^{\text{GDS}}(t)]_i - \vartheta_{q\_\text{accumulation}}(t), \tag{9}$$

where M_{GDS} is the number of GDSs actually supplying gas to the gas distribution network during the time interval of interest.

In accordance with Section 2, for each time slice, we solve an identification problem in the quasi-steady-state definition with the gas quantity entering the pipeline network of interest being equal to the gas quantity consumed from it. Actual variation of gas accumulation in network pipelines may violate this condition and result in a lower identification level and supply/consumption of fictitious quantities from associated branches. This will inevitably decrease the level of credibility in detecting the source of discrepancy and estimating its strength.

It is therefore reasonable to correct initial data of GDS outlet flow meters, which are used as boundary conditions, or sources of defining the boundaries of admissible solution ranges for subproblems from Section 3 at each time slice of simulations using the following formula:

$$[Q_{\mathrm{corr}}^{\mathrm{GDS}}(t_j)]_k = \Theta_{\mathrm{RC}}(t_j)\left\{\sum_{i=1}^{M_{\mathrm{GDS}}}[Q_{\mathrm{meas}}^{\mathrm{GDS}}(t_j)]_i\right\}^{-1}[Q_{\mathrm{meas}}^{\mathrm{GDS}}(t_j)]_k, \quad k=\overline{1,M_{\mathrm{GDS}}}. \tag{10}$$

Note that the introduction of corrected GDS flow rates generally results in a certain mismatch with corresponding gas pressure measurements used for identification. In order to minimize the negative effect of the resulting mismatch, one can introduce a correction factor, ψ, $0 \le \psi \le 1$ which is chosen empirically and specified a priory during transient discrepancy monitoring. As a result, formula (10) takes the form (see (9)):

$$[Q_{\mathrm{corr}}^{\mathrm{GDS}}(t_j)]_k = \left(1-\psi\vartheta_{q\_\mathrm{accumulation}}(t)\left\{\sum_{i=1}^{M_{\mathrm{GDS}}}[Q_{\mathrm{meas}}^{\mathrm{GDS}}(t_j)]_i\right\}^{-1}\right)[Q_{\mathrm{meas}}^{\mathrm{GDS}}(t_j)]_k, \tag{11}$$

where $k=\overline{1,M_{\mathrm{GDS}}}$. For $\psi=0$, search for the sources of discrepancy is performed subject to complete trust in GDS data, while for $\psi=1$, simulations are performed based on the actual gas consumption. All intermediate cases are intended to increase the identification level by reducing the negative influence of the mismatch between corrected flow rates and field measurements of gas pressure at GDS outlets.

5. **Conclusion**

Over the period of 2008-2011, this method of numerical monitoring of the supplier share in gas deliveries has demonstrated its efficiency as applied to the production simulations of Mosregiongaz for the analysis of the mechanisms of discrepancy occurrence in the natural gas supplies through Moscow Ring Gas Distribution Pipeline System (MRGDPS) (Figure 1). The numerical monitoring of natural gas distribution discrepancy at Gazprom Company made it possible to reduce discrepancy in MRGDPS on more than 30%. The method and CAS can be fully computerized based on ordinary computers available to gas industry specialists.

Figure 1. Numerical monitoring of natural gas transmission along MRGDPS (an example of CAS application in Gazprom regiongaz Moscow Control Room)

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