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Research Article

Variable Viscosity of Casson Fluid Flow Over A Stretching Sheet in Porous Media with Newtonian Heating

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Abstract. Casson fluid flow with variable viscosity in porous media over a heated stretching sheet is investigated. The partial differential equations representing the flow motion are first transformed to ordinary differential equations by similarity transformation before being solved numerically by the finite-difference method. The effects of the viscosity variation parameter (Ω), the permeability number (κ), Prandtl number (Pr), Biot number (Bi) and non-Newtonian fluid parameter (β) on the fluid flow and heat transfer, along with the temperature and velocity profiles, are presented graphically for some arbitrary values.

Keywords. Casson fluid; Stretching sheet; Newtonian heating; Variable viscosity

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1. Introduction

Industrially, the stretching process and heat transfer rate are important factors in determining the quality of finished products. Such a situation arises in the manufacturing of plastic and rubber sheets, glass blowing, coating and spinning of fibres, extrusion processes and others.

Crane [10] was the first to consider momentum boundary layer of linear stretching sheet, which was later extended by other researchers for various cases, eg. Chiam [8], McLeod and Rajagopal [21], Vajravelu and Rollins [34], Rao [27], Cortell [9], Mahmoud [17], Ahmad *et al*. [2], Rasekh *et al*. [28], Mastroberardino [20], Ahmad and Wahid [3]. Recently, Reza *et al*. [29] studied the boundary layer flow and heat transfer of Casson fluid over a nonlinearly permeable stretching sheet with chemical reaction in the presence of a variable magnetic field, and Narayana *et al*. [25] discussed the combined effects of Soret and Dufour on MHD boundary layer flow of Jeffrey fluid past a stretching sheet with chemical reaction and heat source, to name a few.

All the above mentioned studies dealt with prescribed surface temperature or prescribed heat flux. It is worth mentioning that there is another class of convective flow, heat and mass transfer problem where the surface heat transfer depends on the surface temperature (Salleh *et al*. [30]). This phenomenon occurs when heat is transported to the convective fluid through a bounding surface with finite heat capacity, which is known as Newtonian heating. In general, there are four common heating processes; i.e. wall-to-ambient temperature distribution, prescribed surface heat flux, conjugate conditions and finite heat capacity, and Newtonian heating (Chaudhary and Jain [6]). Newtonian heating processes arise in several important engineering devises, such as in the heat exchanger (Merkin *et al*. [23]) and around fins (Chaudhary and Jain [7], Salleh *et al*. [30]), which later evoked the need for such a study. Mohamed *et al*. [17] presented a numerical solution for stagnation point flow over a stretching surface generated by Newtonian heating; Uddin *et al*. [32] investigated laminar boundary layer slip flow from a stretching surface in a nanofluid-saturated homogeneous, isotropic porous medium; and, recently, Uma Devi and Anjali Devi [33] compared the heat transfer characteristics of traditional nanofluid with that of emerging hybrid nanofluid over a stretching surface subject to Newtonian heating; Lavanya *et al*. [15] investigated the phenomenon of Newtonian heating under the application of a uniform porous medium when heat generation and chemical reaction appears in the energy and volumetric species equations in the flow of nanofluid; and, Ahmad *et al*. [1] studied the effects of Newtonian heating on MHD Casson nanofluid flow passing a wedge.

Lots of studies have been dedicated to constant physical properties of fluid. However, it is known that the physical properties of fluid may change significantly with temperature (Herwig and Gersten [12], Lai and Kulacki [13]). Studies by Gary *et al*. [11] and Mehta and Sood [22] have shown that the flow characteristics change substantially when the flow regime is subjected to variable viscosity to that of uniform viscosity. As such, to accurately predict flow behaviour, it is necessary to take the variation of viscosity and thermal conductivity into account. Makinde [18] presented an analysis for variable viscosity hydromagnetic boundary layer flow with thermal radiation and Newtonian heating and Olanrewaju *et al*. [26] studied the effects of variable viscosity and magnetic field on flow and heat transfer to a continuous flat plate in the presence of heat generation and radiation with a convective boundary condition. The study of Casson fluid flow with variable viscosity has been conducted by Lare [14] and Malik *et al*. [19].

However, their works were limited to prescribed surface temperatures.

In view of all the above mentioned works and keeping in mind the significant effects of fluid viscosity, the objective of this study is to investigate Casson fluid flow over a stretching sheet in porous media subject to Newtonian heating and variable viscosity. Based on our close observation, such a study has never been considered before.

2. Problem Formulation

Consider a steady two-dimensional boundary layer flow of Casson fluid with variable viscosity flowing over a stretching sheet embedded in porous medium with a permeability κ_1. The plate is heated due to Newtonian heating with heat transfer coefficient h_s. Under the usual boundary layer approximations, the governing equations that represent the problem under consideration are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\left(1+\frac{1}{\beta}\right)\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\mu}{\rho\kappa_1}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

along with boundary conditions:

$$\begin{aligned}&u=cx,\quad v=0,\quad \frac{\partial T}{\partial y}=-h_sT\ (\text{NH})\ \text{ at }\ y=0,\\ &u\to 0,\quad T\to T_\infty\quad \text{as}\quad y\to\infty.\end{aligned} \tag{4}$$

where u and v are the velocity components in x and y directions, respectively. ρ is the density of the fluid, β is the non-Newtonian (Casson) fluid parameter, T is the fluid temperature, α is the thermal diffusivity and $c>0$ is the stretching constant.

The fluid viscosity μ is assumed to vary as an inverse linear function of temperature, given by (Ling and Dybbs [16], Lai and Kulacki [13], Ali [4], and Makinde [18])

$$\mu=\frac{\mu_\infty}{[1+\gamma(T-T_\infty)]} \tag{5}$$

where $\mu_\infty$is cold fluid viscosity and γ is constant.

The stream function ψ is defined in usual fashion, i.e.

$$u=\frac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\frac{\partial\psi}{\partial x} \tag{6}$$

and the similarity solution of eqs. (2)–(6) is given by

$$\psi=\sqrt{c\,v}\,xf(\eta),\quad \eta=y\sqrt{\frac{c}{v}}y,\quad \theta(\eta)=\frac{T-T_\infty}{T_\infty}. \tag{7}$$

Utilizing (5)-(7) into Eqs. (2)-(4), we obtain

$$\left(1+\frac{1}{\beta}\right)[(1+\Omega\theta)f''' - \Omega f''\theta'] - (1+\Omega\theta)^2[(f')^2 - ff''] - \kappa(1+\Omega\theta)f' = 0, \tag{8}$$

$$\theta'' + Pr\, f\theta' = 0, \tag{9}$$

together with the boundary conditions

$$\begin{aligned} &f(\eta)=0, \quad f'(\eta)=0, \quad \theta'(\eta)=-Bi[1+\theta(0)] \quad \text{at} \quad \eta=0,\\ &f'(\eta)\to 1, \quad \theta(\eta)\to 0 \quad \text{as} \quad \eta\to\infty, \end{aligned} \tag{10}$$

where prime denotes differentiation with respect to η, $\Omega = \gamma T_\infty$ is the viscosity variation parameter, $\kappa = \frac{\upsilon}{\kappa_1 c}$ is the permeability number, $Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number and $Bi = h_s\sqrt{\frac{\upsilon}{c}}$ is the conjugate parameter for the Newtonian heating. It is worth mentioning that when $\Omega = \kappa = 0$, Eq. (8) subject to (10) reduced to Newtonian fluid flow, which gives closed form solution

$$f(\eta) = 1 - e^{-\eta}. \tag{11}$$

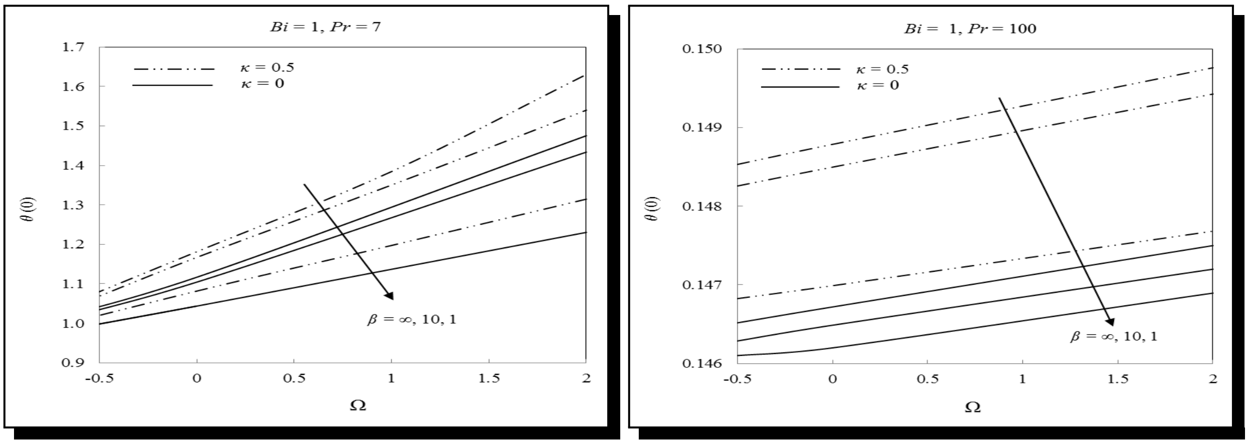
3. Results and Discussion

Eqs. (8)-(10) have been solved by implementing the method introduced by Cebeci and Bradshaw (1988) for several values of viscosity variation parameter (Ω), permeability number (κ), Prandtl number (Pr), Biot number (Bi) and non-Newtonian fluid parameter (β). It should be pointed out that $\Omega > 0$ represents a decrease in fluid viscosity (Makinde [18]). The influence of Ω, κ, Pr, Bi and β on the fluid flow and thermal fields are presented in form of graphs. Numerical results obtained for $\theta(0)$ when $\Omega = \kappa = 0$ and $\beta = \infty$ for several values of Pr and Bi numbers are compared with Sarif *et al*. [31] and found to be satisfactory, as tabulated in Table 1.

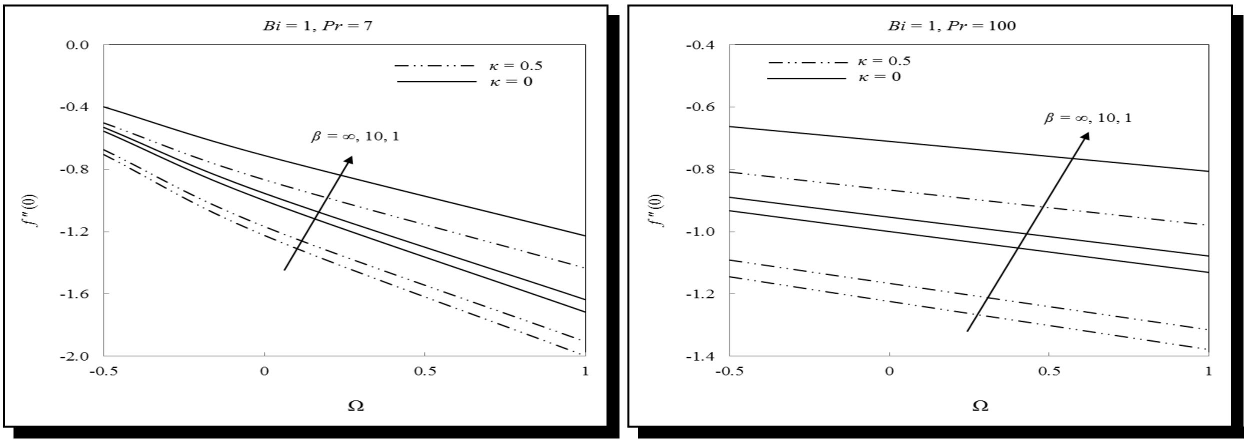
Table 1. Values of $\theta(0)$ obtained for various values of Pr and Bi numbers. Those in ( ) are results from Sarif *et al*. [31]

Pr	$Bi = 0.5$	$Bi = 2$
7	0.3587 (0.35835)	- -
10	0.2763 (0.27658)	6.5024 (6.50263)
100	0.0690 (0.06875)	0.3466 (0.34690)

Figure 1 shows the effects of the viscosity parameter Ω on the (a) surface temperature $\theta(0)$ and (b) the surface shear stress $f''(0)$ when $Bi = 1$, $Pr = 7$ and 100 with permeability parameter $\kappa = 0$ and 0.5, respectively. It is seen that, as viscosity decreases ($\Omega > 0$), surface temperature $\theta(0)$ increases and surface shear stress $f''(0)$ decreases. The effects of Casson fluid β is seen to decrease $\theta(0)$ and increase surface shear stress $f'(0)$. The increment of the Pr number is found to decrease both the surface temperature $\theta(0)$ and surface shear stress $f''(0)$. However, as Ω increases, the effects of the Pr number on the surface temperature $\theta(0)$ is less profound for $Pr = 100$ in comparison with $Pr = 7$ due to small variations of temperature. Nevertheless, the surface temperature $\theta(0)$ (Figure 1a) is found to be higher when the fluid is embedded in porous media, given by the parameter κ, which is in contrast to the behaviour of the surface shear stress $f''(0)$ (Figure 1b).



(a)

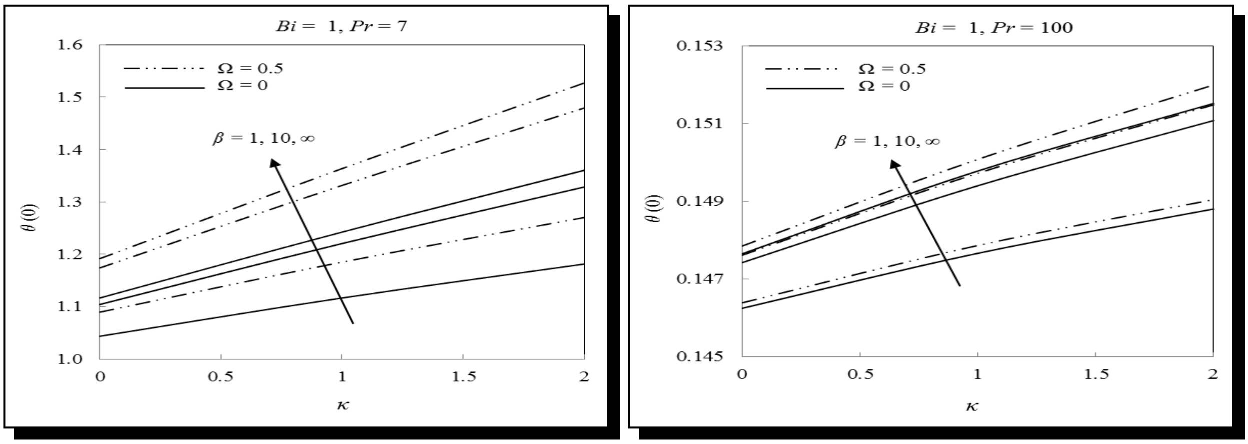


(b)

Figure 1. Values of (a) $\theta(0)$ and (b) $f''(0)$ for various values of Ω and β when $Bi = 1$, $\kappa = 0$ and 0.5 for $Pr = 7$ and 100, respectively

Figure 2 depicts the impact of the porosity κ for $Pr = 7$ and 100 when $\Omega = 0$ and 0.5, respectively, towards (a) the surface temperature $\theta(0)$ and (b) the surface shear stress $f''(0)$. The surface temperature $\theta(0)$ increases with the increase of porosity κ. An opposite trend is observed for the surface shear stress $f''(0)$. This is in line with the graph obtained in Figure 1. The increment of the surface temperature $\theta(0)$ is more pronounced when $Pr = 7$ as compared with $Pr = 100$. It is also noticed that the effects of the non-Newtonian fluid ($\beta \neq \infty$) is to decrease the surface temperature $\theta(0)$ and provides contrary characteristics towards surface shear stress

$f''(0)$. On top of that, the impact of Ω is said to increase surface temperature $\theta(0)$ and decrease surface shear stress $f''(0)$, which further validate the results obtained in Figure 1a. However, the influence of Ω is more outstanding towards surface temperature when $Pr = 7$ to that of $Pr = 100$. It should be highlighted that when $\Omega = 0$, the fluid viscosity is constant and eq. (8) is independent of Pr, which leads to identical skin friction coefficient $f''(0)$ obtained for $Pr = 7$ and $Pr = 100$ (Figure 2b, straight line).



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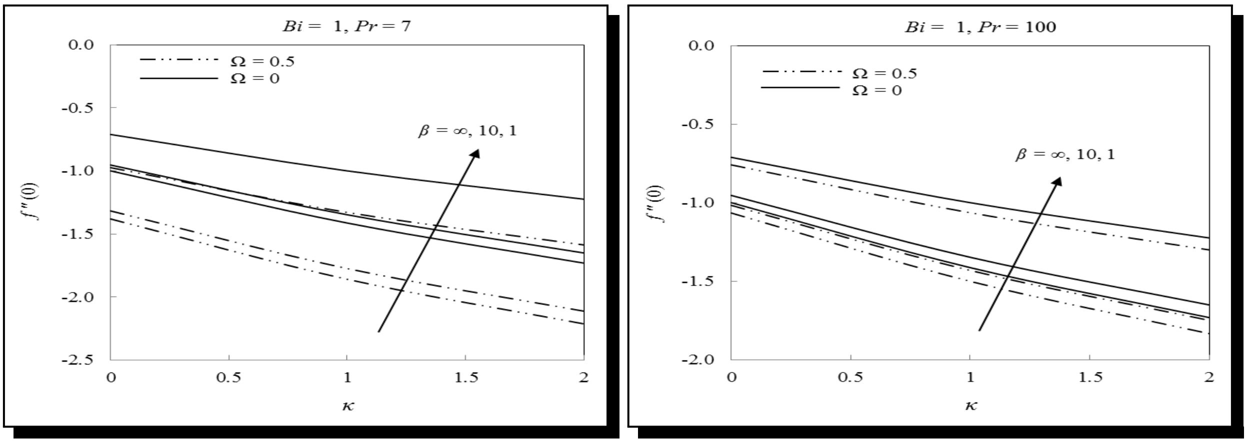
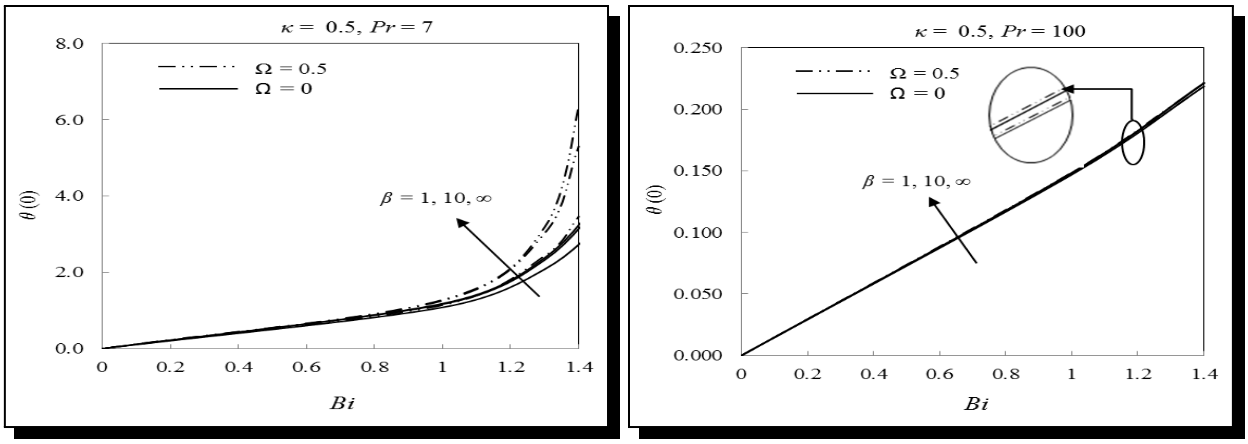


(b)

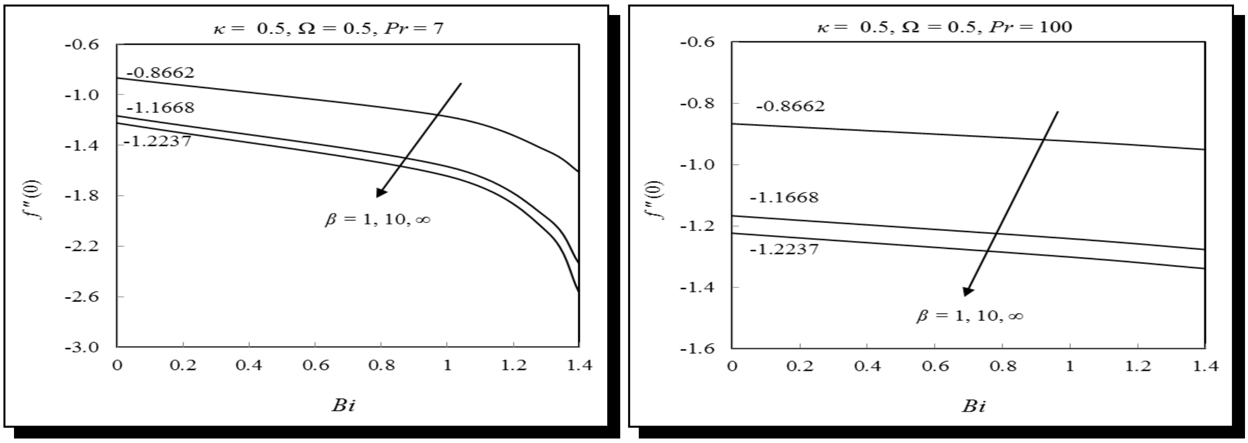
Figure 2. Values of (a) $\theta(0)$ and (b) $f''(0)$ for various values of κ and β when $Bi = 1$ and $\Omega = 0$ and 0.5 for $Pr = 7$ and 100, respectively.

Figure 3 displays the effects of Newtonian heating parameter given by Bi number. The increment of the Bi number is expected to increase surface temperature $\theta(0)$. The surface temperature forms a linear relation with the Bi number when $Pr = 100$. It is found that the effects of the viscosity variation Ω and the non-Newtonian fluid parameter β are less significant when $Pr = 100$ and when $Bi < 1$ for $Pr = 7$. As for $Bi > 0$ and $Pr = 7$, the increment of Ω is seen to increase surface temperature $\theta(0)$. Here also, the Newtonian fluid β is found to decrease surface temperature. Figure 3b is plotted to see the behaviour of surface shear stress $f''(0)$ as Bi

and β increase. For $Bi = 0$ and $\kappa = \Omega = 0.5$, surface shear stress $f''(0)$ are -0.8662, -1.1668 and -1.2237 for $\beta = 1, 2$ and ∞, respectively. These values decrease as Bi. However, in the absence of the viscosity variation Ω $(= 0)$ and $\kappa = 0.5$, the Bi and Pr numbers no longer affect surface shear stress $f''(0)$, which results in the fixed values of $f''(0)$; i.e. -0.8662, -1.1668 and -1.2237 for $\beta = 1, 2$ and ∞, respectively. Nevertheless, the influence of β is found to increase surface shear stress $f''(0)$.



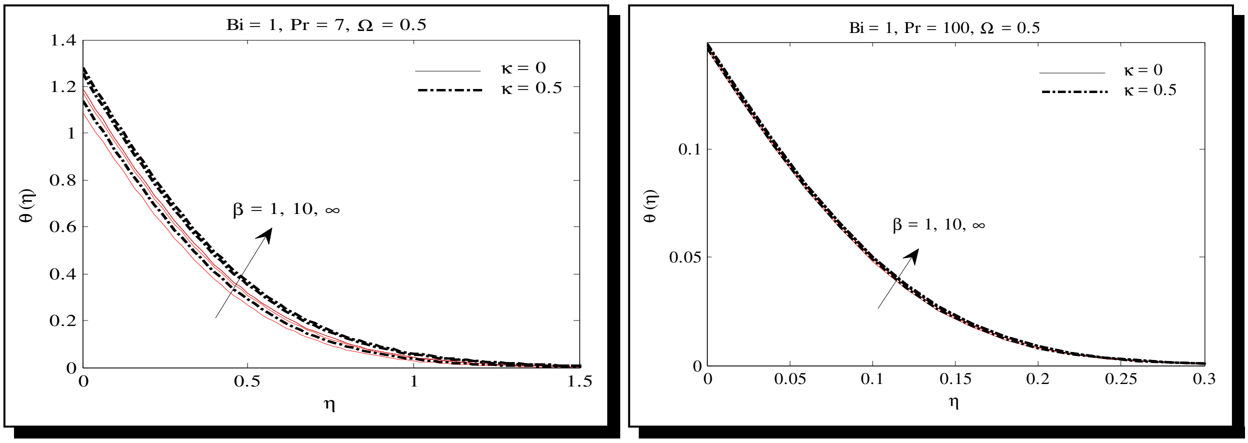
(a)



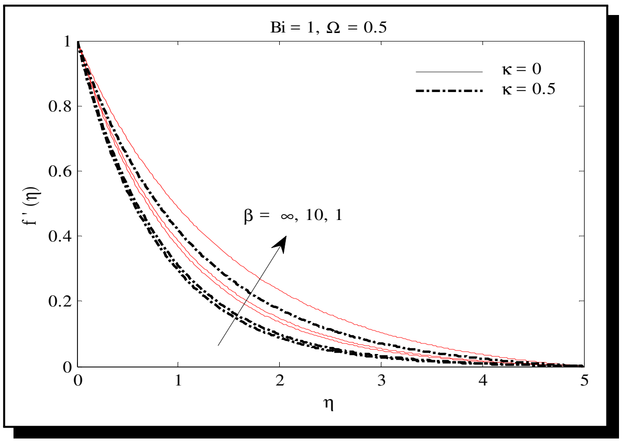
(b)

Figure 3. Values of (a) $\theta(0)$ and (b) $f''(0)$ for various values of Bi and β for $Pr = 7$ and 100, respectively.

The temperature and velocity distributions for various values of the non-Newtonian fluid parameter β when $Bi = 1$, $\Omega = 0.5$, the permeability number $\kappa = 0$ and 0.5, and $Pr = 7$ and 100, are displayed in Figure 4. As $Pr = 7$, temperature distribution θ (η) is enhanced as κ increases while the opposite behaviour is observed for velocity distribution $f'(\eta)$. Also, it is noted that there is a slight decrement in temperature distribution as the flow becomes more non-newtonian ($\beta \ll \infty$) and contrary behaviour is observed for the velocity profile $f'(\eta)$. As Pr increases $(= 100)$, the effects of κ and β are no longer remarkable in the distribution of temperature. On top of that, the distribution of the velocity remains unchanged for both $Pr = 7$ and 100. It is also noted that as Pr increases, both thermal boundary layer thickness and temperature distribution were reduced. This is because as Pr increases, the thermal diffusivity becomes weak, which creates a reduction in temperature distribution and thermal boundary layer thickness.



(a)



(b)

Figure 4. (a) Temperature and (b) velocity profiles for various values of β when $\Omega = 0.5$ and $\kappa = 0$ and 0.5.

Figure 5 illustrates the impact of the Bi and β values towards thermal and fluid flow. Increasing the Bi number means increasing the temperature. The non-Newtonian fluid β has a remarkable effect on temperature distribution for the higher value of the Bi number (= 1.4) when $Pr = 7$. Whereas for $Bi = 0.6$, the distribution of the temperature within the boundary layer is independent of the non-Newtonian parameter β. The same phenomenon occurred for $Pr = 100$, regardless of the Bi number; i.e. no impact of β for both values of Bi numbers. Increment of the Bi number decreases the velocity $f'(\eta)$ for $Pr = 7$ and the effects of the Bi number is no longer seen when $Pr = 100$. The existence of non-Newtonian fluid β increases the velocity.

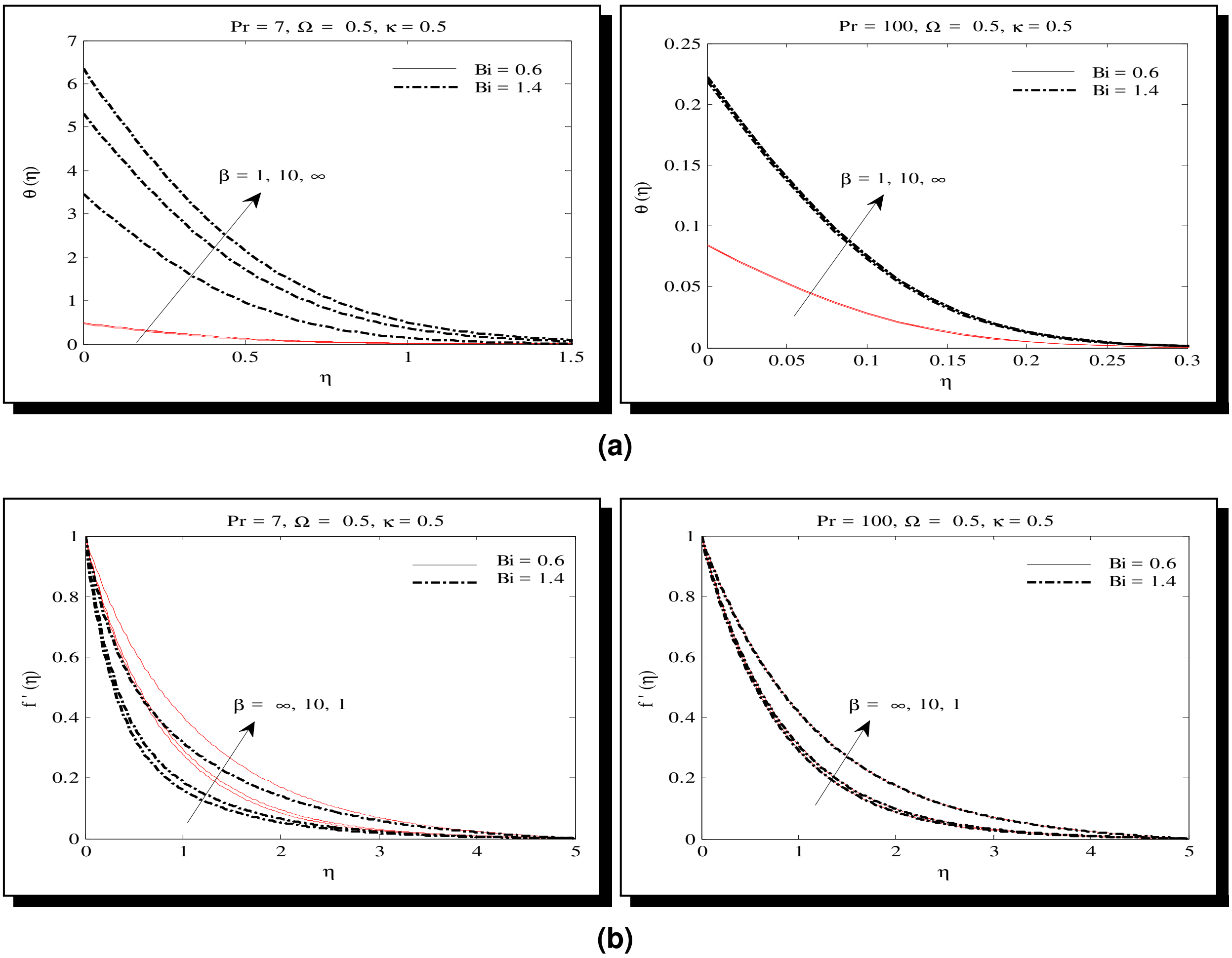


Figure 5. (a) Temperature and (b) velocity profiles for various values of β and Bi number when $\Omega = \kappa = 0.5$.

4. Conclusions

The effect of variable viscosity in porous media over a heated stretching sheet in Casson fluid was investigated for few values of viscosity variation parameter (Ω), the permeability number (κ), Biot number (Bi) and non-Newtonian fluid parameter (β). Only two values of Pr number were taken into consideration; i.e. $Pr = 7$ and 100. Numerical output showed that as viscosity decreases ($\Omega \gg 0$), the surface temperature $\theta(0)$ increases and the surface shear stress $f''(0)$ decreases. The same trend was observed when the sheet being embedded in porous media. Increment values of Pr from 7 to 100 is seen to give less impact towards the surface temperature $\theta(0)$.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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