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The Non-existence of Extremal Objects of Set Theory and the Continuum Problem

Mikhail Valentinovich Antipov

Abstract. It is proved that the classical extremal objects of set theory are non-existent. It is also proved that the concepts of uncountability and continuum are erroneous and the first Hilbert problem is incorrect. The infinity axiom and all unlimited objects are unfounded as well.

1. Introduction

The mathematical insufficiency of the axiom of boundlessness can not be with indifference to the cognition. Complicated constructions of the idealized imagination need in more intend attention. However, the convention of the existence of the phantom of unlimited countability leads to conclusion about the mistaken creation of objects of the highest capacity. The incontestable attributes testify about heavy times is settled in sphere of fundamental scientific researches. Mathematics has not avoided first of all from their. And though such opinion is not supported number of the leading experts convinced in temporal difficulties and natural illness of fast growth, others are capable to note deepening of crisis. Patent means of treatment obviously only drive illness deep into and it is impossible to hope that former diligent development and the continuation travel all over ideas and cognition $\mathscr{PL} \Rightarrow \mathscr{PL}^{\|\infty\|}$ will bring qualitative changes.

In result the $SI_{TT}^{\|\infty\|}$ (System of Idealized Theories) was created, and it is absolute from antiquity to our time. In spite of sharp opposition of system, the other basis (principle PR) and concept of system SA (System of Adequacy) suggested in works [1–7]. This new system SA foresees the foundation of formed knowledge about real world.

Definition 1.1. The infinity axiom $Ax^{\|\infty\|}$ as initial thesis of system $SI_{TT}^{\|\infty\|}$ and scientific cognition $\mathscr{PL}^{\|\infty\|}$ contains:

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(I) The postulate of boundless of the surrounding world, its objects and characteristics.
(II) The permission for cognition to use elements, objects and algorithms as image and unlimited.
(III) The declaration is as founded proofs, which constructive and ineradicable use idea and model of infinity.

Some rules, constructions and conclusions of the set theory are obliged to involve intent interest – though because of sharp paradoxity. However the main attention needs to be paid to the continuum problem. Yet earlier it should realize that such infinity in its actual and potential form. In turn this last enormous mystery for which so far is not present of the intelligible answer is obliged to generate a next chain of puzzled questions. Here problem of statements proveness and thus foundation of cognition is considered also. In a result the offered series not very successfully permitted or even absolutely avoided problems is closed in concept of foundation. The accumulated knowledge turn into entertaining training of a thought outside of this concept.

Theorem 1.2. *If infinity axiom $Ax^{\|\infty\|}$ do not corresponds to the reality RR, non-existence of complicated (second order and higher) formations $\Omega^{\|\infty\|}$ of infinity quantifier $\forall^{\|\infty\|}$ is obliged to reveal by the system $SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|})$ and its methods.*

$$\{Ax^{\|\infty\|} \Rightarrow SI_{TT}^{\|\infty\|}(\forall^{\|\infty\|})\} \not\Rightarrow RR : \Omega^{\|\infty\|}\{\|\infty\|, \|\infty\|\} \xrightarrow{SI_{TT}^{\|\infty\|}} \|\bar{\&}\|(\Omega^{\|\infty\|}). \tag{1.1}$$

Proof. We shall consider real object Ω^{RR} of cognition. It can belong even to zone of imagination but can not go out from dynamic observed reality *RR*. We shall present polyextremal model $\Omega^{\|\infty\|}\{\|\infty\|, \|\infty\|\}$ only for idealized system as allowable

$$SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|}, \Omega^{\|\infty\|}) : \lim_{n\to\infty} \lim_{m\to\infty} (\Omega^{\|n\|})^{\|m\|} \xrightarrow{SI_{TT}^{\|\infty\|}} \begin{cases} \emptyset \Rightarrow \|\bar{\&}\|(\Omega^{\|\infty\|}), \\ (\Omega^{\|\infty\|})^{\|\infty\|} \equiv \Omega^{\|\infty\|}. \end{cases} \tag{1.2}$$

Inevitable repeated limit (1.2) at transition from the real forms of a kind $\Omega^{\|n\|}$ to polyextremal transforms to impossibility of such formation, i.e. falsehood $\|\bar{\&}\|(\Omega^{\|\infty\|})$ or coincidence with former monoextremal form. It follows from estimation unreality of results of the repeated limit as and unary limit is uncontrollable. There is consideration about non-existence of objects $\Omega^{\|\infty\|}$ further. But from (1.2) follows that the illusiveness of objects $\Omega^{\|\infty\|}$ if it really takes place, is obliged to be reflected in falsehood of constructions proved creation of polyextremal objects. Such effect should be fixed by itself system $SI_{TT}^{\|\infty\|}$ and obstacles for this can not present even logic $Lg^{\|\infty\|}$. From here there are an expression (1.1) about falsehood of complicated cognition formations follows. It is possible to compare statement and theorems of [1–7, 11, 12]. Really, if laws $\mathscr{Z}\mathscr{Z}^{\|\infty\|}(\Omega_0^{\|\infty\|}, \Omega_1^{\|\infty\|})$, which are connect fixed objects, are obtained not as a result of limit's transition (and this is so owing to the status of objects), they are not only non-existent, but also are absolutely arbitrary. In that case, at their creation were admitted any (false) methods or direct mathematical errors. The system SA can assert that sensible logic Lg^{RR} is obliged to reveal these sins. Somewhat unusual views for odious infinity axiom $Ax^{\|\infty\|}$ and foundation of the acting cognition system $SI_{TT}^{\|\infty\|}$ require of deep attention. The mathematization of modern knowledge produces opportunity for this purpose. The particular attention deserves the set theory as the fundamental direction of scientific cognition and especially mathematics. □

Theorem 1.1 permits to pay attention to blunders of classical set theory.

2. **Enumerable and non-enumerable sets**

The set theory should be consider as first scientific direction for immediate study of infinity characteristics. Ancient set of natural numbers should be consider as initial unlimited objects. Later scientific cognition tried to investigate indefinable phenomenon.

The enumeration concept causes number of questions. But only the representations about infinite enumeration and unlimited aspiration with meant boundless resources fill objects by properties of (idealized) suitability for further study. The concept of infinite enumeration in a result of process of closure and has determined fixed set of cardinality $\aleph_0$. Invariability of this set is given by concept of actual infinity.

However Cantor has offered the proof of existence of non-enumerable objects and sets. According to his proposals such continuum-sets have cardinality $\aleph_1 = 2^{\aleph_0}$. Here it is necessary to stay. If to take into account that being inside the idealized system and the concept of infinity axiom it is impossible to refute or to confirm existence of objects of the infinite characteristics, the Cantorian theorem [8] about non-enumerable sets deserves special attention. It is in need of analysis according to Theorem 1.1.

Theorem 2.1. *The Cantorian proof of existence of sets of non-enumerable numbers $\mathcal{N}(\aleph_1)$ in unit interval $[0;1]$ contains the initial logic contradiction it means direct mistake.*

$$SI_{TT}^{\|\infty\|} : \{Tt^{\|\infty\|}(C) \subset Doc^{\|\infty\|}, \mathscr{M}^{\|\infty\|}, Ax^{\|\infty\|}\} \overset{Lg}{\Longrightarrow} \|\bar{\&}\|\{Tt^{\|\infty\|}(C)\}, \qquad (2.1)$$

and this statement is conducted in acting system $SI_{TT}^{\|\infty\|}$. Cantor's theorem designated here as $Tt^{\|\infty\|}(C)$ proven with the help of environment $Doc^{\|\infty\|}$, method $\mathscr{M}^{\|\infty\|}$ and axiom $Ax^{\|\infty\|}$. But logic analysis (Lg) results in a conclusion about erroneous ($\|\bar{\&}\|$) initial construction and consequently the Cantorian theorem.

Proof. Cantorian theorem about existence of points of unit interval which not included in enumerable sets, should be considered as one of central in a modern system $SI_{TT}^{\|\infty\|}$ with its ineradicable support of infinity axiom $Ax^{\|\infty\|}$. Therefore irreproachableness of the theorem should be absolute within the framework of the system. The Cantorian proof is constructed for model of interval decomposition $[0;1]$, that is

$$Cantor: \quad \{n \geq 1, n \to \infty\} : [0,1] \Rightarrow \left[0, \frac{1}{3}\right] \cup \left[\frac{1}{3}, \frac{2}{3}\right] \cup \left[\frac{2}{3}, 1\right] \Rightarrow \dots$$
$$\Rightarrow \left[\frac{k}{3^{n-1}}, \frac{3k+1}{3^n}\right] \cup \left[\frac{3k+1}{3^n}, \frac{3k+2}{3^n}\right] \cup \left[\frac{3k+2}{3^n}, \frac{k+1}{3^{n-1}}\right] \Rightarrow \dots; \quad 0 \leq k < 3^{n-1}. \quad (2.2)$$

Indicated in work [8] decomposition contains insignificant feature actually playing decisive role. The construction of interval system (2.2) is actually not decomposition but the formation of an unit interval in which introduces an additional enumerable net of all rational numbers of three-signed representation $0.\gamma_1\gamma_2\gamma_3\dots\gamma_k\dots$, where $\gamma_k = \{0;1;2\}$. Rather the sharp sense of such violent entering is explained by separable function of the net.

It is confirmed according to idea of the proof on each step of representation (2.2) it is always possible to specify such which does not contain the any determined enumerable point x inside and in a border. Just the last circumstance is necessary to note especially as far as the additional points of intervals of decomposition turn such statement as indisputable. But at all inclusion of additional points in the unit interval does not look as indisputable. An unique interval which doesn't include the point (even in a border) is selected. On Cantorian idea just the points of this net should separate enumerate objects from those which are inaccessible to enumeration. In real decomposition of the interval $[0;1]$ such additional net is included by artificial construction and it does not exist. That's why all proof inspires with quite reasonable suspicion as incorrect.

If to take into account qualitative ineradicableness of the additional enumerable net from Cantorian construction (2.2) for the proof of existence of hypothetic numberless points then the whole Cantor's theorem in the given formulation and at given artificial construction should be recognized not proven and the method faulty (2.1). □

The significance of the Cantorian theorem for all subsequent constructions and conclusions is very high.It means that the less obliged defect is inadmissible. At the same time it is necessary to check up whether the Cantorian proof in force will be saved if to remove annoying omission with inclusion of the additional net.

Theorem 2.2. *The restoration of strict validity in Cantorian model of decomposition and reception of non-enumerable points* $\alpha \in [0;1)$ *excludes conclusion of Cantor* $Ww^{\|\infty\|}$. *System of cognition and mathematics loses of it's own superior basis.*

$$SI_{TT}^{\|\infty\|}: \quad \{Tt^{\|\infty\|}(C) \Rightarrow Tt^{\|\infty\|}(A)\} \overset{Lg}{\Longrightarrow} \nexists\{Ww^{\|\infty\|}[Tt^{\|\infty\|}(C)]\}, \quad (2.3)$$

where it is also given for the system $SI_{TT}^{\|\infty\|}$. Cantorian theorem designated as well as earlier $Tt^{\|\infty\|}(C)$ receives founded and incontrovertible modification of model construction $Tt^{\|\infty\|}(A)$ but in a result the logic analysis (Lg) leads to liquidation $(\nexists)$ of Cantorian conclusion $Ww^{\|\infty\|}$ about non-enumerable points of the half-interval.

Proof. We shall preserve Cantorian model of decomposition with one natural amendment. Namely, we shall consider the unit half-interval and it's decomposition we shall make for three-signed half-intervals instead Cantorian intervals.

$$Alter: \quad \{n \geq 1, n \to \infty\}: \ [0,1) \Rightarrow \left[0, \frac{1}{3}\right) \cup \left[\frac{1}{3}, \frac{2}{3}\right) \cup \left[\frac{2}{3}, 1\right) \Rightarrow \ldots$$
$$\Rightarrow \left[\frac{k}{3^{n-1}}, \frac{3k+1}{3^n}\right) \cup \left[\frac{3k+1}{3^n}, \frac{3k+2}{3^n}\right) \cup \left[\frac{3k+2}{3^n}, \frac{k+1}{3^{n-1}}\right) \Rightarrow \ldots;$$
$$0 \leq k < 3^{n-1}. \qquad (2.4)$$

Comparison of Cantorian representation (2.2) and decomposition $Alter$ shows that except strictness and absence of superfluous points in the second case in purposes of unification as initial unit half-interval $[0;1)$ instead interval $[0;1]$ as earlier is taken.

The secret sense of the famous Cantorian theorem reflects in two on naive equalities: $3-2=1$, $3-1=2$. To sorrow for all mathematics just in them the author has enclosed non-child's perfidious hidden motive. The analysis of representations (2.2, 2.4) is necessary for the explanatory.

Between two variants of decomposition such insignificant difference would seem. But it changes all reasons in a root. In the observed scheme $Alter$ (2.4) each decomposition determines equally two half-intervals in which researched point can not be include (in Cantorian model there will be one interval). Completely obviously, that in each of these half-intervals will be found the rational points. Than we receive growth of quantity of half-intervals (where precisely rational values are) also formalized classical but mysterious limit $\lim_{n\to\infty} 2^n \Rightarrow \{2^{\aleph_0}\}$ for any given point. The continuum of enumerable set of rational points follows from the scheme $Alter$. Such conclusion is odd should be refuted by the algorithm, but this algorithm was constructed by Cantor for the proof opposite. Cantor has introduced the additional enumerable net of interval borders, intended only for reception of uniqueness of selected chain of intervals. But the correct scheme $Alter$ (2.4) testifies about unfounded artificial Cantorian construction (2.2).

Really, as more logical and reasonable scheme demonstrates $Alter$ in construction $Cantor$ de facto one point is chosen from continuum quantity of enumerable and it is announced by non-enumerable which is belong to set of cardinality $\aleph_1$. Differently disguised comparison occurs of value α_0 of algorithm $ALG(\lim_{n\to\infty} 2^n)$ (as if ensuring cardinality $\aleph_1$) and some other value α_1 of algorithm $ALG_0(\lim_{n\to\infty} n)$ presenting to other set of cardinality $\aleph_0$. In a result the unreasonable conclusion that the point α_1 is non-enumerable and belongs to continuum set is formed. Obvious and non-refuted opportunities of selected algorithm are attributed illegally to points and numbers of the unit interval. The determining role of algorithm $ALG_k^{\|\infty\|}$ at formation of infinite sets together with the estimation of their capacity is reasonably clear already from indicated.

The doubtful scheme (2.2) becomes from acting actually equality of interval exception $3-1=2$ to illegally used equality $3-2=1$ which false conclusion provides. Thus, the strong-willed inclusion extraneous (superfluous) rational net of interval borders in the scheme $Cantor$ is given to separate enumerable points from some fantastic which was named irrational. In fact between rational and irrational points it is impossible *to insert* any more point though just such bogus point was found in the incorrect Cantorian construction (2.2). For final confirmation of received conclusion we shall determine, consider and reduce to classical limit the binary decomposition of the same half-interval.

$$Binar: \quad \{n \geq 1, \lim(n \to \infty)\}: \ [0,1) \Rightarrow \left[0, \frac{1}{2}\right) \cup \left[\frac{1}{2}, 1\right) \Rightarrow \ldots$$
$$\Rightarrow \left[\frac{k}{2^{n-1}}, \frac{2k+1}{2^n}\right) \cup \left[\frac{2k+1}{2^n}, \frac{k+1}{2^{n-1}}\right) \Rightarrow \ldots; \quad 0 \leq k < 2^{n-1}. \tag{2.5}$$

Here binary decomposition is not the hindrance for uniqueness of fitting of any point. We allow that the half-interval has non-enumerable points. We shall find at least one (α). We can let it has got in one of half-intervals for the first decomposition. This non-enumerable point α will be found in a result of division intervals according to the scheme $Binar$ (2.5). The problem seems to be resolved and the required point is found but this point α coincides with rational point x as far as $\lim_{n\to\infty} |x_n^{(-)} - x_n^{(+)}| = \lim_{n\to\infty} |x_n^{(-)} - \alpha| = 0$, that is their limiting value is the same, differently $\lim x_n = x = \alpha$.

So the limiting rational point is also irrational, i.e. the enumerable set of those and other points belong to the united and conterminous class $\aleph_0$. The scheme of the last proof completely repeats Cantorian form. But in a result there are not any fantastic non-enumerable points in scientific reality and all mysticism of continuum disappears. Potential enumerable rational points is equivalent actual enumerable irrational points.

It is impossible not to pay attention to the proof of the binary decomposition (2.5) is valid for the schemes any correct decomposition, for example three-signed (2.4). Obviously the Cantorian proof by virtue of the stated reason of substitution acts only for the scheme (2.2). In expressions (2.1, 2.3) of the acting system $SI_{TT}^{\|\infty\|}$ the Cantorian theorem $Tt^{\|\infty\|}(C)$ receives proof $Doc^{\|\infty\|}$ within bounds of mathematical method $\mathscr{M}^{\|\infty\|}$ with an ineradicable kernel – infinity axiom $Ax^{\|\infty\|}$. However this proof uses incorrect extraneous inclusion of the net of borders of interval decomposition predetermined reception of false $\|\bar{\&}\|\{Tt^{\|\infty\|}(C)\}$ conclusion. But even such conclusion $Ww^{\|\infty\|}$ cannot be received for any correct scheme. It may adduce as examples decompositions (2.4, 2.5). $\square$

Theorems 2.1 and 2.2 which are refute the way of the Cantorian proof of existence of the non-enumerable factor nevertheless do not give the guarantee that such proof can not be found for other method or in other scheme. The objection is for it.

Theorem 2.3. *The set of all points* $\{\alpha\}$ *of the unit interval* $[0;1]$ *is enumerable.*

$$SI_{TT}^{\|\infty\|}\left(Ax^{\|\infty\|}\right) : \alpha_i \in [0,1] \stackrel{SI}{\Longrightarrow} \langle \mathcal{N}\{\alpha\} = \mathcal{N}\{\aleph_0\}\rangle \stackrel{SI}{\Longrightarrow} ALG[\alpha], \tag{2.6}$$

where the statement belongs to the system $SI_{TT}^{\|\infty\|}\left(Ax^{\|\infty\|}\right)$*, set* $\mathcal{N}\{\alpha\}$ *is natural and the potency of formed set is determined by algorithm* $ALG[\alpha]$*.*

Proof. Mistakes of the Cantorian theorem about non-enumerable points of the unit interval $[0,1]$ demonstrated in the proof of the previous theorems, is valid in the acting system $SI_{TT}^{\|\infty\|}$ ineradicably dependent from infinity axiom $Ax^{\|\infty\|}$. The difference between rational and irrational points only in opportunity whether or not finite fraction-rational representation of the given point. In such case for the first variant there are some notations where they are recorded by finite quantity of the digits and for the second variant such notation is not present. From here follows that the infinite enumerable factor is contained already in representation of any irrational point as infinite fraction. However the set of rational points as well as irrational is enumerable as far as after liquidation of the Cantorian proof the non-enumerable concept should be rejected as illusion, fantasy and not possessing substantiation even in idealized system. At the same time any formed set is determined by algorithm of its task. Thus and the potency of such set acts by function of algorithm. Naturally, algorithm of formation always enumerable sets (any objects) in condition to ensure set of any class of capacity including essentially superior potency of points of interval $[0,1]$. The cardinality of this set is $\aleph_0$.

The negation of non-enumerable points of unit interval has deep roots of connection with the factor of convergence. It is necessary to liquidate error about basic and insuperable difference between the potential and actual forms of infinity. The recognition of existence of irrational number is equivalent to recognition of actual infinity as similar value α can be present only by boundless digits of decomposition. An assumption of the opportunity of unrestricted approach to any number including irrational with the help of sequence of rational values is equivalent to the recognition of potential infinity.

Then the validity of the Cantorian theorem is equivalent to negation of convergence of sequence of fraction-rational numbers to the value α that is possible to reflect as

$$|\&|\left\{Tt^{\|\infty\|}(C)\right\}: \quad m,n \in \mathcal{N}, \quad \inf_{m,n<N}\left|\frac{m}{n}-\alpha\right| \stackrel{N\to\infty}{\not\Longrightarrow} 0, \; \lim_{m,n\to\infty}\frac{m}{n} \neq \alpha, \tag{2.7}$$

where α is irrational number. But set of irrational numbers, contrary to (2.7) is closure of rational numbers set. Or else the consent with the theorem of

Cantor directly means that limit of potential infinity is not actual infinity. At the same time the standard scheme of convergence (2.7) appears wrongful. Such obvious contradictions including the reality of convergence destroy the pretentious Cantorian and any other proof of non-enumerable points of the unit interval.

In the consent with idealized systems SI already not speaking about real approximations, the expressions (2.6, 2.7) ascertain and specify the conclusion of the previous theorem. Validity $|\&|\{Tt^{\|\infty\|}(C)\}$ of the Cantorian theorem contradicts those ways and rules which have earlier resulted in its proof. It is here necessary to include and non-convergence $\lim\limits_{m,n\to\infty} \frac{m}{n} \neq \alpha$ that destroys the beginning of the basic calculus.

We shall address to the binary notation and traditionally we compare points of half-interval $[0;1)$ with their representations. Obviously all binary-rational points with finite quantity of the digits are enumerable, the theory of sets does not object against that. We shall present stereotypedly any point x in kind of infinite fraction

$$SI_{TT}^{\|\infty\|}: \quad x \in [0;1);\ x = 0.\varepsilon_1\varepsilon_2\varepsilon_3\ldots\varepsilon_i\ldots\varepsilon_n\ldots;\ \varepsilon_n = \{0;1\}. \tag{2.8}$$

The potency of points set $\{x\}$ is continuum (?) and it coincides with set of all points of half-interval $[0;1)$ as far as in decomposition (2.8) includes irrational points. With the help and in the basis of this decomposition we form sequence

$$\{N_1\} \cup \{N_2\} \cup \ldots \cup \{N_k\} \cup \ldots : \{N_1\} = \left\{0; \frac{1}{2}\right\}, \{N_{k+1}\} = \left\{\frac{N_k}{2}\right\} \cup \left\{\frac{N_k+1}{2}\right\}, \tag{2.9}$$

in which obviously all elements are enumerable and in each subsequence $\{N_k\}$ quantity of growing elements from 0 up to $1 - \frac{1}{2^k}$ equal 2^k.

For any number x in (2.8) the first k binary digits $0.\varepsilon_1\varepsilon_2\varepsilon_3\ldots\varepsilon_k$ will be found always in $\{N_k\}$ for any $k \geq 1$. In such case we are forming the following row

$$\{0.\varepsilon_1; 0.\varepsilon_1\varepsilon_2; 0.\varepsilon_1\varepsilon_2\varepsilon_3; \ldots; 0.\varepsilon_1\varepsilon_2\ldots\varepsilon_k; \ldots\} \Rightarrow 0.\varepsilon_1\varepsilon_2\varepsilon_3\ldots\varepsilon_k \in \{N_k\}, \tag{2.10}$$

and sequence $0.\varepsilon_1\varepsilon_2\varepsilon_3\ldots\varepsilon_k$ for $k \to \infty$ that is why and the row (2.10) aims to value x. If to take into account that this value x in expression (2.8) completely any number including and irrational value, the sequence (2.9) sort out all numbers of the half-interval and potency at all of this enumerable values is $\aleph_0$ and not continuum.

Thus, the enumerable construction (2.9) includes binary rational finite and infinite fractions, irrational and even transcendental numbers such as e^{-1} or $\pi/4$. There is all imagination including and ridiculous Sierpinski number with record in the digits all sequence of primes. But all points of the half-interval $[0;1)$ are enumerable that immediately follows from offered schemes (2.8 – 2.10). All disassembled scheme is not stacked in the initial preconditions of Cantorian enumerable concept but if we will expand a little (with using certain liberty introduced doubtful logic and non-existence of processing results) and such way of reasons about enumerable which denies non-enumerable concept as even limiting discreteness is enumerable nevertheless is probable.

It is possible to reflect received facts appealing to binary representation (2.10):

$$\lim_{k\to\infty}\{0.\varepsilon_1\varepsilon_2\varepsilon_3\ldots\varepsilon_k000\ldots0_\infty\}\Leftrightarrow\{0.\varepsilon_1\varepsilon_2\varepsilon_3\ldots\varepsilon_k\varepsilon_{k+1}\ldots\varepsilon_\infty\};\ \varepsilon_k,\varepsilon_\infty\in(0;1), \tag{2.11}$$

where the first limit is *extreme* rational number that is standard rational value with finite $k<\infty$ for which the mathematical method $\mathscr{M}^{\|\infty\|}$ admits to be the parameter k any large. In such case the first representation (2.11) reflects assumption of potential infinity and the second representation appears as an actual. But the capacities of point sets of these two types (2.11) coincide. In a result even for $SI_{TT}^{\|\infty\|}$ set of points $\alpha_i\in[0,1]$ is enumerable. It means that potency of such set $\mu(\mathscr{N}\{\alpha\})=\aleph_0$ instead of $\aleph_1$. Such conclusion throws additional light upon the proof of the continuum-hypothesis insolubility [9]. But the algorithm $ALG[\alpha]$ is the determining factor of set potency. □

As far as the Cantorian attempt to find geometrical or arithmetic non-enumerableness ($\aleph_1$) has appeared unusable it has remained to evaluate opportunities of escalating of potency of enumerable sets with the help of theoretic-set ways.

Theorem 2.4. *Any proof of continuum existence (even in zone of imagination) is unfounded, it means that it is not possess of logical (Lg) reasons.*

$$SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|}):\quad Doc^{\|\infty\|}\{\exists[\aleph_1]\}\ \Rightarrow\ \{\|\bar{\&}\|(Ww^{\|\infty\|})\equiv\nexists[\aleph_1]\}, \tag{2.12}$$

i.e. even in $SI_{TT}^{\|\infty\|}$ the proof of continuum objectivity $Doc^{\|\infty\|}\{\exists[\aleph_1]\}$ results to its falsehood $\|\bar{\&}\|(Ww^{\|\infty\|})$, otherwise it results to non-existence $\nexists[\aleph_1]$.

Proof. Such conclusion followed by expression (2.12) is direct follows from the Theorem 2.3 as far as it proves that non-enumerable objects of cardinality $\aleph_1$ does not exist that the Theorems 2.1 and 2.2 were preconditions. The imaginary success in search of non-enumerable phenomenon $\mathscr{N}(\aleph_1)$ is equivalent to find of absolutely non-existence ($\nexists$). In any event if object $\mathscr{N}(\aleph_0)$ is existed, $\mathscr{N}(\aleph_1)$ is not existed. □

Theorem 2.1, and especially Theorems 2.3 and 2.4 compel to form the next conclusion.

Theorem 2.5. *Any solution of the continuum-hypothesis (first Hilbert problem) is unfounded because from non-existence ($\nexists$) of the component objects $\mathscr{N}^{\|\infty\|}$.*

$$SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|},\mathscr{N}^{\|\infty\|}):\ \{\aleph_0\Rightarrow2^{\aleph_0}=\aleph_1\}\overset{\mathscr{M}^{\|\infty\|}}{\Longrightarrow}\nexists\{2^{\aleph_0}=\aleph_1\}, \tag{2.13}$$

where continuum-hypothesis is designated by transition $\aleph_0\Rightarrow\aleph_1$ in system $SI_{TT}^{\|\infty\|}$.

Proof. Statement about unsolvability of the continuum-hypothesis within the framework of mathematical method $\mathscr{M}^{\|\infty\|}$ and acting system $SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|})$ of idealized cognition follows from the Theorems 2.3 and 2.4. This conclusion is valid for all real conditions, all mathematical and cognition systems SS. It majorizes and explains the conventional conclusion of Cohen [9], since continuum and other objects $\Omega^{\|\infty\|}$ of infinity quantifier $\forall^{\|\infty\|}$, for example $\mathscr{N}^{\|\infty\|}$ are absent (2.13) from non-existence of objects of class $\mathscr{N}(\aleph_1)$ even in system $SI_{TT}^{\|\infty\|}$. It is submitted below in detail. □

3. The set of all subsets and cardinalities

The determining statement from work [8] leading to concepts of non-equivalence of non-enumerable sets $\{N^{\|\infty\|}\}$ and cardinalities are considered here.

Theorem 3.1. *The Cantorian proof of non-equivalence of the set* $\{\mathbf{N}\}$ *and set of its subsets is erroneous if the set* $\{\mathbf{N}\}$ *is enumerable (of cardinality* $\aleph_0$*).*

$$SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|}) : \left\{[\aleph(\{\mathbf{N}\}\overset{SI}{=}\aleph_0]\overset{Lg}{\not\cong}2^{\aleph_0}\equiv\aleph_1\right\}\overset{\mathscr{M}^{\|\infty\|}}{\Longrightarrow}\|\bar{\&}\|(Ww^{\|\infty\|}), \quad (3.1)$$

and acting system $SI_{TT}^{\|\infty\|}$ *with the method* $\mathscr{M}^{\|\infty\|}$ *contains ineradicable logic* (Lg) *errors in the proof leading to falsehood* $\|\bar{\&}\|(Ww^{\|\infty\|})$ *of the main conclusion.*

Proof. If the capacities of compared sets are equal accordingly n and 2^n than classical Cantorian theorem about set of all subsets does not cause doubts for finite and enumerable $n \geq 1$. The situation becomes complicated in case of actual infinite $(\|\infty\|)$ quantities of elements and formation of set of cardinality $2^{\aleph_0} \equiv \aleph_1$. □

First of all it is necessary to dwell upon frequent voluntarism of mathematics and its method $\mathscr{M}^{\|\infty\|}$ in process of formation of knowledge.

Lemma 3.1. *It is illegal and incorrect to include in proofs the mathematical constructions* $\mathscr{M}^{\|\infty\|} \to \Omega^{\|\infty\|} \to \mathscr{N}^{\|\infty\|}$ *in conditions of unsolved logical paradoxes.*

$$SI_{TT}^{\|\infty\|} : \ \Omega^{\|\infty\|}(\mathscr{M}^{\|\infty\|}, Ax^{\|\infty\|}) \overset{Lg^{\|\infty\|}}{\Longrightarrow} \Omega^{\|\infty\|}(Px^{\|\infty\|}) \overset{RR}{\Longrightarrow} \|\bar{\&}\|(Doc^{\|\infty\|}). \quad (3.2)$$

Proof. Acting system $SI_{TT}^{\|\infty\|}$ in it's own idealized constructions $\Omega^{\|\infty\|}(\mathscr{M}^{\|\infty\|}, Ax^{\|\infty\|})$ of method $\mathscr{M}^{\|\infty\|}$ and axiom $Ax^{\|\infty\|}$ uses unsolved logical paradoxes $Px^{\|\infty\|}$ with support logic $Lg^{\|\infty\|}$. However this way can not guarantee the foundation of conclusions. Moreover it obliges to bring to falsehood $\|\bar{\&}\|(Doc^{\|\infty\|})$ of the proof. Really, an interpretation of conclusions on the base of paradox with opposite inferences is especially arbitrary. It fixes reality *RR* clearly. Lemma is private case of theorem [11, 12] and expression (3.2) follows from this work. □

In the proof of Cantor [8] the initial thesis about existence of the subset $A_x = \emptyset$ in which even alone element x does not include, is faulty as the element $x = \emptyset$ is included in $\emptyset$. If elements $x \in A$ appropriated A_x and included in this set to designate x^+, that elements are not included will become x^-. Set is $Z = \cup x_i^-$. The structure of set Z is various. If the set of all subsets begins from set A, all elements $x \in A$ are elements x^+ hence $Z = \emptyset$. Simple in pairs rearrangement of next elements in the previous variant results in that all elements $x \in A$ become the kind x^- that is $Z = A$.

Further as derivative artificially created model the set $Z = \cup x_i^-$ all (just *all*) elements x^- is offered. The Cantorian element $\zeta \in A$ equivalent Z can not belong to the type x^+ and type x^-. But in variant $Z = \emptyset$ this conclusion loses the sense and for $Z = A$ actuality of set is impossible. Here it is necessary to find the main reason of incorrect conclusion from aimed paradoxical construction. It is absolute illegality of use of representation *all* in relation to quantity of elements. The actual infinity $\|\infty\|$ has not realization in such fixed *value*. In kind of the explanatory shall specify impossibility to create the heaviest natural number and to have all numbers $\{\mathbf{N}\}$. Paradox of existence of *whole* infinite set is discuss below. Besides for such sets restored in essence concept *more* is unfit. Here under the already known way and on the basis of use (by system $SI_{TT}^{\|\infty\|}$) the quantifier of idealized generality there were in cognition well investigated conditions of many logically unsolvable paradoxes of type *liar*. Unfortunately they can not serve the proof and the disproof of any statement. This contradiction can not enter to Cantorian proof according to Lemma 3.1 and theorem [12]. It serves to itself only. But mathematics has no rights to use extremely suspicious logic constructions. In such case the unsolvability of paradox should be transferred to the main proof. Cantor didn't do it.

Well studied conditions of creation of many logical unsolved paradoxes arose in the cognition according to known way and on the base of system $SI_{TT}^{\|\infty\|}$ and quantifier of idealized community $\forall^{\|\infty\|}$. Unfortunately, Cantorian proof $Doc^{\|\infty\|}(C)$ includes situation of such paradox, though mathematical requirements must not permit it.The next statement makes clear paradox of the important idea.

Lemma 3.2. *The idea all enumerable set contains ineradicable logical* (Lg) *contradiction, connected with non-existence of object* $\Omega^{\|\infty\|}(\mathscr{N})$.

$$SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|}, \mathscr{M}^{\|\infty\|}) : \quad \Omega^{\|\infty\|}(\mathscr{N}) \overset{Lg^{\|\infty\|}}{\Longrightarrow} \|\bar{\&}\|(\mathscr{N}^{\|\infty\|}). \tag{3.3}$$

Proof. The acting system $SI_{TT}^{\|\infty\|}$ of infinity axiom $Ax^{\|\infty\|}$ and method $\mathscr{M}^{\|\infty\|}$ can not prove or found existence (in any sense) *all enumerable set* $\Omega^{\|\infty\|}(\mathscr{N})$. An impossibility to find *largest number* which is acknowledged by system is the confirmation to this.Then idea *all enumerable set* $\mathscr{N}^{\|\infty\|}$ must become source of false conclusions $\|\bar{\&}\|(\mathscr{N}^{\|\infty\|})$ with unrealizableness limit. □

Theorem 3.2. *The liquidation of the logic contradictions in the Cantorian proof of non-equivalence of set and all its subsets destroys the known conclusion:*

$$SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|}) : \quad \left\{ \aleph_0 \overset{(Lg^{RR} \neq Lg^{\|\infty\|})}{\ncong} 2^{\aleph_0} \equiv \aleph_1 \right\} \overset{\mathscr{M}^{\|\infty\|}}{\Longrightarrow} \nexists(Ww^{\|\infty\|}), \tag{3.4}$$

i.e. transition from idealized and erroneous Cantorian logic $Lg^{\|\infty\|}$ *to founded logic* Lg^{RR} *transforms conclusion* $Ww^{\|\infty\|}$ *of Cantorian proof into opposite* (*nonexistent* $\nexists$).

Proof. The analysis of Cantorian logic model in the proof of the previous theorem shows that the reduction of its conditions to foundation, (i.e. to destruction of the special role of empty set and concept *all*) can not ensure Cantorian conclusion, thus justifying the conclusion (3.2). Lemma 3.2 which is effective in any system testifies about it. All conditions considered by Theorem 3.1 were constructive necessary to Cantor for the creation of proof. □

Error of the Cantorian proof is not means the incorrect formulation. It is necessary to create the refuting thesis for the liquidation of such uncertain situation within the framework of the same mathematical method $\mathscr{M}^{\|\infty\|}$. We should define the opportunity of the numerical imagination.

Lemma 3.3. *Realized and implied opportunities of aspiration* (lim) *to unboundedness always yield to assumption of idealization and phantom* $\|\infty\|$.

$$SI_{TT}^{\|\infty\|}\left([Ax,\mathscr{M}]^{\|\infty\|}\right):\quad \left[\forall\, F(n)\subset M^{RR}\right] \overset{Lg^{\|\infty\|}}{\not\Longrightarrow} \left[\lim_{n\to\infty}\{F(n)\to F(n+1)\to\infty\}\right]. \tag{3.5}$$

Proof. The acting system $SI_{TT}^{\|\infty\|}$ with the omnipotent axiom $Ax^{\|\infty\|}$ and method $\mathscr{M}^{\|\infty\|}$ operates by rule (doctrine) *Possibility of Impossible* self-confidently (though non-obviously). It is the permission to operate in spheres, fields, domains and classes, though real attainability and existence of objects even is not discussed.

However operators $(n\to\infty)$ and $\{F(n)\to\infty\}$ demonstrates that any given function $F(n)$ do not approach to problematic element $\|\infty\|$. The doctrine of mathematical method $\mathscr{M}^{\|\infty\|}$ does not refute that function $G(n)\gg F(n)$ can be found, but it can not claim on absolute maximum. Idealized suppositions is obliged to substitute real mathematical opportunities. The logic of system $Lg^{\|\infty\|}$ can not really help here. Our idealized system do not overcome the factor of non-existence of limits (3.5). □

Theorem 3.3. *In acting idealized system of cognition* $SI_{TT}^{\|\infty\|}$ *any set* $\{\mathbf{N}\}$ *of enumerable cardinality* $\aleph_0$ *is equivalent to set of all its subsets.*

$$SI:\ \left[\{\mathbf{N}\}\equiv\{\mathbf{N}\}(\aleph_0),\{\mathbf{M}\}\equiv\{\mathbf{M}\}(2^{\aleph_0})\right] \overset{SI^{\|\infty\|}}{\Longrightarrow} \left[\{\mathbf{N}\}(\aleph_0)\overset{SI^{\|\infty\|}}{\cong}\{\mathbf{M}\}(2^{\aleph_0})\right], \tag{3.6}$$

where set $\{\mathbf{M}\}$ *of cardinality* $2^{\aleph_0}$ *is the set of all subsets of set* $\{\mathbf{N}\}$.

Proof. Let's admit that enumerable set $\{\mathbf{N}\}$ and any finite value $n_1\geq 1$ are given. Then the set of all subsets formed by n_1 initial elements of given set $\{\mathbf{N}\}$ has potency 2^{n_1}. Because of the infinity of set $\{\mathbf{N}\}$ the elements which are equivalent to just created elements can be found always here. We shall proceed to value $n_2>n_1$ and we shall also construct the series of conformity with the potency

2^{n_2} and so on to n_k. Continuing this process, we shall receive the sequence $n_1 < n_2 < n_3 < \ldots < n_k < \ldots$ with the infinite limit. As far as for any k will be found such l that $n_l \geq 2^{n_k}$, for each step mutually unequivocal conformity is maintained. It means that the limit executes equivalence in the complete consent with the method $\mathscr{M}^{\|\infty\|}$.

The generalization of this proof is allowable. Let's present that $F(n)$ is any increasing function of potency of the set Ω_n accepted only the finite values. Then such natural N will be always found when between these interval of natural numbers and the set Ω_n mutually unequivocal conformity is established. Now, with the help of the limit $n \to \infty$ we shall receive equivalence of enumerable natural set and the set $\Omega = \lim\limits_{n\to\infty} \Omega_n$. Or else, the finite value $N > F(n)$ will be found always in spite of the value n. It proves the efficiency of the equivalence operator. As far as the function of set of all subsets qualifies to the condition of function $F(n)$ the theorem is proven.

The operator of set of all subsets forms equivalent set of cardinality $\aleph_0$ and not at all $\aleph_1$, which does not exist.It is contradicts to the Cantorian statement but corresponds to Theorems 2.1-3.2. It proves basic ineradicableness of Cantor's error. In the expression (3.6) the scheme of the offered statement 1, which is proven in the system $SI^{\|\infty\|}_{TT}$ with the help of the method $\mathscr{M}^{\|\infty\|}$ is submitted. The enumerable set $\{\mathbf{N}\}(\aleph_0)$ and the set of its subsets $\{\mathbf{M}\}(2^{\aleph_0})$ are equipotent ($\cong$), i.e. the second variant is also set of class $\{\mathbf{N}\}(\aleph_0)$. Depth sources of this phenomenon are reflected in [1–7]. □

Mathematical constructions and extrapolations can not be arbitrary. It concerns especially to mental formations with out the opportunity of control.

Theorem 3.4. *Any proof of non-equivalence of enumerable set* $\{\mathbf{N}\}$ *and set of all its subsets* $\{\mathbf{M}\}$ *contains an ineradicable error.*

$$SI^{\|\infty\|}_{TT}\left(Ax^{\|\infty\|}\right): \quad \left[\{\mathbf{N}\}(\aleph_0) \overset{SI^{\|\infty\|}}{\ncong} \{\mathbf{M}\}(2^{\aleph_0})\right] \overset{SI^{\|\infty\|}}{\Longrightarrow} \|\bar{\&}\|\left(Ww^{\|\infty\|}\right), \tag{3.7}$$

i.e. for the acting system $SI^{\|\infty\|}_{TT}$ *the proof of non-equivalence* ($\ncong$) *of sets* $\{\mathbf{N}\}$ *and* $\{\mathbf{M}\}$ *is not capable to get rid of the false conclusion* $\|\bar{\&}\|\left(Ww^{\|\infty\|}\right)$.

Proof. This statement follows not from the mistaken Cantorian scheme (Theorem 3.1) but from the Theorem 3.3. Boundlessness of enumerable infinity $\{\mathbf{N}\}(\aleph_0)$ exceeds all opportunities at the impunity to manipulate with it. □

The unreasonable Cantorian conclusion about non-equivalence of enumerable set and set of all its subsets can not escape the problem of cardinalities.

Theorem 3.5. *The Cantorian concept of cardinalities* $\aleph_k \neq \aleph_{k+1}$ *is unfounded.*

$$SI^{\|\infty\|}_{TT}: \quad \left[\{\mathbf{N}\}(\aleph_k) \overset{SI^{\|\infty\|}}{\ncong} \{\mathbf{M}\}(2^{\aleph_k}) \equiv \{\mathbf{M}\}(\aleph_{k+1})\right] \overset{SI^{\|\infty\|}}{\Longrightarrow} \|\bar{\&}\|\left(Ww^{\|\infty\|}\right). \tag{3.8}$$

Proof. As far as the Cantorian proof of existence of cardinalities $\aleph_k$ sets are united for all natural $k \geq 1$, the false conclusion $\|\bar{\&}\|\big(Ww^{\|\infty\|}\big)$ about these sets of higher cardinalities $\{\mathbf{M}\}(\aleph_{k+1})$ will appear identical with the Theorems 3.3 and 3.4. The acting system $SI_{TT}^{\|\infty\|}$ of mathematics can hope and consider only symbol of phantom $\|\infty\|$ in standard conditions. But problematic character of this phantom is quite ineradicable.

Expression (3.8) reflects inability of fundamental mathematics to create and prove construction of sets of cardinalities $\aleph_k$ for parameter $k \geq 1$. □

Statements about unfounded continuum concept and equivalence of enumerable set and the sets of its subsets which was proven in $SI_{TT}^{\|\infty\|}$ and mathematical method $\mathscr{M}^{\|\infty\|}$ completely confirm with the conclusion of the Theorem 1.1 about existence arguments which are deny unreasonable model constructions. This results had demonstrated again the unfounded formal logic $\mathscr{I}\mathscr{L}\mathscr{G}^{\|\infty\|}$ which is inconclusive leaning on infinity axiom $Ax^{\|\infty\|}$. However, the axiom demands trust which measure overcame all reasonable bounds, because there are more and more unpleasant objections from cognizable reality *RR*. The received conclusions can not testify about theirs accident on the background of many determining failures of scientific cognition. In such case it is possible to ask the question about foundation of the mathematical method $\mathscr{M}^{\|\infty\|}$ as a whole, which was inconceivable before.

4. **Consequences of unrestricted idealization**

The mathematization of cognition is objective process of formation of knowledge sphere. The idealization of representations, which accompanied this process, was very conveniently stacked in the mathematical schemes. Youthful scientific disciplines with readiness respond on granted opportunities which seemed boundless. And really, the followed successes in a sphere of natural knowledge have simultaneously strengthened belief in validity of the elected way, which in essential degree is obliged to the mathematization factor and infinity axiom $Ax^{\|\infty\|}$. The negative tendencies of a mathematical method $\mathscr{M}^{\|\infty\|}$ have also begun to be observe for a very long time but at the last time the contradictions has become to outweigh counteraction of the system. The results submitted above should be considered by an appreciable part of burden, such undesirable for the system too. The proven theorems have demonstrated clearly those that ripens in a sphere of scientific cognition for a very long time. Omnipotence of infinity axiom $Ax^{\|\infty\|}$ has appeared only seeming, imaginary and this distinctive property is imposed to cognition from the outside and violently because the axiom of infinity always lacked validity.

Theorem 4.1. *The infinity axiom $Ax^{\|\infty\|}$ is not capable to supply objectivity of any constructions $\Omega^{\|\infty\|}$ even in the zone of imagination.*

$$SI_{TT}^{\|\infty\|}\big(\mathscr{M}^{\|\infty\|}\big): \quad \Big\{Ax^{\|\infty\|} \overset{SI^{\|\infty\|}}{\not\Longrightarrow} \exists\ \big(\forall\ \Omega^{\|\infty\|}\big) \overset{SI^{\|\infty\|}}{\Longrightarrow} \|\bar{\&}\|\big(\Omega^{\|\infty\|}\big)\Big\}. \tag{4.1}$$

Proof. Not any assumptions do not permit to hope that cognition and system $SI_{TT}^{\|\infty\|}$ can form any of mental constructions with impunity. As considered examples shows it is quite probably to come across on final falsehood, which is not capable to hide even proof covered illusion or modelling of objects $\Omega^{\|\infty\|}$ of infinity quantifier $\forall^{\|\infty\|}$. The created constructions doubtless enter in a sphere of imagination but it can not help to system $SI_{TT}^{\|\infty\|}$. Though the idealization led objects in such distances, even in such case the objective analysis and the logic rules of the proofs of the acting system permit find errors in the offered schemes. Thus achievement of the system $SI_{TT}^{\|\infty\|}$ can act against itself. It is necessary to think they thus in a result will be deprived of infinity quantifier. Extrapolative opportunities of cognition and scientific cognition are at not boundless and conglomeration of complicated idealized constructions is capable to result to extremely negative consequences. The expression (4.1) reflects this situation of the acting system $SI_{TT}^{\|\infty\|}(\mathscr{M}^{\|\infty\|})$ and mathematical method. The concept of infinity axiom $Ax^{\|\infty\|}$ can not prevent occurrence in a class of every possible objects $\forall\,\Omega^{\|\infty\|}$ faulty ($\|\bar{\&}\|$) constructions $\|\bar{\&}\|(\Omega^{\|\infty\|})$, which nevertheless the system of idealization considers as possible, existent and effective in cognition. □

As far as the system $SI_{TT}^{\|\infty\|}$ defines more precisely the modelling character of similar complicated constructions and cognition objects $\Omega^{\|\infty\|}$, they are obliged to be referred to a zone of imagination. At the same time their fitting to infinity quantifier $\forall^{\|\infty\|}$ forces to recognize similar objects as idealized. And all these reasons and the explanations can not remove the problem existence as compulsory step on the way to foundation – urgent requirement of human and scientific cognition.

Theorem 4.2. *Boundlessness, unattainableness, unpredictableness, unrealizableness, unobtainableness, antidynamics, impossibility and that's why the non-existence of concepts of any infinity form $\|\infty\|$ are obliged to cause a chain of the false conclusions $\|\bar{\&}\|(Ww^{\|\infty\|})$ at attempts of creation of laws $ZZ^{\|\infty\|}$ in idealized zones.*

$$SI_{TT}^{\|\infty\|} : \nexists\Big\{\forall(\|\infty\|) \overset{Lg}{\Longrightarrow} \Omega^{\|\infty\|}\big(\forall^{\|\infty\|}\big)\Big\} \overset{SS^{RR}}{\Longrightarrow} \Big\{\|\bar{\&}\|\big(Ww^{\|\infty\|} \equiv Ww^{\|\infty\|}\big[ZZ^{\|\infty\|}\big]\big)\Big\}. \tag{4.2}$$

Proof. That is complex of all listed attributes of idealized objects in the system $SI_{TT}^{\|\infty\|}(Ax^{\|\infty\|})$ leading to non-existence ($\nexists$) of all infinity forms $\forall(\|\infty\|)$, so and concepts (objects) $\Omega^{\|\infty\|}(\forall^{\|\infty\|})$ of infinity quantifier, from the point of view of any real system SS^{RR} is obliged to conduct to falsehood of conclusions $\|\bar{\&}\|(Ww^{\|\infty\|})$ at creation of the idealized laws $ZZ^{\|\infty\|}$. At the same time from states which are proven above follows that even from views of $SI_{TT}^{\|\infty\|}$ many constructions of the infinity quantifier appear nonexistent and proofs of their efficiency in cognition is false. This fact intensifies position and contents of the expression (4.2) which remains fair even with such conditions. But the indicated variant is much more important because it is specifies direct the responsibility of axiom $Ax^{\|\infty\|}$ being not limited similarly to the Theorem 1.1 complicated constructions of a kind $\Omega^{\|\infty\|}\{\|\infty\|, \|\infty\|\}$. In given scheme (4.2) the proof can lean on the Theorem 4.1.

It is especially necessary to note the absence of dynamics in all concepts and objects $\Omega^{\|\infty\|}$ of infinity quantifier even of the potential modification of this phantom. The amplification of the determining characteristic ($\|\infty\|$) of such forms is impossible. □

And all of them are necessary for formation of model idealized laws. The proof of validity of the operator of an aspiration to unrestrictedness could serve as the justification of their introduction.

Theorem 4.3. *The aspiration to phantom of infinity $\|\infty\|$ is unrealized and its achievement is impossible even in mathematization imagination $\mathscr{M}^{\|\infty\|}(TT^{\mathscr{L}})$. Thus infinity axiom $Ax^{\|\infty\|}$ is unfounded and the objects $\Omega^{\|\infty\|}$ of infinity quantifier $\forall^{\|\infty\|}$ should be considered as non-existent ($\nexists$). For acting system*

$$SI_{TT}^{\|\infty\|} : \left\{\mathscr{M}^{\|\infty\|} \supset [n; F(n) \to \infty]\right\} \overset{\mathscr{M}^{\|\infty\|}}{\Longrightarrow} \nexists \left\{Ax^{\|\infty\|}, \Omega^{\|\infty\|}, [F(n) \to \infty], \forall F\right\}. \tag{4.3}$$

Proof. We should consider the examples of appealing to the idea of infinity $\|\infty\|$.

1. The operator of increase and the approach to limit $Op(A_n + a_n \Rightarrow A_{n+1})$ is not leading to achievement $\|\infty\|$ with any (A_n, a_n) because of the possibility of substitution $A_n + a_n = A_n$.
2. Let's consider B_1 as the utmost number, B_2 as the last value of numerical axis and $B_3 = \lim\limits_{x\to\infty} x$. There is no real opportunity to find any differences between them.
3. It is impossible to obtain the declared result $\|\infty\|$ in operator of limit B_3 without using of infinity element ∞ with the condition $x \to \infty$.
4. Any maximal number is not approach it to declared phantom of infinity, i.e. always $\forall (B \gg A) : (\|\infty\| - A \equiv \|\infty\| - B)$.
5. If we will admit that value $\|\infty\|$ is arrived, than the return to the begin of aspiration or enumeration is impossible because $\|\infty\| - 1 = \|\infty\|$.
6. If the half-axis $(0, \infty)$ is existent linear object, than there are middle (intermediate) point must be. However, it is impossible to find it with any assumptions.
7. Unsolvable paradoxes $Px^{\|\infty\|}$ of quantifier $\forall^{\|\infty\|}$ is well known to science.
8. There is ambivalence of lightness and hardness: In the operator $Op(x \to \infty)$, there are the imagine (illusive) lightness of enumeration or the removal from begin and impossibility of approach to the limit and especially of it's achievement.
9. Even if we will image the existence $\exists(\infty)$, than aspiration $\|\infty\| \to a$, $\|\infty\| \to 0$ is impossible.

10. In [15] the notion $\|\infty\|$ is explained as *improper element* ∞ (it's not clear what is it doing on the homogeneous numerical axis) with the allowing some operations $F(\infty,\infty)=\infty$ but with the recognition of some else *without any sense*.
11. All of points are absolutely unsolvable in framework of system $SI_{TT}^{\|\infty\|}$. And the single exit is the appeal to idea of non-existence ($\nexists$), which is obtain to substitute the illusion $\|\infty\|$.

All of considered contradictions acquire synonymous and quite sufficient explanation there are in this conclusive responsible inference, for example

$$\{F(\infty)=(\nexists)\}\equiv\{F(\nexists)=(\nexists)\};Px^{\|\infty\|}(\forall^{\|\infty\|})=Px^{(\nexists)}=(\nexists);\lim_{x\to\infty}x=(\nexists). \tag{4.4}$$

Vague, but very important operator $\{n\to\infty\}$, which is included to lots of mathematical constructions are not takes into account the opportunity of approach to potential or actual forms of infinity. The postulation of absolute possibility of the approach with the limit and attainableness of problematic element is unfounded, i.e. the contrary one is founded. There are explanations in known schemes of the method $\mathscr{M}^{\|\infty\|}$. The aspiration to the increasing is really limited. In expression (4.3) operator of unlimited aspiration $[n;F(n)\to\infty]$ of method $\mathscr{M}^{\|\infty\|}$ and axiom $Ax^{\|\infty\|}$ lead to non-existence of the widest strip of values right up to phantom $\|\infty\|$. Conclusions as (4.4) from the offered list tell us about the same. □

The consciousness permits to itself to go out from borders of idealized constructions of accomplishment, but this does not pass for cognition without consequences. Mathematics is obliged to agree with the unrealizableness of approach to values with characteristics of boundlessness, but not infinity. However the impossibility of objects $\Omega^{\|\infty\|}$ of quantifier $\forall^{\|\infty\|}$ straight follows from non-existence ($\nexists$) of the broadest strip near the phantom $\|\infty\|$.

Theorem 4.4. *The factor of non-existence* ($\nexists$) *of objects* $\Omega^{\|\infty\|}$ *disseminates to the neighbourhood* $\|\mathfrak{J}\div\infty\|$ *of the illusive infinity* $\|\infty\|$*:* $(\nexists)\Omega^{\|\mathfrak{J}\div\infty\|}$.

$$\forall SS:(\nexists)\Omega^{\|\infty\|}\overset{SS}{\Longrightarrow}(\nexists)\Omega^{\|\mathfrak{J}\div\infty\|};\Omega^{\|\mathfrak{J}\div\infty\|}=\left\{\mathscr{T}\mathscr{T}^{\|\mathfrak{J}\div\infty\|},\mathscr{P}\mathscr{P}^{\|\mathfrak{J}\div\infty\|},Doc^{\|\mathfrak{J}\div\infty\|}\right\}. \tag{4.5}$$

Proof. All of attempts of approach to phantom $\|\infty\|$ direct condemn to the non-existence both the limited point and some zone $\|\mathfrak{J}\div\infty\|$ of infinite measure in any system SS (4.5). The board $\mathfrak{J}$ is indefinable because of the unattainableness and the restriction of cognition $\mathscr{P}\mathscr{L}$. All of characteristics of any real object Ω^{RR}, including algorithms, notions and proofs are restricted. It means that for real structure RS always $\|\Omega^{RR}\|_{RS}<C_{rs}$, i.e. there is its own zone of unattainableness is implied $\|\mathfrak{J}_{rs}\div\infty\|$ in $RS\subset SS$. It means $\mathfrak{J}=\sup_{rs}\mathfrak{J}_{rs}$. In such case in (4.5) the non-existence of theories, $\mathscr{T}\mathscr{T}$, statutes $\mathscr{P}\mathscr{P}$, proofs Doc and all of other components. □

Theorem 4.5. *The non-existence of the objects* $\Omega^{\|\infty\|}$ *causes illusion, illegality, unsatisfactorness, unfounded state and the falsehood of formed knowledge as a result.*

$$\forall SS: \quad \nexists\Big\{\Omega_1^{\|\infty\|} \cup \Omega_2^{\|\infty\|} \cup \ldots \cup \Omega_n^{\|\infty\|}\Big\} \overset{\mathscr{P}\mathscr{L}^{RR}}{\Longrightarrow} \nexists\Big\{\mathscr{P}\mathscr{L}^{\|\infty\|}, ZZ^{\|\infty\|} \subset TT^{\|\infty\|}\Big\}. \quad (4.6)$$

Proof. The construction of knowledge on the basis of accepted by agreement objects $\Omega^{\|\infty\|}$ of the infinity quantifier is the contents of the system $SI_{TT}^{\|\infty\|}$. But such objects do not exist and it's impossible to present or imagine it. Besides, from the Theorem 4.3 follows the impossibility to approach to it as far as theirs determining parameter is unattainable and unrealized. Marks of concepts are not the reality of representations.

A lots of development scientific cognition have demonstrated [1–7] the extreme danger of unattended manipulations with objects of axiom $Ax^{\|\infty\|}$ in search of adequacy of knowledge and reality. Imaginary objects $\Omega^{\|\infty\|}$ are direct threat to any proofs, knowledge and theories $ZZ^{\|\infty\|} \subset TT^{\|\infty\|}$ (4.6). Illusion of objects is obliged to call illusion of knowledge and imaginary unrestrictedness of aspirations is not capable to stop before the reception of frankly false conclusions. Really in conditions of impossibility and non-existence of cognizable set of objects it is inconveniently even to imagine the existence of any objective, founded criteria which could guarantee high-grade, reasonable cognition in a system SS. Rather long in science $Sc^{\|\infty\|}$ the belief existed that the logic is capable to act as the supreme arbitrator of knowledge but it has appeared as the elementary error. The logic $Lg \equiv Lg^{\|\infty\|} \subset SI_{TT}^{\|\infty\|}$ of idealized system is unfounded [1, 5] because of the same infinity axiom $Ax^{\|\infty\|}$.

The inevitable conglomeration of the idealized constructions (the example: polyextremal models [1, 3, 12] of system $SI_{TT}^{\|\infty\|}$) is direct conducts to irreparable and sharp increase of the uncontrollableness and non-existence factors. It means that the potential falsehood of arbitrary knowledge is growing. The references to logic and extrapolation do not assist because the reality at the first opportunity proves the nonsense of hopes for unexpected true. These annoying misunderstandings stop the progress of cognition simple. □

The appeal to complicated constructions of system $SI_{TT}^{\|\infty\|}$ demonstrates unfounded hopes to the deep cognition because of proven non-existence of such objects $\Omega^{\|\infty\|}$. Thus, new refined knowledge becomes as illusive, which are acting in zones of absolute impossibility. Such unfounded situation is quite unsteady and imagine knowledge losing the neutrality fast becoming to the false. If the radical reason of considered failures of cognition is principle non-existence of initial objects $\Omega^{\|\infty\|}$, then all objects of the infinity quantifier $\forall^{\|\infty\|}$ without exception get in this category and infinity axiom $Ax^{\|\infty\|}$ appears under the shock as the primary source of idealization and dangers. Theorems 4.1–4.5 confirm validity of such reasons.

5. **The conclusion**

The acting system of cognition with basic infinity axiom $Ax^{\|\infty\|}$ is oriented on boundless opportunities of mathematical method $\mathscr{M}^{\|\infty\|}$. Therefore the theorems which are limit method can testify only about the unfounded state of system in its extreme phenomena. In such case the zone of catastrophe is the fundamental scientific directions. However the system of cognition is obliged to prove force and to overcome arisen severe difficulties.

It is promptly to blame the set theory $TS^{\|\infty\|} \subset SI_{TT}^{\|\infty\|}$ for created alarming situation in the sphere of cognition. It has too multilateral character. Comprehensive and careful analysis [1–7] has allowed to reveal much more essential, effective and initial reason. It is the axiom $Ax^{\|\infty\|}$, which has extremely resolutely and irrevocably influencing on all phenomena of the field of knowledge including the theory of sets. But the axiom $Ax^{\|\infty\|}$ is not founded [1–3,12] by nothing. And this fact has the most negative effect for a fate and state of the theories of scientific cognition. It is impossible to be guided by illusory idea of unlimitedness in creative researches. But it means immediately that the basis of the system cognition is unsatisfactory and requires the obligatory replacement.

The continuum-hypothesis and number of integrated problems is not unique, that acting idealized system was stumble over. There are many negative examples [1,3] and all of them are characterized by weak (unfounded) provable base. Those fact that for problems as the continuum-hypothesis such basis is impossible to intensify on principle is the most unpleasant. The solemn proclamation by system $SI_{TT}^{\|\infty\|}$ the existence of object should not consider and to concern seriously as far as before the system has declared about foundation of unreasonable axiom $Ax^{\|\infty\|}$, where objects requiring for the proclamations appeared. The concept of infinity axiom $Ax^{\|\infty\|}$ determining creation of the acting cognition system $SI_{TT}^{\|\infty\|}$ presents permanent proofs of falsehood at the reference to idealizations. There are great difficulties in this elevated but doubtful sphere. Such effect [3,12] notices also here and not only at the analysis of Cantorian theorems.

The opportunities of adaptation of achievements $SI_{TT}^{\|\infty\|}$ to a reality *RR* are not boundless and they are connected with the admissibleness of growing rapprochements of objects $\Omega^{\|\infty\|}$ and real analogues. There are examples when it is impossible here and extending polyextremal modeling [1] in general inaccessibly to the adequate approximation.

Absolute proofs is not present for any system *SS* especially over borders of impossibility [3]. However the measure of proof standing up for the concept of approach to the reality *RR*, is incomparable higher than measure of proof of knowledge $ZZ^{\|\infty\|} \subset SI_{TT}^{\|\infty\|}$. The serious doubts to the address of the acting system and axiom $Ax^{\|\infty\|}$ have occurred not yesterday. With the creation of reasonable alternative to the acting thesis [1–3] the doubts has transformed in founded provable base. The system $SI_{TT}^{\|\infty\|}$ with the help of manipulations got over difficulties earlier, but nowadays had demonstrated growing weakness. Theorems of work [12] about the role of non-existent in imagination permit to generalize consequences of uncontrollable fantasies of the existent cognition.

Theorem 5.1. *The penetration of law* $\mathscr{Z}\mathscr{N}_{(+)}^{\|\infty\|} \subset SI_{TT}^{\|\infty\|}$ *of acting system of the formation of knowledge in zones of impossibility* $\|\mathfrak{I}\div\infty\|$ *determines the simultaneous existence of the opposite law* $\overline{\mathscr{Z}\mathscr{N}}_{(-)}^{\|\infty\|} \subset SI_{TT}^{\|\infty\|}$*, and that's why the ambiguity and falsehood* $\|\bar{\&}\|$ *of idealized cognition* $\mathscr{P}\mathscr{L}^{\|\infty\|}\big(Ax^{\|\infty\|}, \Omega_k^{\|\infty\|}, \mathscr{I}\mathscr{L}\mathscr{G}^{\|\infty\|}\big)$.

$$SA: \quad \Big\{ZN_{(+)}^{\|N\to\infty\|} \overset{SI^{\|\infty\|}}{\Longrightarrow} \mathscr{Z}\mathscr{N}_{(+)}^{\|G\div\infty\|}\Big\} \overset{\mathscr{P}\mathscr{L}^{RR}}{\Longrightarrow} \Big\{\overline{\mathscr{Z}\mathscr{N}}_{(-)}^{\|\infty\|}\Big\} \overset{\mathscr{P}\mathscr{L}^{RR}}{\Longrightarrow} \|\bar{\&}\|\big(\mathscr{P}\mathscr{L}^{\|\infty\|}\big). \quad (5.1)$$

Proof. The system $SI_{TT}^{\|\infty\|}$ is absolutely sure in naturalism and uniqueness of idealized way $ZN_{(+)}^{\|N\to\infty\|} \Rightarrow \mathscr{Z}\mathscr{N}_{(+)}^{\|G\div\infty\|} \Rightarrow \mathscr{Z}\mathscr{N}^{\|\infty\|}$ with the reception of laws, therefore, and knowledge, which are solemnly announced as positive and fundamental. This position is in need of the serious correction because of the non-existence of all objects $\Omega^{\|G\div\infty\|}$ and equivalence of laws $\mathscr{Z}\mathscr{N}_{(+)}^{\|\mathfrak{I}\div\infty\|} \equiv \overline{\mathscr{Z}\mathscr{N}}_{(-)}^{\|\mathfrak{I}\div\infty\|}$ with the falsehood $\|\bar{\&}\|\big(\mathscr{Z}\mathscr{N}^{\|\infty\|}, \Omega^{\|\infty\|}, \mathscr{T}\mathscr{T}^{\|\infty\|} \subset \mathscr{P}\mathscr{L}^{\|\infty\|}\big)$. The expression (5.1) illustrates this statement of system *SA* and real cognition. □

Statements about unfounded concepts of non-enumerables, continuum and the first Hilbert problem, in the result of the offered theorem are obtain to attract the intend attention like others mistakes of fundamental cognition.

Theorem 5.2. *The exception of sphere of impossibility* $\|\mathfrak{I}\div\infty\|$ *from cognition* $\mathscr{P}\mathscr{L}^{RR} \to \mathscr{P}\mathscr{L}^{\|\infty\|}$ *and existence determines the matter of the restriction principle PR.*

$$SA: \quad \Big\{\Omega^{\|\mathfrak{I}_{rs}\div\mathfrak{I}\div\infty\|} \not\subset \mathscr{P}\mathscr{L}^{RR}\Big\} \overset{DD-RR}{\Longrightarrow} \Big\{\mathfrak{I}^{-1} \ll A_{rs} < \big\|\Omega_{rs}^{RR}\big\|_{RS} < B_{rs} \ll \mathfrak{I}\Big\} \equiv PR. \quad (5.2)$$

Proof. It is inadmissibly to put into area of cognition of objects which are belong to the zone of unattainableness, but the system $SI_{TT}^{\|\infty\|}$ ignores this logic regularly. On the other hand the reality $DD-RR$ and the adequacy system *SA* [1–7] hold on to it unconditionally. But fixed estimations of characteristics of all objects $\Omega_{rs}^{RR} \subset RS$ obtained in (5.2) reflect the essentiality of principle *PR* as the basis of the system *SA*. □

The criticism requires in special explanations of philosophical character of the article and some entered concepts which are unusual on the first view.

The scientific cognition $Sc^{\|\infty\|}$ founds on the infinity axiom and the fundamental scientific directions structurally and ineradicable contain this axiom $Ax^{\|\infty\|}$ in central constructions. But axiom $Ax^{\|\infty\|}$ is not only especially philosophical, but it is absolutely unfounded, direct contradicts to all of real supervision and requires in the unconditional faith and the declarative agreement. Taking into account a role of axiom in becoming of whole extremely plentiful set of objects $\Omega^{\|\infty\|}$ of cognition, the philosophical character of the researched constructions inconveniently to dispute.

The restriction principle *PR* has not avoided a philosophical sense, as far as it has acted as basis of the new cognitive system, including and mathematics. But the axiom $Ax^{\|\infty\|}$ and the principle *PR* there are on opposite poles of validity, as far as the restriction factor is self-justified.The illusive part of objects $\Omega^{\|\infty\|}$ is so essential, that it must deforms the cognition. However, all mathematical constructions in and the conclusions are maintained in strictly classical style, according to hypothesis of existence of enumerable infinity of capacity $\aleph_0$, which was accepted here.

And nevertheless, they have led to conclusions, which are evidently contradict to basic ideas and views of acting mathematics.

Naturally, new reasonable concepts which are adequate to received data can not lean on the former apparatus of cognition, including mathematical. Therefore here in the preliminary form the system of concepts and designations [1–7, 11, 12] is offered. It liquidates contradictions and schedules ways of exit from the deadlock.

The cycle of works about the conception of restriction principle in the published look chronologically was caused by the necessity of solution of the problem of randomness [1, 13, 14]. It become clear later, that another important problems of mathematics, physics and philosophy [1-7, 10-14] need in the limitation of the huge influence of infinity axiom.

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Mikhail Valentinovich Antipov, *Russian Academy of Sciences, Siberian Branch, Novosibirsk, Russia*.
E-mail: amv@osmf.sscc.ru

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