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Research Article

On b-Regularity and Normality in Intuitionistic Fuzzy Topological Spaces

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Abstract. In this paper, the concept of fuzzy b-open (fuzzy b-closed)sets to introduce some new types of intuitionistic fuzzy b separation axioms, intuitionistic fuzzy b-T_i space (for $i = 3,4$) and intuitionistic fuzzy b-regular and b-normal spaces are introduced, some theorems about them are investigated. Stronger forms of intuitionistic fuzzy b-regular and intuitionistic fuzzy b-normal spaces are introduced. Moreover, the relationships between these separation axioms and others are investigated.

Keywords. Intuitionistic fuzzy b separation axioms; Intuitionistic fuzzy b-regular space; intuitionistic fuzzy b-normal space

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1. Introduction

In 1996, Adnadjevic [1] introduced the concept of b-open sets in general topology, and Othman [23] extend that concept to fuzzy topological space. Several authors introduced the concepts of fuzzy separation axioms using the notion of fuzzy open set see (Ghanim, Kerre and Mashhour [17]). Singal and Rajvanshi [26], Balasubramanian [8], Mahmoud, Alla and Ellah [21], and Othman and Latha [22] by using the notions of fuzzy regular open sets, fuzzy β-open sets, fuzzy α-open sets and fuzzy semi α-open sets respectively. Singal and Prakash [25] have introduced the concept of fuzzy pre-separation axioms. Qahtani and Al-Qubati [2] have introduced and studied new kinds of fuzzy pre-separation axioms. Several notions based on fuzzy pre-separation axioms have been studied. After defining the concept of intuitionistic fuzzy set by Atanassov [4] and intuitionistic fuzzy topological spaces, by Coker in [15, 16], some authors studied the concept of separation axioms in intuitionistic fuzzy topological spaces. In (2001), Bayhan and Coker [9] gave some characterizations of T_1 and T_2 separation axioms in intuitionistic topological spaces, they gave interrelations between several types of separation axioms and some counterexamples. In 2003 [18] Lupianez defined new notions of Hausdorffness in the intuitionistic fuzzy topological spaces. In (2005) [11], Bayhan and Coker studied pairwise separation axioms in double intuitionistic topological spaces. For more studies, we can find them in ([10], [14], [19], [24]). Al-Qubati and Al-Qahtani [3] have introduced and studied new types of b-separation axioms (b-T_i-space, for $i = 0,1,2$) in intuitionistic fuzzy topological spaces. The main purpose of this paper is to introduce and study some new types of intuitionistic fuzzy b-separation axioms, which is intuitionistic fuzzy b-T_i space (for $i = 3,4$) and (intuitionistic Fuzzy b-Regular and Fuzzy b-normal spaces by using the concept of fuzzy b-open (fuzzy b-closed) sets. Also, we will introduce stronger forms of (intuitionistic fuzzy b-regular) and (intuitionistic fuzzy b-normal) spaces with relationships between these separation axioms and others.

2. Preliminaries

Throughout, this paper by (X,τ) or simply by X, we mean an intuitionistic fuzzy topological space (Ifts, Shorty).

Definition 2.1 ([4–7]). Let X be a nonempty fixed set. An intuitionistic fuzzy set A (IFS for short) in X is an object having the form $A = \{\langle x,\mu_A(x),\gamma_A(x)\rangle : x \in X\}$, where the function $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x)+\gamma_A(x) \le 1$, for each $x \in X$.

Definition 2.2 ([5]). Let X be a nonempty set and the form $A = \{\langle x,\mu_A(x),\gamma_A(x)\rangle : x \in X\}$, $B = \{\langle x,\mu_B(x),\gamma_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if for all $x \in X[\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)]$,

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$,

(c) $A = \{\langle x,\gamma_A(x),\mu_A(x)\rangle : x \in X\}$,

(d) $A \bigcap B = \{\langle x,\mu_A(x) \wedge \mu_B(x),\gamma_A(x) \vee \gamma_B(x)\rangle : x \in X\}$,

(e) $A \bigcup B = \{\langle x,\mu_A(x) \vee \mu_B(x),\gamma_A(x) \wedge \gamma_B(x)\rangle : x \in X\}$,

(f) $0_{\tilde{X}} = \{\langle x,,0,1\rangle : x \in X\}$ and $1_{\tilde{X}} = \{\langle x,1,0\rangle : x \in X\}$.

Definition 2.3 ([5]). Let $\{A_i : i \in I\}$ be an arbitrary family of IFS in X. Then,

(a) $\bigcap A_i = \{\langle x,\wedge\mu_{A_i}(x),\vee\gamma_{A_i}(x) : x \in X\}$,

(b) $\bigcup A_i = \{x,\vee\mu_{A_i}(x),\wedge\gamma_{A_i}(x) : x \in X\}$.

Definition 2.4 ([13]). Let $\alpha,\beta\in[0,1],\alpha+\beta\leq 1$. An intuitionistic fuzzy point (IFP for short) of nonempty set X is an IFS of X denoted by $P=x_{(\alpha,\beta)}$ and defined by:

$$P=x_{(\alpha,\beta)}(y)=\begin{cases}(\alpha,\beta) & \text{if } x=y,\\ (0,1) & \text{if } x\neq y.\end{cases} \tag{2.1}$$

In this case, x is called the support of $x(\alpha,\beta)$ and α, β are called the value and no value of $x(\alpha,\beta)$, respectively.

Clearly, an intuitionistic fuzzy point can be represented by an ordered pair of fuzzy point as follows:

$$x_{(\alpha,\beta)}=(x_\alpha,1-x_{(1-\beta)}).$$

In IFP, $x_{(\alpha,\beta)}$ is said to belong to an IFS $A=\{\langle x,\mu_A(x),\gamma_A(x)\rangle : x\in X\}$ denoted by $P=x_{\alpha,\beta}\in A$ (or $P\subseteq A$), if $\alpha\leq\mu_A(x)$ and $\beta\geq\gamma_A(x)$.

We identify a fuzzy point x_r in X by the intuitionistic fuzzy point $x_{(r,(1-r))}$ in X.

Proposition 2.5. *An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points belonging to A.*

The proof is on similar lines as in [20].

Definition 2.6 ([4]). Let X and Y be two nonempty sets and $f:X\to Y$ be a function. Then,

(a) if $B=\{\langle x,\mu_B(y),\gamma_B(y)\rangle : y\in Y\}$ is an intuitionistic fuzzy set in Y, then the preimage of B under$f$ denoted by$f^{-1}(B)$ is the IFS in X defined by

$$f^{-1}(B)=\{\langle x,f^{-1}(\mu_B)(x),f^{-1}(\gamma_B)(x)\rangle : x\in X\}.$$

(b) if $B=\{\langle x,\lambda_B(y),\nu_B(y)\rangle : x\in X\}$ is an intuitionistic fuzzy set in X, then the image of A under f denoted by $f(A)$ is the intuitionistic fuzzy set in Y defined by

$$f(A)=\{\langle y,f(\lambda_A(y)),1-f(1-\nu_A)(y)\rangle : y\in Y\},$$

where

$$f(\lambda_A)(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)}\{\lambda_A(x)\} & \text{if } f^{-1}(y)\neq\emptyset,\\ 0, & \text{otherwise}\end{cases}$$

and

$$1-f(1-\upsilon_A)(y)=\begin{cases}\inf\limits_{x\in f^{-1}(y)}\{\nu_A(x)\} & \text{if } f^{-1}(y)\neq\emptyset,\\ 1, & \text{otherwise.}\end{cases}$$

Proposition 2.7 ([6]). *Let A,A_i $(i\in I)$ be IFSs in X,B,B_j $(j\in J)$ IFSs in Y*

(a) $A_1\subseteq A_2\Rightarrow f(A_1)\subseteq f(A_2)$,

(b) $B_1\subseteq B_2\Rightarrow f^-1(B_1)\subseteq f^{-1}(B_2)$,

(c) $A\subseteq f^-(f(A))$ *and if f is injective, then* $A=f(f^{-1}(A))$,

(d) $f^-(f(B))\subseteq B$ *and if f is surjective, then* $f(f^{-1}(B))=B$,

(e) $f^{-1}(1_{\tilde{Y}}) = 1_{\tilde{X}}$,

(f) $f^{-1}(0_{\tilde{Y}}) = 0_{\tilde{X}}$,

(g) $f(1_{\tilde{X}}) = 1_{\tilde{Y}}$ *if f is surjective,*

(h) $f(0_{\tilde{X}}) = 0_{\tilde{Y}}$.

Replacing fuzzy sets [27] by intuitionistic fuzzy sets in Chang's definition of a fuzzy topological space [12], we get the following.

Definition 2.8 ([15])**.** An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family of intuitionistic fuzzy sets in X satisfy the following axioms:

(T1) $0_{\tilde{X}}, 1_{\tilde{X}} \in X$.

(T2) If $A_1, A_2 \in \tau$, then $A_1 \bigcap A_2 \in \tau$.

(T3) If $A_\lambda \in \tau$ for each λ in Λ, then $\bigcup\limits_{\lambda \in \Lambda} A_\lambda \in \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS for short) and each intuitionistic fuzzy set in τ is known as an intuitionistic fuzzy open set (IFOS for short) of X, and the complement of an intuitionistic fuzzy closed set (IFCS for short).

Example 2.9 ([15])**.** Let $X = \{a,b,c\}$ and M_1, M_2, M_3 and M_4 be an intuitionistic fuzzy sets on X defined as follows:

$$M_1 = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle, \quad M_2 = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.3}, \frac{c}{0.3}\right)\right\rangle,$$

$$M_3 = \left\langle x, \left(\frac{a}{0.5}, \frac{b}{0.6}, \frac{c}{0.4}\right), \left(\frac{a}{0.2}, \frac{b}{0.3}, \frac{c}{0.3}\right)\right\rangle, \quad M_4 = \left\langle x, \left(\frac{a}{0.4}, \frac{b}{0.5}, \frac{c}{0.2}\right), \left(\frac{a}{0.5}, \frac{b}{0.4}, \frac{c}{0.4}\right)\right\rangle.$$

Then the family $\tau = \{0_{\tilde{X}}, 1_{\tilde{X}}, M_1, M_2, M_3, M_4\}$ is an IFT on X.

Definition 2.10. An IFS A of an IFTS (X,τ) is an

(a) intuitionistic fuzzy b-open set (IFbOS in short) if $A \subseteq int(cl(A)) \bigcup cl(int(A))$

(b) ntuitionistic fuzzy b-closed set (IFbCS in short) if $cl(int(A)) \bigcap int(cl(A))$

Definition 2.11 ([15])**.** Let A any intuitionistic fuzzy set. Then,

$Ibcl(A) = \bigcap\{F : A \subseteq F,\ F$ is IFbCS in $X\}$ is called intuitionistic fuzzy b-closure.

$Ibint(A) = \bigcup\{U : U \subseteq A,\ U$ is IFbOS in $X\}$ is called intuitionistic fuzzy bInterior.

Example 2.12. In Example 2.9, if

$$A = \left\langle x, \left(\frac{a}{0.55}, \frac{b}{0.55}, \frac{c}{0.45}\right), \left(\frac{a}{0.3}, \frac{b}{0.4}, \frac{c}{0.3}\right)\right\rangle,$$

then,

$$int(A) = \bigcup\{U : U \subseteq A,\ U \text{ is } \mathit{IFbOS} \text{ in } X\} = M_4$$

and

$$cl(A) = \bigcap\{F : A \subseteq F,\ F \text{ is } \mathit{IFCS} \text{ in } X\} = 1_{\tilde{X}}.$$

Corollary 2.13 ([23])**.** *Any union of intuitionistic fuzzy b-open sets is a fuzzy b-open set.*

Remark 2.14 ([23])**.** The intersection of intuitionistic fuzzy b-open sets need not be fuzzy b-open set.

Definition 2.15. A mapping $f:(X,\tau)\to(Y,\rho)$ from an intuitionistic fuzzy topological space (X,τ) to another intuitionistic fuzzy topological space (Y,ρ) is said to be intuitionistic fuzzy b-continuous if $f^{-1}(M)$ is an intuitionistic fuzzy b-open set in X for each intuitionistic fuzzy open set M in Y.

- Intuitionistic fuzzy b^*-continuous if $f^{-1}(M)$ is intuitionistic fuzzy b-open set in X for each intuitionistic fuzzy b-open set M in Y.
- Intuitionistic fuzzy b^{**}-continuous if $f^{-1}(M)$ is intuitionistic fuzzy open set in X for each intuitionistic fuzzy b-open set M in Y.

Definition 2.16. Let (X,τ) be an IFTS and $Y\subseteq X$. Then $(Y,\tau|Y)$ is called a subspace of (X,τ) where

$$\tau|Y=\{G|Y=(\mu_G|Y,\nu_G|Y):G\in\tau\}.$$

3. The Main Results

Definition 3.1. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy b-T_0 (or in short *IFb*-T_0) if for every pair of intuitionistic fuzzy points $p=x_{(\alpha,\beta)}$, $q=y_{\gamma,\eta}$ with different supports, there exists an intuitionistic fuzzy b-open set M such that either $(p\subseteq M, q\nsubseteq M)$ or $(q\subseteq M, p\nsubseteq M)$.

Definition 3.2. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy b-T_1 (or in short *IFb*-T_1) if for every pair of intuitionistic fuzzy points $p=x_{(\alpha,\beta)}$, $q=y_{\gamma,\eta}$ with different supports, there exists an intuitionistic fuzzy b-open sets M and N such that $(p\subseteq M, q\nsubseteq M)$ and $(q\subseteq N, p\nsubseteq N)$.

Definition 3.3. An intuitionistic fuzzy topological space (X,τ) is said to be aintuitionistic fuzzy stronger b-T_1 (or in short *IFb*-Ts) if every intuitionistic fuzzy point is an intuitionistic fuzzy b-closed set.

Remark 3.4. Clearly, an intuitionistic fuzzy stronger b-T_1-space is an intuitionistic fuzzy b-T_1 but the converse need not be true as shown by the following example.

Example 3.5. Let $X=\{a,b\}$ and M_1,M_2,M_3 and M_4 be an intuitionistic fuzzy sets on X defined as follows:

$$M_1=\left\langle x,\left(\frac{a}{1.0},\frac{b}{0.0}\right),\left(\frac{a}{0.0},\frac{b}{1.0}\right)\right\rangle,\quad M_2=\left\langle x,\left(\frac{a}{0.0},\frac{b}{1.0}\right),\left(\frac{a}{1.0},\frac{b}{0.0}\right)\right\rangle,$$

$$M_3=\left\langle x,\left(\frac{a}{0.7},\frac{b}{0.0}\right),\left(\frac{a}{0.8},\frac{b}{0.6}\right)\right\rangle,\quad M_4=\left\langle x,\left(\frac{a}{0.8},\frac{b}{1.0}\right),\left(\frac{a}{0.3},\frac{b}{0.2}\right)\right\rangle.$$

Then the family $\tau = \{0_{\tilde{X}}, 1_{\tilde{X}}, M_1, M_2, M_3, M_4\}$ is an IFT on X. All crisp intuitionistic fuzzy points are intuitionistic fuzzy b-closed sets, so the space (X,τ) is an intuitionistic fuzzy b-T_1 space but not every intuitionistic fuzzy points is intuitionistic fuzzy b-closed sets.

Definition 3.6. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy b-*Hausdorff* (or in short IFb-T_2) if for every pair of intuitionistic fuzzy points $p = x_{(\alpha,\beta)}$, $q = y_{(\gamma,\eta)}$ with different supports, there exists an intuitionistic fuzzy b-open sets M and N such that $(p \subseteq M, q \nsubseteq M)$ and $(q \subseteq N, p \nsubseteq N)$ and $M \nsubseteq N$.

Definition 3.7. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy b-*Uryshon* (or in short IFb-$T_{2\frac{1}{2}}$) if for every pair of intuitionistic fuzzy points $p = x_{(\alpha,\beta)}$, $q = y_{(\gamma,\eta)}$ with different supports, there exists an intuitionistic fuzzy b-open sets M and N such that $(p \subseteq M, q \nsubseteq M)$ and $(q \subseteq N, p \nsubseteq N)$ and $Ibcl(M) \nsubseteq Ibcl(N)$.

Definition 3.8. An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy b-*regular* (or in short IFb-R) if for every pair of intuitionistic fuzzy points p and intuitionistic fuzzy b-closed N such that $p \nsubseteq N$ there exists an intuitionistic fuzzy b-open sets M_1 and M_2 such that $(p \subseteq M_1, N \subseteq M_2$ and $M_1 \nsubseteq M_2)$.

Definition 3.9. A fuzzy topological space $(X,\tau)$is said to be an intuitionistic fuzzy b-T_3 space if it is a intuitionistic fuzzy b-regular space as well as intuitionistic fuzzy b-stronger T_1 space.

Theorem 3.10. *If (X,τ) an intuitionistic fuzzy b-regular space (IFb-RS), then for any intuitionistic fuzzy point $p = x_{(\alpha,\beta)}$ and an intuitionistic fuzzy open set N such that $p \subseteq N$, there exists an intuitionistic fuzzy b-open set M_1 such that $p \subseteq M_1 \subseteq Ibcl(M_1) \subseteq N$.*

Proof. Suppose that X be an IFb-RS. Let $N = \langle x, \mu_N(x), \gamma_N(x)\rangle$ be an IFOS of X and p be an IF point in X such that $p \subseteq N$. Then $N^c = \langle x, \gamma_N(x), \mu_N(x)\rangle$ is an IFbCS in X. Since X is an IFb-RS, therefore there exist two IFbOSs M_1 and M_2 such that $p \in M_1, N^c \subseteq M_2$ and $M_1 \nsubseteq M_2$. Now M_2^c is an IFbCS such that $M_1 \subseteq M_2^c \subseteq N$, thus, $p \in M_1 \subseteq Ibcl(M_1)$ and $Ibcl(M_1) \subseteq M_2^c \subseteq N$, so $Ibcl(M_1) \subseteq N$. Hence $p \subseteq M_1 \subseteq Ibcl(M_1) \subseteq N$. □

Proposition 3.11. *Every subspace of IFb-Regular space is IFb-regular.*

Proof. Let (X,τ) be a IFb regular-space and Y be subspace of X. To prove that Y is an IFb-regular, where $\tau_Y = \{G_Y = \langle x, \mu_{G|Y}(x), \nu_{G|Y}(x)\rangle,\ x \in Y, G \in \tau\}$ and $G = \langle x, \mu_G(x), \nu_G(x)\rangle$. Let $p = x_{(\alpha,\beta)}$ be an IFP in Y, and NY be an IF-b CS in Y such that $p \nsubseteq NY$. Since Y is a subspace of X, so $p \in X$ and there exist an IFb-CS F in X such that the closed set generated by it for Y is F_Y. Since X is IFb-Rs such that $p \nsubseteq F$, there exist two intuitionistic fuzzy b-open sets (IFbOSs) M_1, M_2 such that $p \subseteq M_1 = \langle x, \mu_{M_1}(x), \gamma_{M_1}(x)\rangle$ and $N \subseteq M_1 = \langle x, \mu_{M_2}(x), \gamma_{M_2}(x)\rangle$ and $M_1 \nsubseteq M_2$ $M_{1Y} = \langle x, \mu_{M_{1|Y}}(x), \gamma_{M_{1|Y}}(x)\rangle$ and $M_{2Y} = \langle x, \mu_{M_{2|Y}}(x), \gamma_{M_{2|Y}}(x)\rangle$ are IF-bOS in Y such that that $p \subseteq M_{1Y}$, $N \subseteq M_{2Y}$ and $M_{1Y} \nsubseteq M_{2Y}$. Hence Y is an IFb-regular space. □

Theorem 3.12. *An intuitionistic fuzzy topological space (X,τ) is said to be an intuitionistic fuzzy b-regular space if and only if for an intuitionistic fuzzy point $p = x_{(\alpha,\beta)}$ and an intuitionistic fuzzy closed set N such that $p \nsubseteq N$, there exist two intuitionistic fuzzy b-open sets M_1, M_2 such that $p \subseteq M_1$, $N \subseteq M_2$, and $Ibcl(M_1) \nsubseteq (Ibcl(M_2))$.*

Proof. Since $p \nsubseteq N$ and (X,τ) is (IFb-R), there exist two intuitionistic fuzzy b-open sets M_1, M_2 such that $p \subseteq N$, $N \subseteq M_2$ and $M_1 \nsubseteq M_2$ and by Theorem 3.10 $p \subseteq M_2 \subseteq Ibcl(M_2) \subseteq M_1$. Hence $Ibcl(M_2) \subseteq M_1$. Also, $N \subseteq M_2 \subseteq Ibcl(M_2)$. But since $Ibcl(M_1) \nsubseteq Ibcl(M_2)$, then $N \subseteq M_2 \subseteq Ibcl(M_2) \subseteq Ibcl(M_1) \subseteq M_1^c$ and hence $p \subseteq M_1$, $N \subseteq M_2$ and $Ibcl(M_1) \nsubseteq (Ibcl(M_2))$.

The converse is clear. □

Theorem 3.13. *Let (X,τ) be an IFb-R space which is also IF-T_0. Then (X,τ) is IFb-$T_{2\frac{1}{2}}$.*

Proof. Let $p = x_{(\alpha,\beta)}$ and $q = x_{(\gamma,\eta)}$ are intuitionistic fuzzy points with different supports. Since (X,τ) is an IF-T_0 space, then there exists an intuitionistic fuzzy open set N such that $p \subseteq N$, $q \nsubseteq N$ or $q \subseteq N$, $p \nsubseteq N$. Consider the part $p \subseteq N$, $q \nsubseteq N$. This implies that $p \nsubseteq N^c$ where N^c is an intuitionistic fuzzy closed set. Since (X,τ) is an IFb-R, so by Theorem 3.12, there exist an intuitionistic fuzzy b-open sets M_1 and M_2 such that $p \subseteq M_1$ and $N^c \subseteq M_2$ and $Ibcl(M_1) \nsubseteq (Ibcl(M_2))$ or $p \subseteq M_1$, $q \subseteq M_2$ and $Ibcl(M_1) \nsubseteq (Ibcl(M_2))$. Hence (X,τ) is IFb-$T_{2\frac{1}{2}}$. □

Theorem 3.14. *If $f : (X,\tau) \to (Y,\rho)$ is a closed injective intuitionistic fuzzy b-continuous mapping and (Y,ρ) is an intuitionistic fuzzy regular space, then (X,τ) is an intuitionistic fuzzy b-regular space.*

Proof. Let p be an intuitionistic fuzzy point and N an intuitionistic fuzzy closed set in X. Then $f(p)$ is an intuitionistic fuzzy point and $f(N)$ is an intuitionistic fuzzy closed set in Y. Since (Y,ρ) is an intuitionistic fuzzy regular space then there exist two intuitionistic fuzzy open sets M_1 and M_2 such that $f(N) \subseteq M_2$, $f(p) \subseteq M_1$ and $M_1 \nsubseteq M_2$. It follows that $N \subseteq f^{-1}(M_2)$, $p \subseteq f^{-1}(M_1)$ and $f^{-1}(M_1) \subseteq f^{-1}(M_2)$ where $f^{-1}(M_1)$ and $f^{-1}(M_2)$ are intuitionistic fuzzy b-open sets in X. Hence $f : (X,\tau)$ is an intuitionistic fuzzy b-regular space. □

Theorem 3.15. *If $f : (X,\tau) \to (Y,\rho)$ be an intuitionistic fuzzy b^{**}-continuous and closed injection mapping. Then X is an intuitionistic fuzzy regular space if Y is intuitionistic fuzzy b-regular space.*

Theorem 3.16. *Every IFb-R and IFT_2 space is an IFb-$T_{2\frac{1}{2}}$ space.*

Proof. Let (X,τ) be an intuitionistic fuzzy space which is IFb-R and IF-T_2. Let p and q be two intuitionistic fuzzy points with different supports in X. Since X is an IF-T_2, there exist two intuitionistic fuzzy open sets M and N such that $p \subseteq M$, $q \nsubseteq M$, $q \subseteq N$, $p \nsubseteq N$ and $M \nsubseteq N$ or $Ibcl(M) \nsubseteq N$. So that $q \nsubseteq (M)$. Since $Ibcl(M)$ is an intuitionistic fuzzy closed set and (X,τ) is an IFb-R. So by Proposition 3.11, there exist an intuitionistic fuzzy b-open sets M_1 and M_2 such that $Ibcl(M) \subseteq M_2$, $q \subseteq M_1$ and $Ibcl(M_1) \nsubseteq Ibclu(M_2)$. Since $p \subseteq Ibcl(M)$, we have $p \subseteq M_2$, $q \subseteq M_1$, and $Ibcl(M_1) \nsubseteq Ibcl(M_2)$. Hence the space (X,τ) is IFb-$T_{2\frac{1}{2}}$ space. □

Definition 3.17. An intuitionistic fuzzy topological space (X,τ) is said to be

- intuitionistic fuzzy b^*-regular space (or in short IFb^*-R) if for an intuitionistic fuzzy point p and an intuitionistic fuzzy b-closed set N such that $p \nsubseteq N$, there exist two intuitionistic fuzzy b-open sets M_1 and M_2 such that $N \subseteq M_2$, $p \subseteq M_1$ and $M_1 \nsubseteq M_2$.
- intuitionistic fuzzy b^{**}-regular space (or in short IFb^{**}-R) if for an intuitionistic fuzzy point p and an intuitionistic fuzzy b-closed set N such that $p \nsubseteq N$, there exist two intuitionistic fuzzy open sets M_1 and M_2 such that $N \subseteq M_2$, $p \subseteq M_1$ and $M_1 \nsubseteq M_2$.

Theorem 3.18. *An intuitionistic fuzzy topological space (X,τ) is said to be*

(1) *intuitionistic fuzzy b^*-regular space (or in short IFb^*-R) if for an intuitionistic fuzzy point p and an intuitionistic fuzzy b-open setM_1 such that $p \subseteq M_1$, there exist an intuitionistic fuzzy b-open set M_2 such that $p \subseteq M_2 \subseteq IbclM_2 \subseteq M_1$.*

(2) *intuitionistic fuzzy b^{**}-regular space (or in short IFb^{**}-R) if for an intuitionistic fuzzy point p and an intuitionistic fuzzy b-open set M_1 such that $p \subseteq M_1$, there exist an intuitionistic fuzzy open set M_2 such that $p \subseteq M_2 \subseteq IbclM_2 \subseteq M_1$.*

Proof. (1) The proof is similar to (2).

(2) Let M_1 be an intuitionistic fuzzy b-open set such that $p \subseteq M_1$, then $p \nsubseteq M_1^c$ and M_1^c is an intuitionistic fuzzy b-open set. Therefore, there exist two an intuitionistic fuzzy open sets N and K such that $p \subseteq N$, $M_1^c \subseteq K$ and $N \nsubseteq K$. Now, we can get $Ibl(N)k^c \subseteq (M_1^c)^c = M_1$ and $P \subseteq N \subseteq Ibcl(N) \subseteq M_1$. It is clear that $p \subseteq N \subseteq Ibint\ (Ibcl(N) \subseteq Ibcl(N) \subseteq M_1)$. Therefore, if we put $Ibint(Ibcl(N) = M_2$, then $p \subseteq N \subseteq M_2 \subseteq Ibcl(M_2) \subseteq M_1$, where M_2 is an intuitionistic fuzzy open set. The converse is clear. □

Theorem 3.19. *An intuitionistic fuzzy topological space (X,τ) is said to be*

- *intuitionistic fuzzy b^*-regular space (or in short IFb^*-R) if for an intuitionistic fuzzy point p and an intuitionistic fuzzy b-closed set N such that $p \nsubseteq N^c$, there exist two intuitionistic fuzzy b-open sets M_1 and M_2 such that $p \subseteq M_1$, $N \subseteq M_2$, and $Ibcl(M_1) \nsubseteq Ibcl(M_2)$.*
- *intuitionistic fuzzy b^{**}-regular space (or in short IFb^{**}-R) if for an intuitionistic fuzzy point p and an intuitionistic fuzzy b-closed set N such that $p \nsubseteq N$, there exist two intuitionistic fuzzy open sets M_1 and M_2 such that $p \subseteq M_1$, $N \subseteq M_2$, and $Ibcl(M_1) \nsubseteq Ibcl(M_2)$.*

Proof. Similar to that of Theorem 3.18. □

Theorem 3.20. *Let f be an injective, intuitionistic fuzzy b^*-closed and intuitionistic fuzzy b^*-continuous mapping from an intuitionistic fuzzy topological space (X,τ) into an intuitionistic fuzzy topological space (Y,ρ). Then X is b^*-R if Y is b^*-R.*

Theorem 3.21. *Let (X,τ) be an intuitionistic fuzzy b^*-regular space which is also intuitionistic fuzzy b-T_0. Then (X,τ) is fuzzy b-$T_{2\frac{1}{2}}$.*

Proof. Let p and q are intuitionistic fuzzy points with different supports. Since (X,τ) is a fuzzy b-T_0 space, then there exists an intuitionistic fuzzy b-open set M such that $p \nsubseteq M, q \nsubseteq M$ or $q \subseteq M, p \nsubseteq M$. Consider the part $p \nsubseteq M$, $q \nsubseteq M$. This implies that $p \subseteq (M^c)^c$, where M^c is an intuitionistic fuzzy b-closed set. Since (X,τ) is *IFb**-*R*, so by Theorem 3.19, there exist intuitionistic fuzzy b open sets N and K such that$p \subseteq N$, $M^c \subseteq K$ and $Ibcl(N) \nsubseteq Ibcl(K)$, or $p \subseteq N, q \subseteq K$ and $Ibcl(N) \nsubseteq Ibcl(K)$.

Hence (X,τ) is an intuitionistic fuzzy b-$T_{2\frac{1}{2}}$. □

Theorem 3.22. *Let (X,τ) be an intuitionistic fuzzy b^{**}-regular space which is also intuitionistic fuzzy b-T_0. Then (X,τ) is fuzzy b-$T_{2\frac{1}{2}}$.*

Theorem 3.23. *Let (X,τ) be an intuitionistic fuzzy b^{**}-regular space which is also intuitionistic fuzzy b-T_2. Then (X,τ) is fuzzy b-$T_{2\frac{1}{2}}$.*

Definition 3.24. An intuitionistic fuzzy topological space (X,τ) is said to be

- intuitionistic Fuzzy b^*-T_3 space if it is intuitionistic fuzzy b^*-regular space as well as intuitionistic fuzzy b-stronger T_1 space.
- intuitionistic Fuzzy b^{**}-T_3 space if it is intuitionistic fuzzy b^{**}-regular space as well as intuitionistic fuzzy b-strongerT_1 space.

Definition 3.25. An intuitionistic fuzzy topological space (X,τ) is said to be

- Iintuitionistic fuzzy b-**normal** space (or in short IFb-N) if for every pair of intuitionistic fuzzy closed sets N_1 and N_2 such that $N_1 \nsubseteq N_2$, there exist two intuitionistic fuzzy b-open sets(IFbOSs) M_1 and M_2 such that $N_1 \subseteq M_1$, $N_2 \subseteq M_2$ and $M_1 \nsubseteq M_2$.
- Intuitionistic fuzzy b-T_4 space if it is an intuitionistic fuzzy b-**normal** space as well as intuitionistic fuzzy b-stronger T_1 space.

Theorem 3.26. *An intuitionistic fuzzy topological space (X,τ) is an intuitionistic fuzzy b-normal space if and only if for every intuitionistic fuzzy closed set N_1 and every intuitionistic fuzzy open set N_2 such that $N1 \subseteq N_2$ there exists an intuitionistic fuzzy b-open set M such that $N_1 \subseteq M \subseteq Ibcl(M) \subseteq N_2$.*

Proof. Let (X,τ) be an intuitionistic fuzzy b-normal and let $N_1 \subseteq N_2$, where N_1 is a intuitionistic fuzzy closed and N_2 is an intuitionistic fuzzy open set, then $(N_2)^c$ is an intuitionistic fuzzy closed set but (X,τ) be an intuitionistic fuzzy b-normal hence there exist two intuitionistic fuzzy b-open setsM_1 and M_2 such that $N_1 \subseteq M_1$, $(N_2)^c \subseteq M_2$ and $M_1 \nsubseteq M_2$, therefore $N_1 \subseteq M \subseteq (M_2)^c \subseteq N_2$ that implies $Ibc(N_1) \subseteq Ibc(M) \subseteq Ibc(M_2)^c \subseteq Ibc(N_2)$, then $N_1 \subseteq Ibc(N_1) \subseteq Ibc(M_1) \subseteq (M_2)^c \subseteq N_2$. Hence $N_1 \subseteq M_1 \subseteq Ibc(M_1) \subseteq N_2$. The converse is clear. □

Theorem 3.27. *If $f : (X,\tau) \to (Y,\rho)$ is closed injective, intuitionistic fuzzy b-continuous mapping and (Y,ρ) is a intuitionistic fuzzy normal space, then (X,τ) is an intuitionistic fuzzyb-normal space.*

Theorem 3.28. *If $f:(X,\tau)\to(Y,\rho)$ be an intuitionistic fuzzy b^{**}-continuous mapping. Then (X,τ) is a intuitionistic fuzzy normal space, if (Y,ρ) is an intuitionistic fuzzy b-normal space.*

Theorem 3.29. *Every IFb-N andIF-T_s space is IFb-R.*

Proof. Let (X,τ) be an intuitionistic fuzzy b-normal and $(IF\text{-}T_s)$ space. Let p and N be respectively, an intuitionistic fuzzy point and intuitionistic fuzzy closed set in X and $p\nsubseteq N$. Since (X,τ) is IFTs, then p is an intuitionistic fuzzy closed set and since (X,τ) is an intuitionistic fuzzy b-normal, there exist intuitionistic fuzzy b-open sets M_1 andM_2 such that $p\subseteq M_1$, $N\subseteq M_2$ and $M_1\nsubseteq M_2$. Hence(X,τ) is an intuitionistic fuzzy b-regular. □

Definition 3.30. An intuitionistic fuzzy topological space (X,τ) is said to be

- intuitionistic fuzzy b^*-normal space (or in short *IFb**-*N*) if for every pair of intuitionistic fuzzy b-closed sets N_1 and N_2 such that $N_1\nsubseteq N_2$, there exist two intuitionistic fuzzy b-open sets M_1 and M_2 such that $N_1\subseteq M_1$, $N_2\subseteq M_2$ and $M_1\nsubseteq M_2$.
- intuitionistic fuzzy b^{**}-normal space (or in short *IFb***-*N*) if for every pair of intuitionistic fuzzy b-closed sets N_1 and N_2 such that $N_1\nsubseteq N_2$, there exist two intuitionistic fuzzy open sets M_1 and M_2 such that $N_1\subseteq M_1$, $N_2\subseteq M_2$ and $M_1\nsubseteq M_2$.

Theorem 3.31. (1) *Every IFb*-N and IFb-Ts space is IFb*-R.*

(2) *Every IFb**-N and IFb-Ts space is IFb**-R.*

Proof. (1) Let (X,τ) be an intuitionistic fuzzy b^*-normal and *IFb**-Ts space. Let p and N be respectively, an intuitionistic fuzzy point and an intuitionistic fuzzy b-closed set in X and $p\nsubseteq N$. Since (X,τ) is IFb-Ts, then p is an intuitionistic fuzzy b-closed set and since (X,τ) is intuitionistic fuzzy b^*-normal, there exist an intuitionistic fuzzy b-open sets M_1 and M_2 such that $p\subseteq M_1$, $N\subseteq M_2$ and $M_1\nsubseteq M_2$.

Hence (X,τ) is an intuitionistic fuzzy b^*-regular.

Proof for case (2) is similarly follow. □

Definition 3.32. An intuitionistic fuzzy topological space (X,τ) is said to be

- intuitionistic Fuzzy b^*-T_4 space if it is intuitionistic fuzzy b^*-normal space as well as intuitionistic fuzzy b-stronger T_1 space.
- intuitionistic Fuzzy b^{**}-T_4 space if it is intuitionistic fuzzy b^{**}-normal space as well as intuitionistic fuzzy b-stronger T_1 space.

Theorem 3.33. (1) *Every IFb*-T_4 space is IFb*-T_3.*

(2) *Every IFb**-T_4 space is IFb**-T_3.*

Proof. Strictly follow from Theorem 3.31. □

4. Conclusion

Separation axioms are one among the most common, important and interesting concepts in Topology. They can be used to define more restricted classes of topological spaces. In this paper, we have studied b-regular and b-normal spaces in intuitionistic fuzzy topological spaces and other strong form by using b-open and b-closed sets. Certain other problems to study these spaces by using other sets which are also worthy for future studies.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

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