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Research Article

Multi-criteria Decision Making Method based on Interval Type-2 Fuzzy Sets for Supplier Selection

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Abstract. Supplier selection is one of the most important activities in supply chain management. The right choice of supplier would help the company to reduce purchasing risk, maximizing the overall profit and increasing customer satisfaction. However, the method of selecting the right supplier is not straightforward as it involves a number of potential companies with diverse conflicting criteria. This paper aims to propose a modified technique for order preference by incorporating the concepts of entropy weight interval type-2 fuzzy sets and linguistic weighted average. The proposed method is illustrated with a problem of supplier selection. The evaluation results using the proposed method are consistent with the original results. This indicates that the proposed method offers a feasible solution to supplier selection problems.

Keywords. Supplier selection; Entropy method; Linguistic weighted average; Decision making

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1. Introduction

Supplier selection has been widely recognized as one of the most important processes in the purchasing activities. Supplier selection is the process by which the buyer identifies, evaluates, and contracts with suppliers. The inaccurate selection of suppliers may affect the whole network of supply chain management. Liao and Kao [11] postulated that inappropriate decision will affect the entire supply chain since the position of supplier selection and its related tasks are at the front end in the supply chain process. Supply chain management is the management of a network of interconnected businesses involved in the provision of product and service packages required by the end customers in a supply chain [8]. Many researchers advocated that the production of raw material in the manufacturing process and the component parts can reach up to seventy percent of product cost [15], [12], [7]. Therefore, choosing the right suppliers become a key strategic step in businesses since the impacts are directly goes to manufactures. The challenge in selecting the appropriate suppliers makes supplier selection a fertile topic for operations and management science disciplines. The objectives of supplier selection process are reducing purchase risk, maximizing the overall profit, increasing customer satisfaction, and building closeness relationships between buyers and suppliers [13]. Chou and Chang [4] suggested that manufacturers cannot predict customer demand precisely, and they preferred to manage suppliers by using different methods such as supplier development, supplier evaluation, supplier selection, supplier coordination, etc. Anthony [1] unveils that about thirty percent of all errors happen during the manufacturing process can be blamed on supplier delivery of defective goods. Therefore, some consideration of supplier capacities such as quality control, operational cost and company culture and the willingness of company to cooperate are needed before inviting supplier into a supply chain. Unqualified supplier may reduce the competitiveness and will evoke a supply chain crisis. On the other hand, a qualified supplier can become a central factor of competitiveness [9]. Supplier selection involves more than one criterion and sometimes there will be conflict with each other [21]. The complexity in supplier selection was further emphasised by Wu and Liu [18]. They relate supplier selection as a multi-criteria problem which includes both quantitative and qualitative factors. Therefore, supplier selection can be considered as a multi-criteria decision making (MCDM) problem.

There have been many different methods of MCDM that have been applied to solving supplier selection problem. An extensive review on MCDM methods and its applications to supplier selection was made by Chai et al. [2]. Establishing weight for criteria or importance of every decision maker is one of the critical steps in most MCDM methods. In relation to this, Wu and Mendel [19] used linguistic weighted average (LWA) to obtain weight. The weights were modeled as interval type-2 fuzzy sets (IT2 FSs). The LWA can be viewed as a generalization of the fuzzy weighted average (FWA) where the type-1 fuzzy inputs are replaced with IT2 FSs. In an attempt to introduce a new MCDM based on IT2 FS, Chen and Lee [3] presented a fuzzy ranking method to calculate the ranking values. Cui et al. [6] proposed entropy weight as to avert vagueness of decision makers. One of the most popular preference approaches in MCDM is The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). In this paper, we present a modified IT2 FS TOPSIS approach by replacing the steps of decision makers' weight, aggregation, and reduction with entropy method, LWA and fuzzy ranking method. The proposed IT2 FTOPSIS is applied to a supplier selection problem. The rest of this paper is organized as follows. In Section 2, we recall some basic notions that are needed in this research including some definitions. In Section 3, we present the steps of our proposed method. An example of the implementation of the proposed method for a case of supplier selection is described in Section 4. Finally, the conclusions of this paper are presented in Section 5.

2. Preliminaries

This section introduces the basic definitions and notions of fuzzy set theory, entropy method, LWA and fuzzy ranking method.

Definition 1 (Type-1 Fuzzy Sets [22]). A fuzzy set A in the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ is defined as follows:

$$A = \{\langle x, \mu_A(x)\rangle \mid x \in X\} \tag{1}$$

which is characterized by membership function $\mu_A(x) : X \to [0,1]$, where $\mu_A(x)$ indicates the membership degree of the element x to the set A.

Definition 2 (Type-2 Fuzzy sets [19]). A type-2 fuzzy set $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}$ shown as follows:

$$\tilde{A} = \left\{((x,u), \mu_{\tilde{A}}(x,u)) \;\middle|\; \forall\ x \in X,\ \forall\ u \in J_x \subseteq [0,1],\ 0 \le \mu_{\tilde{A}}(x,u) \le 1\right\} \tag{2}$$

where, J_x denotes an interval in $[0,1]$. Besides, the type-2 fuzzy set $\tilde{A}$ also can be represented as follows:

$$\tilde{A} = \int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \tag{3}$$

where J_x and $\iint$ denotes the union over all admissible x and u.

Definition 3 (Interval Type-2 Fuzzy Sets [19]). An IT2 FS $\tilde{X}$ is characterized by its membership function (MF) $\mu_{\tilde{X}}(x,u)$, i.e.

$$\begin{aligned}\tilde{X} &= \int_{x\in D_{\tilde{X}}} \int_{u\in J_x\subseteq[0,1]} \mu_{\tilde{X}}(x,u)/(x,u) = \int_{x\in D_{\tilde{X}}} \int_{u\in J_x\subseteq[0,1]} 1/(x,u) \\ &= \int_{x\in D_{\tilde{X}}} \left[\int_{u\in J_x\subseteq[0,1]} 1/(u)\right] \Big/ x\end{aligned} \tag{4}$$

where x called the primary variable, has domain $D_{\tilde{X}}$; $u \in [0,1]$, called the secondary variable, has domain $J_x \in [0,1]$ at each $x \in D_{\tilde{X}}$; J_x is also called the support of the secondary MF, and the amplitude of $\mu_{\tilde{X}}(x,u)$, called a secondary grade of $\tilde{X}$, equals 1 for $\forall \in x \in D_{\tilde{X}}$ and $\forall\ u \in J_x \in [0,1]$.

Definition 4 (Trapezoidal membership functions of IT2 FS [14]). The upper membership function and the lower membership function of an IT2 FS are type-1 membership functions, respectively. Figure 1 shows the trapezoidal membership function of T2 FS.

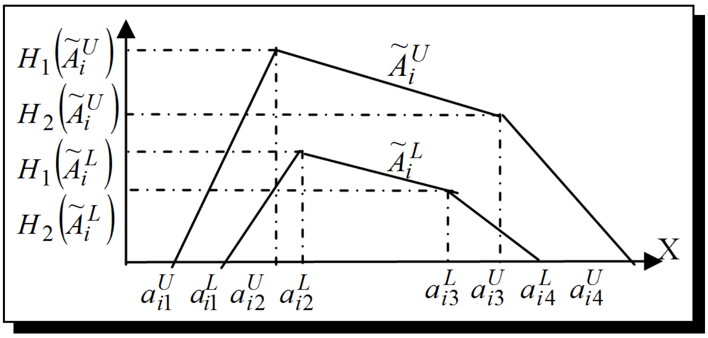

Figure 1. The upper trapezoidal MF$\tilde{\tilde{A}}_i^U$ and the lower MF$\tilde{\tilde{A}}_i^L$ of the IT2 FS$\tilde{A}_i$

Definition 5 (Entropy weight [6]). The entropy value can be defined as

$$e_j = k\sum_{j=1}^{n} p_{ij}\ln p_{ij}, \tag{5}$$

where $k$is a constant, let $k = \frac{1}{\ln m}$.

Definition 6 (Linguistic weighted average [19]). When at least one sub-criterion or weight is modeled as an IT2FS, the resulting weighted average is called a Linguistic Weighted Average. From Wu and Mendel [19], the formula of LWA can be defined as follows:

$$\tilde{Y}_{LWA} = \frac{\sum_{i=1}^{n} \tilde{X}_i \tilde{W}_i}{\sum_{i=1}^{n} \tilde{W}_i}, \tag{6}$$

where $\tilde{X}_i = 1/FOU(\tilde{X}_i) = 1/[\underline{X}_i, \bar{X}]$, $\tilde{W}_i = 1/FOU(\tilde{W}_i) = 1/[\underline{W}_i, \bar{W}]$,
$\tilde{X}_i$ and $\tilde{W}_i$are words modeled by IT2 FSs.

Definition 7 (Fuzzy ranking method [17]). The fuzzy ranking method is defined by considering two interval type-2 trapezoidal fuzzy numbers. Let $\tilde{\tilde{A}}_s^U$ and $\tilde{\tilde{A}}_t^U$ be upper trapezoidal MF of the IT2FS $\tilde{\tilde{A}}_s$ and $\tilde{\tilde{A}}_t$, respectively as shown in Figure 2, where $\tilde{\tilde{A}}_s^U = (a_{s1}^U, a_{s2}^U, a_{s3}^U, a_{s4}^U; H_1(\tilde{\tilde{A}}_s^U), H_2(\tilde{\tilde{A}}_s^U))$ and $\tilde{\tilde{A}}_t^U = (a_{t1}^U, a_{t2}^U, a_{t3}^U, a_{t4}^U; H_1(\tilde{\tilde{A}}_t^U), H_2(\tilde{\tilde{A}}_t^U))$.

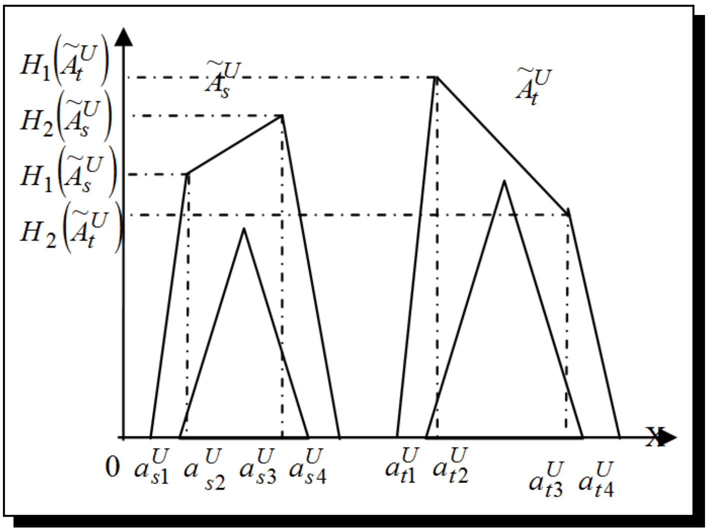

Figure 2. Two Interval Trapezoidal T2 FS$\tilde{\tilde{A}}_s$ and $\tilde{\tilde{A}}_t$.

In order to define the likelihood $p(\tilde{\tilde{A}}_s^U \geq \tilde{\tilde{A}}_t^U)$ of $\tilde{\tilde{A}}_s^U \geq \tilde{\tilde{A}}_t^U$, the strength E_{ts} of $\tilde{\tilde{A}}_t^U$ over $\tilde{\tilde{A}}_s^U$ by considering the difference between $\tilde{\tilde{A}}_{sk}^U$ and $\tilde{\tilde{A}}_{tk}^U$, where $1 \leq k \leq 4$, and by considering the difference between $H_k(\tilde{\tilde{A}}_s^U)$ and $H_k(\tilde{\tilde{A}}_t^U)$ where $1 \leq k \leq 2$ is defined. We define the strength E_{ts} of $\tilde{\tilde{A}}_t^U$ over $\tilde{\tilde{A}}_s^U$ as follows:

$$E_{ts} = \frac{N_{ts}}{D_{ts}}$$

$$= \frac{\sum_{k=1}^{4}\max(a_{tk}^{u}-a_{sk}^{u},0)+(a_{t4}^{u}-a_{s1}^{u})+\sum_{k=1}^{2}\max(H_k(\tilde{\tilde{A}}_t^U)-H_k(\tilde{\tilde{A}}_s^U),0)}{\sum_{k=1}^{4}|a_{tk}^{u}-a_{sk}^{u}|+(a_{s4}^{u}-a_{s1}^{u})+(a_{t4}^{u}-a_{t1}^{u})+\sum_{k=1}^{2}|H_k(\tilde{\tilde{A}}_t^U)-H_k(\tilde{\tilde{A}}_s^U)|} \tag{7}$$

where D_{ts} denotes summation of the absolute difference between a_{tk}^U and a_{sk}^U, where $1 \le k \le 4$, the absolute difference between a_{s4}^U and a_{s1}^U, the difference between a_{t4}^U and a_{t1}^U and the absolute difference between $H_k(\tilde{\tilde{A}}_t^U)$ and $H_k(\tilde{\tilde{A}}_s^U)$, where $1 \le k \le 2$; N_{ts} denotes summation of the absolute difference between a_{t4}^U and a_{s1}^U and the difference between $H_k(\tilde{\tilde{A}}_t^U)$ and $H_k(\tilde{\tilde{A}}_s^U)$, where $1 \le k \le 2$. Because the strength E_{ts} of $\tilde{\tilde{A}}_t^U$ over $\tilde{\tilde{A}}_s^U$ might not lie between 0 and 1, in order to let the likelihood $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U)$ of $\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U$ lie between 0 and 1, in order to let the likelihood $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U)$ of $\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U$ lie between 0 and 1, the likelihood $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U)$ of $\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U$ is defined as follows

$$P(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U)$$

$$= \max(1-\max(E_{ts},0),0) = \max\left(1-\max\left(\frac{N_{ts}}{D_{ts}},0\right),0\right)$$

$$= \max\left(1-\max\left(\frac{\sum_{k=1}^{4}\max(a_{tk}^{u}-a_{sk}^{u},0)+(a_{t4}^{u}-a_{s1}^{u})+\sum_{k=1}^{2}\max(H_k(\tilde{\tilde{A}}_t^U)-H_k(\tilde{\tilde{A}}_s^U),0)}{\sum_{k=1}^{4}|a_{tk}^{u}-a_{sk}^{u}|+(a_{s4}^{u}-a_{s1}^{u})+(a_{t4}^{u}-a_{t1}^{u})+\sum_{k=1}^{2}|H_k(\tilde{\tilde{A}}_t^U)-H_k(\tilde{\tilde{A}}_s^U)|},0\right),0\right). \tag{8}$$

If $E_{ts} \le 0$, then $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U) = 1$, where $E_{ts} \ge 0$ means that $\tilde{\tilde{A}}_t^U$ dominates $\tilde{\tilde{A}}_s^U$ absolutely; if $0 < E_{ts} < 1$, then $0 < p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U) < 1$; If $E_{ts} \le 1$, then $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U) = 0$, where $E_{ts} \ge 1$ means that $\tilde{\tilde{A}}_t^U$ dominates $\tilde{\tilde{A}}_s^U$ absolutely; if $0 < E_{ts} < 1$, then the larger the value of E_{ts}, the smaller the likelihood $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U)$ of $\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U$. It should be noted that the likelihood $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U)$ of $\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U$ has the following property:

(i) $0 \le p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U) \le 1$

(ii) $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U) + p(\tilde{\tilde{A}}_t^U \ge \tilde{\tilde{A}}_s^U) = 1$

(iii) $p(\tilde{\tilde{A}}_s^U \ge \tilde{\tilde{A}}_t^U) = 0.5$

3. The Proposed Method

The primary steps in the proposed method were retrieved from [3]. We propose entropy weight in Step 2, LWA in Step 3, and fuzzy ranking method in Step 4. The step-wise procedures are summarized as follows.

Step 1: Construct the decision Y_p of the p-th decision-maker and create the average decision matrix $\bar{Y}$ defined as follows:

$$\bar{Y} = (\tilde{\tilde{f}}_{ij}^{p})_{m\times n} = (\tilde{\tilde{f}}_{ij}^{p})_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_m \end{array} & \begin{bmatrix} \tilde{\tilde{f}}_{11}^{p} & \tilde{\tilde{f}}_{12}^{p} & \cdots & \tilde{\tilde{f}}_{1n}^{p} \\ \tilde{\tilde{f}}_{21}^{p} & \tilde{\tilde{f}}_{22}^{p} & \cdots & \tilde{\tilde{f}}_{2n}^{p} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\tilde{f}}_{m1}^{p} & \tilde{\tilde{f}}_{m2}^{p} & \cdots & \tilde{\tilde{f}}_{mn}^{p} \end{bmatrix} \end{array}$$

where:

$$\tilde{\tilde{f}}_{ij} = \left(\frac{\tilde{\tilde{f}}^1_{ij} \oplus \tilde{\tilde{f}}^2_{ij} \oplus \cdots \oplus \tilde{\tilde{f}}^k_{ij}}{k} \right), \tag{9}$$

$x_1, x_2, \ldots, x_m$ represent the alternative and $C_1, C_2, \ldots, C_n$ represents criteria. $\tilde{\tilde{f}}_{ij}$ is an interval type-2 fuzzy set, $1 \le i \le m$, $1 \le j \le n$, $1 \le p \le k$ and k denotes the number of decision makers.

Step 2: Calculate the weight, w_j of the decision maker, k with respect to the criteria. We apply entropy method by Cui et al. [6] to avert subjectivity of human preference.

(i) Construct entropy value for criteria as follows:

$$\tilde{\tilde{e}}_j = \frac{\sum_{i=1}^{m} \tilde{\tilde{f}}_{ij} \log \tilde{\tilde{f}}_{ij}}{\log m} \tag{10}$$

(ii) Calculate the weight of criteria w_j using formula as follows:

$$\tilde{\tilde{w}}_j = \frac{1 - \tilde{\tilde{e}}_j}{\sum_{i=1}^{n} (1 - \tilde{\tilde{e}}_j)}, \quad \text{where } 1 \le i \le m,\ 1 \le j \le n. \tag{11}$$

Step 3: Find the LWA with the following equation.

$$\tilde{Y}_i = \frac{\sum_{j=1}^{n} \tilde{\tilde{f}}_{ij} \tilde{\tilde{w}}_j}{\sum_{j=1}^{n} \tilde{\tilde{w}}_j}, \quad \text{where } 1 \le i \le m,\ 1 \le j \le n. \tag{12}$$

Step 4: Use the fuzzy ranking method eq. (8) to reduce type 2 fuzzy set to type 1 fuzzy sets and rank the alternatives. Upper and Lower fuzzy preference matrices are constructed as follows:

$$P^U = \begin{bmatrix} P(\tilde{\tilde{A}}^U_1 \ge \tilde{\tilde{A}}^U_1) & P(\tilde{\tilde{A}}^U_1 \ge \tilde{\tilde{A}}^U_2) & \cdots & P(\tilde{\tilde{A}}^U_1 \ge \tilde{\tilde{A}}^U_n) \\ P(\tilde{\tilde{A}}^U_2 \ge \tilde{\tilde{A}}^U_1) & P(\tilde{\tilde{A}}^U_2 \ge \tilde{\tilde{A}}^U_2) & \cdots & P(\tilde{\tilde{A}}^U_2 \ge \tilde{\tilde{A}}^U_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(\tilde{\tilde{A}}^U_n \ge \tilde{\tilde{A}}^U_1) & P(\tilde{\tilde{A}}^U_n \ge \tilde{\tilde{A}}^U_2) & \cdots & P(\tilde{\tilde{A}}^U_n \ge \tilde{\tilde{A}}^U_n) \end{bmatrix}, \tag{13}$$

$$P^L = \begin{bmatrix} P(\tilde{\tilde{A}}^L_1 \ge \tilde{\tilde{A}}^L_1) & P(\tilde{\tilde{A}}^L_1 \ge \tilde{\tilde{A}}^L_2) & \cdots & P(\tilde{\tilde{A}}^L_1 \ge \tilde{\tilde{A}}^L_n) \\ P(\tilde{\tilde{A}}^L_2 \ge \tilde{\tilde{A}}^L_1) & P(\tilde{\tilde{A}}^L_2 \ge \tilde{\tilde{A}}^L_2) & \cdots & P(\tilde{\tilde{A}}^L_2 \ge \tilde{\tilde{A}}^L_n) \\ \vdots & \vdots & \ddots & \vdots \\ P(\tilde{\tilde{A}}^L_n \ge \tilde{\tilde{A}}^L_1) & P(\tilde{\tilde{A}}^L_n \ge \tilde{\tilde{A}}^L_2) & \cdots & P(\tilde{\tilde{A}}^L_n \ge \tilde{\tilde{A}}^L_n) \end{bmatrix}. \tag{14}$$

Ranking values for $\text{Rank}(\tilde{\tilde{A}}^U_i)$, $\text{Rank}(\tilde{\tilde{A}}^L_i)$ and $\text{Rank}(\tilde{A}_i)$ can be calculated as follows:

$$\text{Rank}(\tilde{\tilde{A}}^U_i) = \frac{1}{n(n-1)} \sum_{k=1}^{n} P(\tilde{\tilde{A}}^U_i \ge \tilde{\tilde{A}}^U_k) + \frac{n}{2} - 1, \tag{15}$$

$$\text{Rank}(\tilde{\tilde{A}}^L_i) = \frac{1}{n(n-1)} \sum_{k=1}^{n} P(\tilde{\tilde{A}}^L_i \ge \tilde{\tilde{A}}^L_k) + \frac{n}{2} - 1, \tag{16}$$

$$\text{Rank}(\tilde{A}_i) = \frac{\text{Rank}(\tilde{\tilde{A}}_i^U) + \text{Rank}(\tilde{\tilde{A}}_i^L)}{2} \tag{17}$$

where $1 \le i \le n$ and $\sum_{i=1}^{n} \text{Rank}(\tilde{A}_i) = 1$.

Step 5: Rank the alternatives in descending order. The larger value of $\text{Rank}(\tilde{A}_i)$, the more the preference of the alternatives.

4. Numerical Example

In this section, we consider the numerical example discussed in Sanayei et al. [14] to illustrate the proposed method. A company desires to select a suitable supplier to purchase the key components of its new product. There are five candidate suppliers (S_1, S_2, S_3, S_4, and S_5) remain for further evaluation. Three decision makers, D_1, D_2 and D_3, have been formed to select the most suitable supplier. The following criteria have been defined: Product quality (C_1), On-time delivery (C_2), Price/cost (C_3), Supplier's technological level (C_4), Flexibility (C_5).

The hierarchical structure of this problem is depicted in Figure 3.

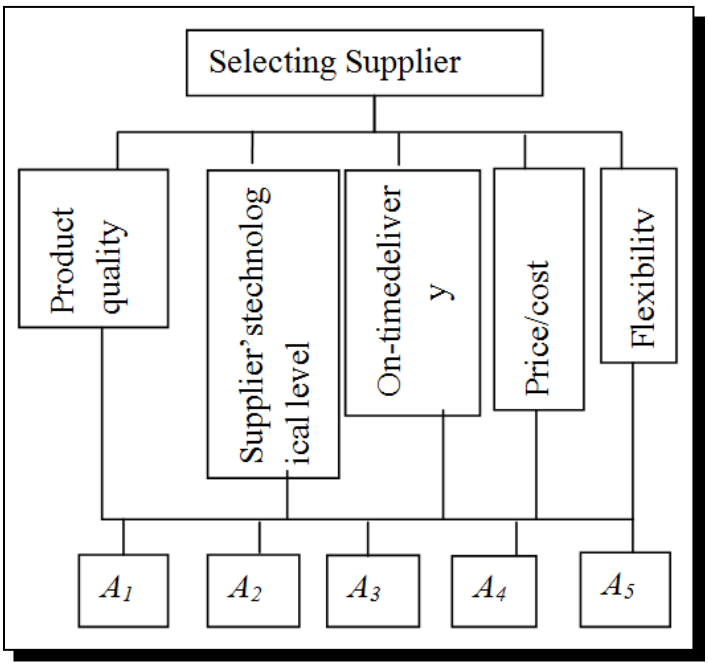

Figure 3. Hierarchical Structure of Supplier Selection

The decision makers (DMs) used the linguistic variables in Table 1 to rate the alternatives with respect to each criterion.

Table 1. Linguistic Terms and Interval Type-2 Fuzzy Set

Linguistic Terms	Interval Type-2 Fuzzy Sets
Very Poor (VP)	((0,0,0,0.1;1,1),(0,0,0,0.05;0.9,0.9))
Poor (P)	((0.0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.2;0.9,0.9))
Medium Poor (MP)	((0.1,0.3,0.3,0.5;1,1),(0.2,0.3,0.3,0.4;0.9,0.9))
Fair (F)	((0.3,0.5,0.5,0.7;1,1),(0.4,0.5,0.5,0.6;0.9,0.9))
Medium Good (MG)	((0.5,0.7,0.7,0.9;1,1),(0.6,0.7,0.7,0.8;0.9,0.9))
Good (G)	((0.7,0.9,0.9,1;1,1),(0.8,0.9,0.9,0.95;0.9,0.9))
Very Good (VG)	((0.9,1,1,1;1,1),(0.95,1,1,1;0.9,0.9))

The ratings given by DMs are shown in Table 2.

Table 2. Ratings of Five Suppliers by Decision Makers

		C_1	C_2	C_3	C_4	C_5
D_1	S_1	G	MG	G	G	G
	S_2	G	VG	MP	G	VG
	S_3	VG	MG	F	VG	G
	S_4	G	G	MG	G	G
	S_5	MG	MG	MG	MG	MG
D_2	S_1	G	MG	G	G	G
	S_2	G	VG	F	VG	MG
	S_3	VG	G	F	VG	VG
	S_4	G	G	MG	G	G
	S_5	MG	G	MG	MG	MG
D_3	S_1	VG	VG	G	G	G
	S_2	G	VG	MP	VG	VG
	S_3	G	G	F	VG	G
	S_4	G	MG	G	G	VG
	S_5	MG	G	MG	G	MG

In this numerical example, the linguistic variables are presented as IT2 FSs. Instead of using trapezoidal fuzzy number of linguistic variables (see Sanayei et al. [14]), we replaced it with linguistic variables from Chen and Lee [20] that used interval type-2 fuzzy numbers.

Implementation of the proposed method is presented in the step-wise procedures as follows.

Step 1: From Table 1 and Table 2, the decision matrices Y_p of the p-th decision-maker can be constructed respectively:

The average decision matrix $\bar{Y}$ is defined as eq. (9).

$$\bar{Y} = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{bmatrix} \tilde{\tilde{f}}_{11} & \tilde{\tilde{f}}_{12} & \tilde{\tilde{f}}_{13} & \tilde{\tilde{f}}_{14} & \tilde{\tilde{f}}_{15} \\ \tilde{\tilde{f}}_{21} & \tilde{\tilde{f}}_{22} & \tilde{\tilde{f}}_{23} & \tilde{\tilde{f}}_{24} & \tilde{\tilde{f}}_{25} \\ \tilde{\tilde{f}}_{31} & \tilde{\tilde{f}}_{32} & \tilde{\tilde{f}}_{33} & \tilde{\tilde{f}}_{34} & \tilde{\tilde{f}}_{35} \\ \tilde{\tilde{f}}_{41} & \tilde{\tilde{f}}_{42} & \tilde{\tilde{f}}_{43} & \tilde{\tilde{f}}_{44} & \tilde{\tilde{f}}_{45} \\ \tilde{\tilde{f}}_{51} & \tilde{\tilde{f}}_{52} & \tilde{\tilde{f}}_{53} & \tilde{\tilde{f}}_{54} & \tilde{\tilde{f}}_{55} \end{bmatrix} \end{array}$$

For example,

$$\tilde{\tilde{f}}_{11} = ((0.7667, 0.9333, 0.9333, 1; 1, 1), (0.85, 0.9333, 0.9333, 0.97; 0.9, 0.9))$$

$$\tilde{\tilde{f}}_{12} = ((0.7, 0.9, 0.9, 1; 1, 1), (0.8, 0.9, 0.9, 0.95; 0.9, 0.9))$$

$$\tilde{\tilde{f}}_{13} = ((0.8333, 0.9667, 0.9667, 1; 1, 1), (0.9, 0.967, 0.967, 0.98; 0.9, 0.9))$$

Step 2: The weight, w_j of the decision maker, k with respect to the criteria.

(1) Using eq. (10), the entropy value for criteria can be obtained as follows:

$$\tilde{\tilde{e}}_1 = \begin{pmatrix} (0.74657, 0.3338, 0.33338, 0.05892; 1, 1), \\ (0.55702, 0.33334, 0.33334, 0.20210; 0.9, 0.9) \end{pmatrix}$$

$$\tilde{\tilde{e}}_2 = \begin{pmatrix} (0.77788, 0.39412, 0.39412, 0.10110; 1, 1), \\ (0.60258, 0.39412, 0.39412, 0.2538; 0.9, 0.9) \end{pmatrix}$$

$$\tilde{\tilde{e}}_3 = \begin{pmatrix} (0.98042, 0.78453, 0.78453, 0.4540; 1, 1), \\ (0.91604, 0.78453, 0.78453, 0.63845; 0.9, 0.9) \end{pmatrix}$$

(2) Using eq. (11), we can get the weight of criteria w_j as follows:

$$\tilde{\tilde{w}}_1 = \begin{pmatrix} (0.23435, 0.23227, 0.23227, 0.2206; 1, 1), \\ (0.23588, 0.23227, 0.23227, 0.22622; 0.9, 0.9) \end{pmatrix}$$

$$\tilde{\tilde{w}}_2 = \begin{pmatrix} (0.20540, 0.21109, 0.21109, 0.21068; 1, 1), \\ (0.21162, 0.21109, 0.21109, 0.21156; 0.9, 0.9) \end{pmatrix}$$

$$\tilde{\tilde{w}}_3 = \begin{pmatrix} (0.01811, 0.07507, 0.07507, 0.12796; 1, 1), \\ (0.04471, 0.07507, 0.07507, 0.10251; 0.9, 0.9) \end{pmatrix}$$

Step 3: The LWA can be obtained using eq. (12).

$$\tilde{Y}_1 = \begin{pmatrix} (0.70193, 0.88663, 0.88663, 0.98596; 1, 1), \\ (0.79416, 0.88663, 0.88663, 0.93614; 0.9, 0.9) \end{pmatrix}$$

$$\tilde{Y}_2 = \begin{pmatrix} (0.78831, 0, 89815, 0.89815, 0.93736; 1, 1), \\ (0.84332, 0.89815, 0.89815, 0.91533; 0.9, 0.9) \end{pmatrix}$$

$$\tilde{Y}_3 = \begin{pmatrix} (0.78794, 0.90451, 0.90451, 0.95459; 1, 1), \\ (0.84773, 0.90451, 0.90451, 0.92672; 0.9, 0.9) \end{pmatrix}$$

Step 4: The likelihood $p(\tilde{\tilde{A}}_s^U \geq \tilde{\tilde{A}}_t^U)$ is calculated using eq. (8). The upper and lower preference matrixes are calculated using on eq. (13) and eq. (14), respectively.

$$P^U = \begin{bmatrix} 0.5 & 0.4166 & 0.3799 & 0.5127 & 0.8104 \\ 0.5834 & 0.5 & 0.4329 & 0.5944 & 0.8771 \\ 0.6201 & 0.5671 & 0.5 & 0.6304 & 0.8819 \\ 0.4873 & 0.4056 & 0.3696 & 0.5 & 0.8066 \\ 0.1896 & 0.1229 & 0.1181 & 0.1934 & 0.5 \end{bmatrix}$$

$$P^L = \begin{bmatrix} 0.5 & 0.3702 & 0.3154 & 0.5311 & 0.9548 \\ 0.6298 & 0.5 & 0.3932 & 0.6571 & 1 \\ 0.6846 & 0.6068 & 0.5 & 0.71 & 1 \\ 0.4689 & 0.3429 & 0.29 & 0.5 & 0.9509 \\ 0.0452 & 0 & 0 & 0.0491 & 0.5 \end{bmatrix}$$

Using eq. (15), eq. (16) and eq. (17), we can calculate Rank$(\tilde{S}_i^U)$, Rank$(\tilde{S}_i^L)$ and Rank$(\tilde{\tilde{S}}_i)$. It is shown in Table 3.

Table 3. Final Evaluation Result

	Rank$(\tilde{S}_i^U)$	Rank$(\tilde{S}_i^L)$	Rank$(\tilde{\tilde{S}}_i)$
S_1	0.2060	0.2086	0.2073
S_2	0.2244	0.2340	0.2292
S_3	0.2340	0.2501	0.2425
S_4	0.2035	0.2026	0.2030
S_5	0.1312	0.1047	0.2280

Step 5: Rank the alternatives by descending order.

From Table 3, we can see that the preference order is $S_3 > S_2 > S_5 > S_4 > S_1$. Therefore, S_3 is the best alternative.

In order to validate the results, we provide three other methods to compare with the proposed methods. A comparison result between the proposed method and three other methods is presented in Table 4.

Table 4. Comparison Result with Other Methods

Methods	Preferences obtained using the methods	Preferences obtained using the proposed method
Chen and Lee [3]	$z_2 > z_1 > z_3$	$z_2 > z_1 > z_3$
Wang and Lee [16]	$A_4 > A_3 > A_1 > A_2$	$A_4 > A_3 > A_1 > A_2$
Chu and Lin [5]	$A_2 > A_3 > A_1$	$A_2 > A_3 > A_1$
Sanayei et al. [14]	S_3	$S_3 > S_2 > S_5 > S_4 > S_1$

The ranking results from proposed method are consistent with Chen and Lee [3], Wang and Lee [16] and Chu and Lin [5]. In Sanayei et al. [14], the best alternative is S_3. Again, this result is consistent with the proposed method. Sanayei et al. [14] only interested to search the best alternative instead of ranking the alternatives.

5. Conclusions

Supplier selection has attracted the attention of companies since it is important to ensure companies survival in competitive market condition and to fulfil customer satisfaction. Unqualified supplier may reduce the competitiveness and will evoke a supply chain crisis. In this paper, we proposed the modified preference method TOPSIS based on IT2 FS approach for solving supplier selection problem. The combination of entropy method, LWA and ranking values has been concurrently used in the proposed method. In the modified version of T2 TOPSIS, we used entropy weight to calculate the weight for decision makers. Then, we aggregated the weight of the criteria using LWA. Finally, we rank the alternatives by using the ranking value. This combination could avert the vagueness of human thought and possibly used to deal with supplier selection problem.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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