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# Speech Around the Twin Primes Conjecture, the Mersenne Primes Conjecture, and the Sophie Germain Primes Conjecture

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**Abstract.** Here, we state a simple Assertion **(A)**, we introduce an original method of induction, and we use it to give a simple and detailed proof that **(A)** is stronger than the twin primes conjecture, the Mersenne primes conjecture and the Sophie Germain primes conjecture; *this helps us to explain why it is not surprising to conjecture that the Mersenne primes conjecture and the Sophie Germain primes conjecture are all special cases of the twin primes conjecture*.

## Prologue

Briefly, our original method of induction is based around the following simple definitions. Let $n$ be an integer $\geq 2$, we say that $y(n)$ is a *cache* of $n$, if $y(n)$ is an integer of the form $0 \leq y(n) < n$ (*Example.* If $n = 6$, then $y(n)$ is a cache of $n$ if and only if $y(n) \in \{0, 1, 2, 3, 4, 5\}$). Now, for every couple of integers $(n, y(n))$ such that $n \geq 2$ and $0 \leq y(n) < n$ (observe that $y(n)$ is a cache of $n$), we define $y(n, 2)$ as follows: $y(n, 2) = 1$ **if** $y(n) \equiv 1 \mod [2]$; and $y(n, 2) = 0$ **if** $y(n) \equiv 0 \mod [2]$. It is immediate that $y(n, 2)$ is well defined, since $n \geq 2$. In this paper, induction will be made on $n$ and $y(n, 2)$ (where $n$ is an integer $\geq 2$ and $y(n)$ is a cache of $n$).

## 1. Introduction

The primes numbers are well-known (see [9] or [10] or [11]); in particular, it is known:

**Theorem 1.0** (see [9]). *Let $n$ be an integer $\geq 1$. Then there exists a prime between $n$ and $2n$.*

The twin primes are also well-known. We recall that an integer $t$ is a twin prime (see [1] or [2], [4] or [5] or [6] or [8] or [13]), if $t$ is a prime number

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$\geq 3$ and if $t-2$ or $t+2$ is also a prime number $\geq 3$. *Example.* 1000000000061 and 1000000000063 are twin primes (see [8]). It is conjectured that there are infinitely many twin primes. A *Mersenne prime* (see [8] or [9] or [13]) is a prime of the form $M_m = 2^m - 1$, where $m$ is prime (the Mersenne primes are known for large values and it is conjectured that there are infinitely many Mersenne primes); we recall (see [3] or [12]) that a prime $q'$ is called a *Sophie Germain prime*, if both $q'$ and $2q'+1$ are prime; the first few Sophie Germain primes are $2, 3, 5, 11, 23, 29, 41, \ldots$, and the *Sophie Germain primes conjecture* says that there are infinitely many couples of the form $(q', 2q'+1)$, where $q'$ and $2q'+1$ are prime. Now, for every integer $n \geq 2$, we define $\mathscr{P}(n)$, $p_n$, $\mathscr{T}(n)$, $t_n$, $\mathscr{M}(n)$, $m_n$, $m_{n,1}$, $m_{n,2}$, $\mathscr{H}(n)$, $h_n$, $h_{n,1}$ and $h_{n,2}$ as follows: $\mathscr{P}(n) = \{p;\ p$ is prime and $1 < p < 2n\}$, $p_n = \max\limits_{p \in \mathscr{P}(n)} p$, $\mathscr{T}(n) = \{t;\ t$ is a twin prime and $1 < t < 2n\}$, $t_n = \max\limits_{t \in \mathscr{T}(n)} t$ (note that $3 \in \mathscr{T}(n)$), $t_n = \max\limits_{t \in \mathscr{T}(n)} t$, $\mathscr{H}(n) = \{h;\ 1 < h < 2n$, and $h$ is a Sophie Germain prime$\}$ (note $3 \in \mathscr{H}(n)$), $h_n = \max\limits_{h \in \mathscr{H}(n)} h$, $h_{n,1} = h_n^{h_n}$, $h_{n,2} = h_{n,1}^{h_{n,1}}$, $\mathscr{M}(n) = \{m;\ 1 < m < 2n$, and $m$ is a Mersenne prime$\}$ (note $3 \in \mathscr{M}(n)$), $m_n = \max\limits_{m \in \mathscr{M}(n)} m$, $m_{n,1} = m_n^{m_n}$ and $m_{n,2} = m_{n,1}^{m_{n,1}}$. Using the previous denotations, let us define:

**Definition 1.0** (Fundamental 1)**.** For every integer $n \geq 2$, we put

$$\mathscr{X}(n,2) = \{h_{n,2}\} \bigcup \{m_{n,2}\}.$$

From Definition 1.0 and the definition of $t_n$, then the following two allegations are immediate.

**Allegation 1.1.** Let $n$ be an integer $\geq 3$; consider $x_{n,2} \in \mathscr{X}(n,2)$, and look at the couple $(x_n, x_{n,1})$ (*Example* 0. If $x_{n,2} = h_{n,2}$, then $x_n = h_n$ and $x_{n,1} = h_{n,1}$. *Example* 2. If $x_{n,2} = m_{n,2}$, then $x_n = m_n$ and $x_{n,1} = m_{n,1}$). Then $0 < x_n < x_{n,1} < x_{n,2}$, and $x_{n-1,2} \leq x_{n,2}$.

**Allegation 1.2.** Let $n$ be an integer $\geq 3$; consider $t_n$, and look at $t_{n-1}$ ($t_{n-1}$ is meaningful since $n-1 \geq 2$). If $2n-1$ is not a twin prime, then $t_n = t_{n-1}$.

Now, using the previous notations and definitions, let **(A)** be the following assertion:

**(A).** For every integer $r \geq 3$, **one and only one** of the following two properties **w(A.*r*)** and **o(A.*r*)** is satisfied.

**w(A.*r*):** There exists not a twin prime $\geq p_r$.

**o(A.*r*):** For every $x_{r,2} \in \mathscr{X}(r,2)$, we have $x_{r,2} > t_r$.

Let us remark (see page 38 for detail) that if for every integer $r \geq 3$, property **o(A.*r*)** of assertion **(A)** is satisfied, then the *Sophie Germain primes conjecture and the Mersenne primes conjecture are simultaneously special cases of the twin primes conjecture*. It is easy to see that property **o(A.*r*)** of assertion **(A)** is satisfied for large values of $\boldsymbol{r}$. In this paper, using only the immediate part of the original method of induction, we prove a Theorem which immediately implies the following result **(Q)**:

**(Q).** *Suppose that assertion* **(A)** *holds. Then the Sophie Germain primes conjecture, the Mersenne primes conjecture and the twin primes conjecture simultaneously hold.*

Result **(Q)** helps us to explain why to conjecture that the Sophie Germain primes conjecture and the Mersenne primes conjecture are simultaneously special cases of the twin primes conjecture is not surprising.

## 2. Proof of a Theorem which Implies the Result (Q)

The following theorem immediately implies our result **(Q)** mentioned above.

**Theorem 2.1.** *Let* $(n, y(n))$ *be a couple of integers such that* $n \geq 3$ *and* $y(n)$ *is a cache of* $n$*. Now suppose that assertion* **(A)** *holds. Then at least one of the following two properties* (i) *and* (ii) *is satisfied by the couple* $(n, y(n))$.

(i) *If* $y(n) \equiv 0 \mod[2]$*, then, there exists a twin prime* $\geq p_n - y(n)$.

(ii) *If* $y(n) \not\equiv 0 \mod[2]$*, then, for every* $x_{n,2} \in \mathscr{X}(n,2)$*, we have*

$$x_{n,2} > 1 + t_n - y(n).$$

To prove Theorem 2.1, we use:

**Lemma 1.** *Suppose that* $n = 3$*. Then Theorem* 2.1 *is contented.*

***Proof.*** Clearly $y(n) \in \{0,1,2\}$, and it suffices to show that Theorem 2.1 is satisfied for all $y(n) \in \{0,1,2\}$. So, we have to distinguish two cases (namely case where $y(n) \in \{0,2\}$, and case where $y(n) = 1$).

*Case* 0. $\boldsymbol{y(n) \in \{0,2\}}$. Clearly $y(n) \equiv 0 \mod[2]$ and we have to show that property (i) of Theorem 2.1 is satisfied by the couple $(n, y(n))$. Recall $n = 3$, so $\mathscr{P}(n) = \{2,3,5\}$, $p_n = 5 = t_n$, and clearly 5 is a twin prime $\geq p_n$; in particular 5 is a twin prime $\geq p_n - y(n)$. So property (i) of Theorem 2.1 is satisfied by the couple $(n, y(n))$, and Theorem 2.1 is contented. Case 0 follows.

*Case* 1. $\boldsymbol{y(n) = 1}$. Clearly $y(n) \equiv 1 \mod[2]$; so $y(n) \not\equiv 0 \mod[2]$, and therefore, we have to show that property (ii) of Theorem 2.1 is satisfied by the couple $(n, y(n))$. Since $n = 3$, then $\mathscr{T}(n) = \{3,5\}$, $t_n = 5$, $\mathscr{M}(n) = \{3\}$, $m_n = 3$, $m_{n,1} = 3^3 = 27$, $m_{n,2} = 27^{27}$, $\mathscr{H}(n) = \{2,3,5\}$, $h_n = 5$, $h_{n,1} = 5^5 = 3125$ and $h_{n,2} = 3125^{3125}$; clearly $\mathscr{X}(n,2) = \{m_{n,2}, h_{n,2}\}$, and via the previous equalities, it becomes immediate to see that, for every $x_{n,2} \in \mathscr{X}(n,2)$, we have $x_{n,2} > t_n$; in particular, for every $x_{n,2} \in \mathscr{X}(n,2)$, we have $x_{n,2} > 1 + t_n - y(n)$. So property (ii) of Theorem 2.1 is satisfied by the couple $(n, y(n))$, and Theorem 2.1 is contented. *Case* 1 follows, and Lemma 1 immediately follows. □

From the meaning of Theorem 2.1, let us define:

**Definition 2.2.** We say that $(n, y(n))$ is a remarkable couple of Theorem 2.1, if the couple $(n, y(n))$ is a counter-example of Theorem 2.1 with $n$ **minimum** and $y(n,2)$ **minimum**.

Using Lemma 1 via the meaning of Theorem 2.1, it becomes easy to see:

**Remark 0.** If Theorem 2.1 is false, then there exists $(n, y(n))$ such that $(n, y(n))$ is a remarkable couple of Theorem 2.1.

**Consequence 0** (Application of Remark 0 and Lemma 1)**.** *Suppose that Theorem* 2.1 *is false, and let* $(n, y(n))$ *be a remarkable couple of Theorem* 2.1*. Then* $n \geq 4$*,* $p_n$ *and* $p_{n-1}$ *are odd primes, and* $p_{n-1} \leq p_n \leq 2n-1$*.*

***Proof.*** $n \geq 4$ (use Lemma 1), so $p_n$ and $p_{n-1}$ are odd primes, and clearly $p_{n-1} \leq p_n \leq 2n-1$ (use the definition of $p_n$ and observe that $n \geq 4$, by the previous). □

**Remark 1.** Suppose that Theorem 2.1 is false, and let $(n, y(n))$ be a remarkable couple of Theorem 2.1. We have the following two simple properties (R.1.0) and (R.1.1).

(R.1.0) (The using of the **minimality** of $n$). Put $d = n-1$ and let $y(d) = j$, where $j \in \{0,1\}$ (note that $d < n$, $d \geq 3$ (use Consequence 0), $y(d)$ is a cache of $d$, and the couple $(d, y(d))$ clearly exists). Now look at the couple $(d, y(d))$; then, by the **minimality** of $n$, the couple $(d, y(d))$ is not a counter-example of Theorem 2.1. Clearly $y(d) \equiv j \mod[2]$ (because $y(d) = j$, where $j \in \{0,1\}$), and therefore property **(j)** of Theorem 2.1 is satisfied by the couple $(d, y(d))$ ((*Example* 1.0. If $j = 0$ (i.e. if $y(d) = j = 0$), then property (i) of Theorem 2.1 is satisfied by the couple $(d, y(d))$; so there exists a twin prime $\geq p_d$. *Example* 1.1. If $j = 1$ (i.e. if $y(d) = j = 1$), then property (ii) of Theorem 2.1 is satisfied by the couple $(d, y(d))$; so, for every $x_{d,2} \in \mathscr{X}(d,2)$, we have $x_{d,2} > t_d$)).

(R.1.1) (The using of the **minimality** of $y(n,2)$: the **immediate** part of the original method of induction). If $y(n) \equiv 1 \mod[2]$, clearly $y(n,2) = 1$. Now let the couple $(n, y'(n))$ such that $y'(n) = 0$. Clearly $y'(n)$ is a cache of $n$ such that $y'(n,2) = 0$ (note that $n \geq 4$ (use Consequence 0)). Clearly $y'(n,2) < y(n,2)$, where $y(n)$ and $y'(n)$ **are two caches** of $n$ (since $y(n,2) = 1$ and $y'(n,2) = 0$, by the previous); then, by the **minimality** of $y(n,2)$, the couple $(n, y'(n))$ is not a remarkable couple of Theorem 2.1; in particular, the couple $(n, y'(n))$ is not a counter-example of Theorem 2.1. Note that $y'(n) \equiv 0 \mod[2]$ (since $y'(n) = 0$, by the definition of $y'(n)$), and therefore, property (i) of Theorem 2.1 is satisfied by the couple $(n, y'(n))$; so there exists a twin prime $\geq p_n - y'(n)$ , and clearly there exists a twin prime $\geq p_n$ (because $y'(n) = 0$).

**Consequence 1** (Application of Remark 1). *Suppose that Theorem* 2.1 *is false, and let* $(n, y(n))$ *be a remarkable couple of Theorem 2.1. Then we have the following four properties.*

(c.1.0) *There exists a twin prime* $\geq p_{n-1}$.

(c.1.1) *For every* $x_{n-1,2} \in \mathscr{X}(n-1, 2)$, *we have* $x_{n-1,2} > t_{n-1}$.

(c.1.2) *For every* $x_{n,2} \in \mathscr{X}(n, 2)$, *we have* $x_{n,2} > t_{n-1}$.

(c.1.3) *If* $y(n) \equiv 1 \mod[2]$, *then for every* $x_{n,2} \in \mathscr{X}(n, 2)$, *we have* $x_{n,2} > t_n$.

***Proof.*** Property (c.1.0) is easy (indeed consider the couple $(d, y(d))$ such that $d = n-1$ and $y(d) = 0$, and apply *Example* 1.0 of property (R.1.0) of Remark 1); property (c.1.1) is also easy (indeed, consider the couple $(d, y(d))$ such that $d = n-1$ and $y(d) = 1$, and apply *Example* 1.1 of property (R.1.0) of Remark 1), and property (c.1.2) immediately follows by using property (c.1.1) and by observing (via Allegation 1.1) that $x_{n-1,2} \leq x_{n,2}$. Now to prove Consequence 1 it suffices to show property (c.1.3). *Fact. For every* $x_{n,2} \in \mathscr{X}(n, 2)$, *we have* $x_{n,2} > t_n$. Indeed, observing that there exists a twin prime $\geq p_n$ (use property (R.1.1) of Remark 1), clearly property **w(A.*n*)** of assertion **(A)** is not satisfied, and recalling that assertion **(A)** holds, then we immediately deduce that property **o(A.*n*)** of assertion **(A)** is satisfied; therefore, for every $x_{n,2} \in \mathscr{X}(n, 2)$, we have $x_{n,2} > t_n$. ☐

***Proof of Theorem* 2.1.** We reason by reduction to absurd. Suppose that Theorem 2.1 is false and let $(n, y(n))$ be a remarkable couple of Theorem 2.1 (such a remarkable couple exists, by using Remark 0). Then we observe the following.

*Observation* 1. $y(n) \not\equiv 1 \mod[2]$.

Otherwise,

(2.1) $\quad y(n) \equiv 1 \mod[2]$,

so

(2.2) $\quad y(n) \not\equiv 0 \mod[2]$,

and clearly there exists $x_{n,2} \in \mathscr{X}(n, 2)$ such that $x_{n,2} \leq t_n$ ((indeed note $y(n) \not\equiv 0 \mod[2]$ (by congruence (2.2)), and in particular, property (ii) of Theorem 2.1 is not satisfied by the couple $(n, y(n))$; therefore there exists $x_{n,2} \in \mathscr{X}(n, 2)$ such that $x_{n,2} \leq 1 + t_n - y(n)$; in particular there exists $x_{n,2} \in \mathscr{X}(n, 2)$ such that $x_{n,2} \leq t_n$ (because $y(n) \geq 1$, since $y(n) \equiv 1 \mod[2]$ (by congruence (2.1)) and $y(n)$ is a cache of $n$))). This contradicts property (c.1.3) of Consequence 1 (by using congruence (2.1) and property (c.1.3) of Consequence 1). Observation 1 follows.

Observation 1 implies that

(2.3) $\quad y(n) \equiv 0 \mod[2]$,

and clearly

(2.4) $\quad$ there exists not a twin prime $\geq p_n - y(n)$

((indeed note $y(n) \equiv 0 \mod[2]$ (by congruence (2.3)), and in particular, property (i) of Theorem 2.1 is not satisfied by the couple $(n, y(n))$; so there exists not a twin prime $\geq p_n - y(n)$)); (2.4) immediately implies that

(2.5) there exists not a twin prime $\geq p_n$

(indeed note that $y(n) \geq 0$, since $y(n) \equiv 0 \mod[2]$ (by Congruence (2.3)) and $y(n)$ is a cache of $n$); consequently,

(2.6) there exists $x_{n,2} \in \mathscr{X}(n,2)$ such that $x_{n,2} \leq t_n$

(indeed, observing that there exists not a twin prime $\geq p_n$ (use (2.5)), clearly property **w(A.*n*)** of assertion **(A)** is satisfied, and recalling that assertion **(A)** holds, then we immediately deduce that property **o(A.**$n$**)** of assertion **(A)** is not satisfied; therefore there exists $x_{n,2} \in \mathscr{X}(n,2)$ such that $x_{n,2} \leq t_n$). Now we have the following simple fact.

*Fact.* $2n-1$ *is not a twin prime*. Otherwise, clearly $p_n = t_n = 2n-1$ (note (via consequence 0) that $n \geq 4$), and so $t_n$ is a twin prime $\geq p_n$ (where $t_n = 2n-1$); in particular there exists a twin prime $\geq p_n$, and this contradicts (2.5). The *Fact* follows.

This simple *Fact* made, observing (by the previous *Fact*) that $2n-1$ is not a twin prime and remarking (via Consequence 0) that $n \geq 4$, then Allegation 1.2 immediately implies that

(2.7) $t_n = t_{n-1}$ .

Now, using equality (2.7) and using (2.6), then we immediately deduce that there exists $x_{n,2} \in \mathscr{X}(n,2)$ such that $x_{n,2} \leq t_{n-1}$; this contradicts property (c.1.2) of Consequence 1. Theorem 2.1 follows. □

**Remark 2.** Note that to prove Theorem 2.1, we consider a couple $(n, y(n))$ such that $(n, y(n))$ is a counter-example with $n$ **minimum** and $y(n,2)$ **minimum** (i.e. $(n, y(n))$ is a *remarkable* couple of Theorem 2.1). In properties (c.1.0), (c.1.1), and (c.1.2) of Consequence 1 (via property (R.1.0) of Remark 1), the minimality of $n$ is used; and in property (c.1.3) of Consequence 1 (via property (R.1.1) of Remark 1), the minimality of $y(n,2)$ is used. Consequence 1 helps us to give a simple and detailed proof of Theorem 2.1.

**Corollary 2.3.** *Suppose that Assertion* **(A)** *holds. Then we have the following five properties.*

(2.3.0) *For every integer* $n \geq 3$*, there exists a twin prime* $\geq p_n$.

(2.3.1) *The twin primes conjecture holds.*

(2.3.2) *For every integer* $n \geq 3$ *and for every* $x_{n,2} \in \mathscr{X}(n,2)$*, we have* $x_{n,2} > t_n$.

(2.3.3) $\lim_{n\to+\infty} m_{n,2} = \lim_{n\to+\infty} h_{n,2} = +\infty$.

(2.3.4) *The Mersenne primes conjecture and the Sophie Germain primes conjecture simultaneously hold.*

***Proof.*** (2.3.0). Indeed, consider the couple $(n, y(n))$ with $y(n) = 0$. The couple $(n, y(n))$ is of the form $0 \leq y(n) < n$, where $n \geq 3$, $y(n) \equiv 0 \mod[2]$, and $y(n)$ is a cache of $n$. Then property (i) of Theorem 2.1 is satisfied by the couple $(n, y(n))$. So there exists a twin prime $\geq p_n$ (since $y(n) = 0$).

(2.3.1). Observing (via Theorem 1.0) that there exists always a prime between $n$ and $2n$ (for every integer $n \geq 1$), clearly $p_n \geq n$ (use the definition of $p_n$ and the fact that there exists always a prime between $n$ and $2n$ (for every integer $n \geq 1$)), and using property (2.3.0), then we immediately deduce that for every integer $n \geq 3$, there exists a twin prime $\geq p_n \geq n$. Consequently, for every integer $n \geq 3$, there exists a twin prime $\geq n$, and the previous is clearly stronger than the twin primes conjecture.

(2.3.2). Let the couple $(n, y(n))$ be such that $y(n) = 1$. The couple $(n, y(n))$ is of the form $0 \leq y(n) < n$, where $n \geq 3$, $y(n) \equiv 1 \mod[2]$, $y(n) \not\equiv 0 \mod[2]$, and $y(n)$ is a cache of $n$. Then property (ii) of Theorem 2.1 is satisfied by the couple $(n, y(n))$. So for every $x_{n,2} \in \mathscr{X}(n, 2)$, we have $x_{n,2} > t_n$ (since $y(n) = 1$).

(2.3.3). Observing (via property (2.3.1)) that the twin primes conjecture holds, clearly

$$\lim_{n \to +\infty} t_n = +\infty . \tag{2.8}$$

Now using equality (2.8) and property (2.3.2) (via the definition of $\mathscr{X}(n, 2)$), then we immediately deduce that $\lim_{n \to +\infty} m_{n,2} = \lim_{n \to +\infty} h_{n,2} = +\infty$.

(2.3.4) It is an immediate consequence of property (2.3.3) (via the definition of $m_{n,2}$ and $h_{n,2}$). □

Using property (2.3.1) and property (2.3.4) of Corollary 2.3, then the following result **(Q)** becomes immediate.

**Result (Q).** *Suppose that Assertion* **(A)** *holds. Then, the twin primes conjecture, the Mersenne primes conjecture and the Sophie Germain primes simultaneously hold.*

**Conjecture 1.** *Assertion* **(A)** *holds.*

**Epilogue.** To conjecture that the Mersenne primes conjecture and the Sophie Germain primes conjecture are simultaneously consequences of the twin primes conjecture is not surprising. Indeed, let **(A′)** be the following assertion:

(A′) For every integer $r \geq 3$, **at most one** of the following two properties **w(A′.*r*)** and **o(A′.*r*)** holds.

**w(A′.*r*)** There exists not a twin prime $\geq p_r$ .

**o(A′.*r*)** For every $x_{r,2} \in \mathscr{X}(r, 2)$, we have $x_{r,2} > t_r$ .

Note that assertion **(A′)**, somehow, resembles to assertion **(A)**. More precisely, assertion **(A)** implies assertion **(A′)** (*Proof.* In particular, the twin primes conjecture holds (use property (2.3.1) of Corollary 2.3); consequently, assertion **(A′)** holds (use the definition of assertion **(A′)** and the previous)).

**Consequence 2.** *Assertion* **(A)** *and assertion* **(A′)** *are equivalent.*

Conjecture 1 implies that the Mersenne primes conjecture and the Sophie Germain primes conjecture are consequences of the twin primes conjecture.

***Proof.*** Suppose that Conjecture 1 holds. If the twin primes conjecture holds, clearly assertion **(A′)** holds; observing that assertion **(A′)** and assertion **(A)** are equivalent, then **(A)** holds, and result **(Q)** implies that the Mersenne primes conjecture and the Sophie Germain primes conjecture simultaneously hold. □

**Conjecture 2.** *Suppose that assertion* **(A′)** *holds. Then the twin primes conjecture, the Mersenne primes conjecture and the Sophie Germain primes conjecture simultaneously hold.*

Conjecture 2 immediately implies that the Mersenne primes conjecture and the Sophie Germain primes conjecture are consequences of the twin primes conjecture.

***Proof.*** Suppose that Conjecture 2 holds. If the twin primes conjecture holds, clearly assertion **(A′)** holds, and in particular the Mersenne primes conjecture and the Sophie Germain primes conjecture simultaneously hold. □

**Conjecture 3.** *For every integer $r \geq 3$, property* $\mathbf{o(A.}\boldsymbol{r}\mathbf{)}$ *of assertion* **(A)** *holds (note that property* $\mathbf{o(A.}\boldsymbol{r}\mathbf{)}$ *of assertion* **(A)** *is exactly property* $\mathbf{o(A'.}\boldsymbol{r}\mathbf{)}$ *of assertion* **(A′)**; *moreover, it is easy to see that property* $\mathbf{o(A'.}\boldsymbol{r}\mathbf{)}$ *of assertion* **(A′)** *is satisfied for very large values of* $\boldsymbol{r}$*).*

Conjecture 3 also implies that the Mersenne primes conjecture and the Sophie Germain primes conjecture are consequences of the twin primes conjecture.

***Proof.*** Suppose that Conjecture 3 holds. If the twin primes conjecture holds, clearly, $\lim_{n\to+\infty} t_n = +\infty$; so $\lim_{n\to+\infty} m_{n,2} = \lim_{n\to+\infty} h_{n,2} = +\infty$ (via the definition of $\mathscr{X}(n,2)$), and it results that the Mersenne primes and the Sophie Germain primes are all infinite. □

Now, using the previous three conjectures, it becomes natural and not surprising to conjecture the following:

**Conjecture 4.** *The Mersenne primes conjecture and the Sophie Germain primes conjecture are consequences of the twin primes conjecture.*

**Conjecture 5.** *Let $(n, r(n))$ be a couple of integers such that $n \geq 3$ and $0 \leq r(n) < n$. We have the following.*

(i) *If $r(n) \equiv 0 \mod[3]$, then $m_{n,2} > t_n - r(n)$.*
(ii) *If $r(n) \equiv 1 \mod[3]$, then $h_{n,2} > 1 + t_n - r(n)$.*

(iii) *If* $r(n) \equiv 2 \mod[3]$*, then there exists a twin prime* $\geq 2 + p_n - r(n)$.

It is easy to see that Conjecture 5 simultaneously implies the twin primes conjecture, the Mersenne primes conjecture and the Sophie Germain primes conjecture, and to attack this conjecture, we must consider the original method of induction.

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