Journal of Informatics and Mathematical Sciences
Vol. 17, No. 1, pp. 9–20, 2025
ISSN 0975-5748 (online); 0974-875X (print)
Published by RGN Publications
DOI: 10.26713/jims.v17i1.3121

http://www.rgnpublications.com

Research Article

A Numerical Study of Heuristics Time Deviation Technique for Job Sequencing to Minimize Total Elapsed Time and Idle Time

Dimpal Vishwasrao Dhote*[1] and Vinod Kumar Singh[2]

[1] *Department of Mathematics, Institutions of Higher Learning, Research and Specialised Studies, Sardar Patel Mahavidyalaya, Chandrapur 442402, Maharashtra, India*
[2] *Department of Mathematics, Chintamani College of Science, Pombhurna, Maharashtra, India*
***Corresponding author:** dimpaldhote71@gmail.com

Received: March 29, 2025 **Accepted:** April 25, 2025 **Published:** April 30, 2025

Abstract. Job sequencing refers to the process of determining the most suitable order for performing a number of different jobs across a limited number of available resources, services or facilities to make effective use of these available facilities and achieve greater output with minimum time delay. This study introduces an innovative heuristic technique called the Time Deviation Technique. This technique helps in obtaining the optimal job schedule and also minimizes the total elapsed time and idle time. Here we first apply the new method to the n-jobs, 2 machines problem and extend it to the n-jobs, 3 machines problem. And lastly, we applied this method to the n-jobs, m-machines problem after converting into 2 machines n jobs problems by using the condition of Johnson's method. The proposed technique is very simple and easy to understand. For this we have given examples numerical illustration of job sequencing problem of n jobs 2 machines, n jobs 3 machines and n jobs m machines problem.

Keywords. Job sequencing, Processing time, Time deviation method, Total elapsed time, Idle time, Heuristics technique

Mathematics Subject Classification (2020). 90B35, 90B30, 90B36, 68M20

1. Introduction

Job Sequencing Problem

Sequencing has been very considerable research in the past few years. Sequencing is a method for arranging jobs in an optimal sequence. There are different types of sequencing adopted in industries, for example, priority-based, job size oriented, first-in-first-out, and processing-time-based methods. The problem of sequencing may have some restriction such as time for each job, order of processing of each machine and main is available resources. In our daily life we face the situation where one needs to save the required time for processing the products on different machines. Job sequencing is important to find the optimal schedule and minimize the total elapsed time and idle time. When more jobs are performed on more than two machine. In job sequencing problem various research article deals with to find an optimal job sequence, total elapsed time and idle time under various situations.

In earlier times, many researchers showed their interest in sequencing problems of their efficient and impactful applications in production and manufacturing. The early focus on sequencing problems was strongly associated with the rise of mass production in 20th century. Johnson's algorithm is a very good example of early and still useful technique, developed by S. M. Johnson's in 1954 [5], it aimed to determine the optimal sequence and minimize the total elapsed time of jobs and idle time of machines in production. Works of Gupta *et al*. [1], Iqbal *et al*. [3], Karthikeyan [7], Rajbhar [10], Rao *et al*. [11], Watekar [14] focuses on determining the best possible order of jobs i job sequencing problem to minimize the total elapsed time and idle time of machines. The single job scheduling problem in a two stage hybrid flow -shop that has m-identical machine at first stage and one machine at the second stage to minimize the makespan studied by Koulamas and Kyparisis [8]. Kalczynski and Kamburowski [6] developed "a heuristic technique for minimizing the expected makespan in two-machine flow shops with consistent coefficient of variation". Singh and Kumar [12] have studied "path optimization algorithm for network problems using job sequencing technique". They proposed a new method to identify the best path from the starting source to the destination using job sequencing method to enhance efficiency and effectiveness. Vachajitpan [13] gives computer program for solving n-job m-machine problems, which is equivalent for finding the optimal route within a finite network.

Some definition and formula to find total minimum elapsed time and idle time:

(1) *Total elapsed Time*: Total elapsed time means the time between the starting of the first job and completing the last job. This also include idle time given by any machine:

$$\sum_{j=2}^{n} t_{2j} + \sum_{j=2}^{n} I_{2j},$$

where t_{2j} = processing time for jth job on M2,

I_{2j} = Idle time between completion of $(j-1)$th job on M1 and starting the jth job on M2.

(2) *Idel time*: Ideal time means the machine remains idle (not working condition) during the total elapsed time. It is denoted by 'I'.

I for Machine-M1 = Total elapsed time − Time of last job done in Machine-M1,

I for Machine-M2 = Time of first job done in Machine-M1 + [Time of the jth job starts on M2 − Time of $(j-1)$th job done in M2].

2. New Heuristics Time Deviation Technique

We now define the following terms which will be used in the proposed new method known as time deviation methods to find the optimal sequence of jobs in job sequencing problems.

The row deviation means gap among the times in corresponding cells to the minimum time in that row. Similarly, column deviation means gap among the times in corresponding cells to the minimum time in that column.

Let a_i be the minimum time of the ith row and b_i be the minimum time of the jth column.

Time deviation for rows (i,j)th cell denoting as r_{ij} is defined as:

$$r_{ij} = t_{ij} - a_i\,.$$

Time deviation for columns (i,j)th cell denoting as c_{ij} is defined as:

$$c_{ij} = t_{ij} - b_i\,.$$

2.1 Heuristics Algorithm for Sequencing n Jobs on Two Machines

(1) Construct the table of time deviation for sequencing problem by calculating row deviation and column deviation in sequencing problems.

(2) Assign the corresponding job first on Machine-M1, which has both time allocation vector set to zero.

(3) Suppose more than one cell has deviation vectors set to zero. Then calculate the total deviation value for the column that contains these cells. Assign the job first to the cell with the greatest variance and assigned next job which have next largest sum deviation and so on.

(4) Similar steps are followed for Machine-M2, but in Machine-M2 the corresponding job is assigned last.

(5) If we have achieved the optimal sequence for all the jobs in the job sequencing problem, otherwise go to next step.

(6) Form a table with reduced time durations that contains only not-assigned jobs.

(7) Again process the step from 1 to 5 for table with reduced time durations. But assigned the jobs next to the previously assigned jobs for cells where each vector is zero.

(8) Finally, stop the process and determine the total elapsed time and idle time of Machine-M1 and Machine-M2.

Problem Involving Sequencing n Jobs on Two Machines by Using the Time Deviation Technique

Example 2.1. In the given job sequencing problem, there are 8 jobs, each of which must be processed on two machines, M1 and M2. The goal of this problem is to find the optimal schedule for the given jobs, also determine the total minimum elapsed time and the idle time for machine M1 and M2.

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7	J8
M1	14	26	17	11	9	26	18	15
M2	21	15	16	21	22	12	13	25

Solution. Calculate the deviation for the given time duration tables for jobs on M1 and M2 is:

Jobs/Machines	J1	J2	J3	J4	J5	J6	J7	J8
M1	(5,0)	(17,11)	(8,1)	(2,0)	(0,0)	(17,14)	(9,5)	(6,0)
M2	(9,7)	(3,0)	(4,0)	(9,10)	(10,13)	(0,0)	(1,0)	(13,10)

For job J5 on Machine-M1, both deviations are zero. Therefore, place job J5 in the first position on the left side of the sequence. Also, for job J6 on Machine-M2, both deviations are zero. Therefore, place job J6 in the first position on the right side of the sequence, as given below.

J5							J6

Remove jobs J5 and J6 from the above table, and then write the updated table for the other jobs, as shown below:

Jobs/Machines	J1	J2	J3	J4	J7	J8
M1	14	26	17	11	18	15
M2	21	15	16	21	13	25

Again calculate the deviation for remaining jobs.

Jobs/Machines	J1	J2	J3	J4	J7	J8
M1	(3,0)	(15,11)	(6,1)	(0,0)	(7,5)	(4,0)
M2	(8,7)	(2,0)	(3,0)	(8,10)	(0,0)	(12,10)

For job J4 on Machine-M1, both deviations are zero. Therefore, place job J4 to the left side of the sequence after job J5. Also, for job J7 on Machine-M2, both deviations are zero. Therefore, place job J7 to the right side of the sequence before job J6, as given below.

J5	J4					J7	J6

Remove jobs J4 and J7 from the above table, and then write the updated table for the other jobs, as shown below:

Jobs/Machines	J1	J2	J3	J8
M1	14	26	17	15
M2	21	15	16	25

Again calculate the deviation for remaining jobs:

Jobs/Machines	J1	J2	J3	J8
M1	(0,0)	(12,11)	(3,1)	(1,0)
M2	(6,7)	(0,0)	(1,0)	(10,10)

For job J1 on Machine-M1, both deviations are zero. Therefore, place job J1 on the left side of the sequence after job J4. Also for job J2 on Machine-M2, both deviations are zero. Therefore, place job J2 to the right side of the sequence before job J7, as given below:

J5	J4	J1			J2	J7	J6

Remove jobs J1 and J2 from the above table, and then write the updated table for the other jobs, as shown below:

Jobs/Machines	J3	J8
M1	17	15
M2	16	25

Again calculate the deviation for remaining jobs:

Job/Machines	J3	J8
M1	(2,1)	(0,0)
M2	(0,0)	(9,10)

For job J8 on Machine-M1, both deviations are zero. Therefore, place job J8 on the left side of the sequence after Job J1. Also for job J3 on Machine-M2, both deviations are zero. Therefore, place Job J3 to the right side of the sequence before job J2, as shown below:

J5	J4	J1	J8	J3	J2	J7	J6

This sequence is required optimal sequence to given sequencing problem. By using this optimal sequence, we find the total Minimum Elapsed Time and Idle Time for the Machine-M1 and Machine-M2 by using usual procedure.

Jobs sequence/ Machines	Machine-M1		Machine-M2		Idle Time for Machine-M2
	Time-In	Time-Out	Time-In	Time-Out	
J5	0	9	9	31	9
J4	9	20	31	52	0
J1	20	34	52	73	0
J8	34	49	73	98	0
J3	49	66	98	114	0
J2	66	92	114	129	0
J7	92	110	129	142	0
J6	110	136	142	154	0

The total minimum elapsed time = 154 hour

For the idle time of Machine-M1 = 154 − 136 = 18 hour

For the idle time of Machine-M2 = 9 + (154 − 154) = 9 hours

2.2 Heuristics Algorithm for Sequencing n Jobs on Three Machines

(1) Construct the table of time deviation for sequencing problem by calculating row deviation and column deviation in sequencing problem.

(2) Find the cell which has time deviation both are zero column wise from first column to last column.

(3) Perform the job first if the time deviation vectors are zero for Machine-1. If both time deviation vectors are zero for Machine-3, perform that job last.

(4) For job with zero time deviation vector on Machine-2 calculate the total deviation vectors for cells above and below those zero cells separately. Then, compare these totals.

(5) If the total deviation vectors above the cell are less than those below, then first start with that job.

(6) If the totals are equal, then you can either perform the job at the beginning or at the end.

(7) If we have achieved the optimal sequence for all the jobs in the job sequencing problem Otherwise go to next step.

(8) From a table with reduced time duration that contains only not assigned jobs.

(9) Again process the step from 1 to 7 for table with reduced time duration. But assigned the jobs next to the previously assigned jobs for cells where each vector is zero.

(10) If any job has time deviation vectors that are zero for both M1 & M2, Execute that job first which will reschedule the order of sequence. Similarly, if any job has time deviation vector that are zero for both M2 & M3, Execute that job last which will also reschedule the order of sequence.

(11) Finally, stop the process and determine the total elapsed time and idle time of Machine-M1, Machine-M2 & Machine-M3.

Problem Involving Sequencing n Jobs on Three Machines by Using Time Deviation Technique

Example 2.2. In the given job sequencing problem, there are 5 jobs, each of which must be processed on three machines, M1, M2 and M3. The goal of this problem is to find the optimal schedule for the given jobs, also determine the total minimum elapsed time and the idle time for machine M1, M2 and M3.

Jobs/Machines	J1	J2	J3	J4	J5
M1	3	9	6	5	4
M2	4	5	1	2	3
M3	8	9	5	7	10

Solution. Calculate the deviation for the given time duration tables for jobs on M1, M2 and M3 is

Jobs/Machines	J1	J2	J3	J4	J5
M1	(0,0)	(6,4)	(3,5)	(2,3)	(1,1)
M2	(3,1)	(4,0)	(0,0)	(1,3)	(2,0)
M3	(3,4)	(4,4)	(0,4)	(2,5)	(5,7)

For job J1 on Machine-M1, both deviations are zero. Therefore, place job J1 in the first position on the left side of the sequence. Also for job J3 on Machine-M2, both deviations are zero. For job J3 this job time on machine-M3 is smaller than Machine-M1, so place job J3 to the right side of the sequence, as given below:

J1				J3

Remove Jobs J1 and J3 from the above table and then write the updated table for the other jobs, as shown below:

Jobs/Machines	J2	J4	J5
M1	9	5	4
M2	5	2	3
M3	9	7	10

Again calculate the deviation for remaining jobs.

Jobs/Machines	J2	J4	J5
M1	(5,4)	(1,3)	(0,1)
M2	(3,0)	(0,0)	(1,0)
M3	(2,4)	(0,5)	(3,7)

For job J4 on Machine-M2, both deviations are zero. For job J4 this job time on machine-M3 is smaller than Machine-M1 so, Job J4 assigned the last second on right side of the sequence, as given below:

J1			J4	J3

Remove job J4 from the above table and write the updated table for thee other jobs, as shown below:

Jobs/Machines	J2	J5
M1	9	4
M2	5	3
M3	9	10

Again calculate the deviation for remaining jobs.

Jobs/Machines	J2	J5
M1	(5,4)	(0,1)
M2	(2,0)	(0,0)
M3	(0,4)	(1,7)

The job J5 on Machine-M2, both deviations are zero and for job J5 time on Machine-M1 is smaller than machine-M3. So, Place the job J5 in second column on left side of sequence and job J2 in free place of the sequence, as given below:

J1	J5	J2	J4	J3

This is required optimal sequence. By using this optimal sequence, we find the total minimum elapsed time and idle time for Machine-M1, Machine-M2 and Machine-M3 by using usual procedure.

Jobs/Machine sequence	Machine-M1		Machine-M2		Machine-M3		Idle time For Machine2	Idle time For Machine3
	Time-In	Time-Out	Time-In	Time-out	Time-In	Time-out		
J1	0	3	3	7	7	15	3	7
J5	3	7	7	10	15	25	0	0
J2	7	16	16	21	25	34	6	0
J4	16	21	21	23	34	41	0	0
J3	21	27	27	28	41	46	4	0

The total minimum elapsed time = 46 hour

For the idle time of Machine-M1 $= 46 - 27 = 19$ hour

For the idle time of Machine-M2 $= 3 + (16 - 10) + (27 - 23) + (46 - 28) = 3 + 6 + 4 + 18 = 31$ hour

For the idle time of Machine-M3 $= 7 + (46 - 46) = 7$ hours

2.3 Heuristics Algorithm for Sequencing n Jobs on m Machines

(1) Convert the m machines problem in to two machines problem, by using below condition:

(i) $\text{Min}(t_{1j}) \geq \text{Max}(t_{ij})$ for $i = 2, 3, \dots, n-1$,

(ii) $\text{Min}(t_{nj}) \geq \text{Max}(t_{ij})$ for $i = 2, 3, \dots, n-1$,

or both and by using the existing procedure.

(2) Apply the time deviation algorithm for the reduced two machines sequencing problem obtained from step 1 and obtain the optimal job sequence and determine the total minimum elapsed time and idle time.

Problem Involving Sequencing n Jobs on m Machines by Using Time Deviation Technique

Example 2.3. In the given job sequencing problem, there are 4 jobs, each of which must be processed on m machines, M1, M2, M3, M4, M5, and M6. The goal of this problem is to find the optimal schedule for the given jobs, also determine the total minimum elapsed time and idle time for machines M1, M2, M3, M4 and M5.

Jobs/Machines	J1	J2	J3	J4
M1	18	17	11	20
M2	8	6	5	4
M3	7	9	8	3
M4	2	6	5	4
M5	10	8	7	8
M6	25	19	15	12

Solution. Convert the six machines problem in two machines problem.

Jobs/Machines	J1	J2	J3	J4
G	45	46	36	39
H	52	48	40	31

Calculate the deviation for each converted jobs on G and H, as shown below:

Jobs/Machines	J1	J2	J3	J4
G	(9,0)	(10,0)	(0,0)	(3,8)
H	(21,7)	(17,2)	(9,4)	(0,0)

For job J3 on machine G and jobs J4 on machine H, both deviations are zero. Therefore, place job J3 in the first position on the left side of the sequence. Also place job J4 in the first position on the right side of the sequence, as given below:

J3			J4

Remove jobs J3 and J4 from the above table, and then write the updated table for the other jobs, as shown below:

Jobs/Machines	J1	J2
G	45	46
H	52	48

Again calculate the deviations for remaining jobs on G and H:

Jobs/Machines	J1	J2
G	(0,0)	(1,0)
H	(4,7)	(0,2)

For job J1 on machine G, both deviations are zero. Therefore, place job J1 on the left side of the sequence after job J3 and job J2 in free place of the sequence, as given below:

J3	J1	J2	J4

This is required optimal sequence. By using this optimal sequence, we find total minimum Elapsed Time and idle time for Machine-M1, Machine-M2, Machine-M3, Machine-M4, Machine-M5 and Machine-M6 by using usual procedure.

Jobs/Machines	Machine-M1		Machine-M2		Machine-M3		Machine-M4		Machine-M5		Machine-M6	
sequence	Time-In	Time-Out	Time-In	Time-Out	Time-In	Time-Out	Time-In	Time-Out	Time-In	Time-Out	Time-In	Time-Out
J3	0	11	11	16	16	24	24	29	29	36	36	51
J1	11	29	29	37	37	44	44	46	46	56	56	81
J2	29	46	46	52	52	61	61	67	67	75	81	100
J4	46	66	66	70	70	73	73	77	77	85	100	112

Idle time for Machine-M2	Idle time for Machine-M3	Idle time for Machine-M4	Idle time for Machine-M5	Idle time for Machine-M6
11	16	24	29	36
13	13	15	10	5
9	8	15	11	0
14	9	6	2	0

The total minimum elapsed time = 112 hour.

For the idle time of Machine-M1 $= 112 - 66 = 46$ hour.

For the idle time of Machine-M2 $= 11 + (29 - 16) + (46 - 37) + (66 - 52) + (112 - 70) = 11 + 13 + 9 + 14 + 42 = 89$ hour.

For the idle time of Machine-M3 $= 16 + (37 - 24) + (52 - 44) + (70 - 61) + (112 - 73) = 16 + 13 + 8 + 9 + 39 = 85$ hour.

For the idle time of Machine-M4 $= 24 + (44 - 29) + (61 - 46) + (73 - 67) + (112 - 77) = 24 + 15 + 15 + 6 + 35 = 95$ hour.

For the idle time of Machine-M5 $= 29 + (46 - 36) + (67 - 56) + (77 - 75) + (112 - 85) = 29 + 10 + 11 + 2 + 27 = 79$ hour.

For the idle time of Machine-M6 $= 36 + (56 - 51) + (112 - 112) = 36 + 5 + 0 = 41$ hours.

3. Conclusion

In this paper, we proposed a new concept of n jobs processed through the 2 machines, n jobs processed through the 3 machines, n jobs processed through the m machines, is introduced. This heuristic method called as time deviation methods is a frame for findings optimal sequence and minimum total elapsed time and idle time of n jobs and 2 machines, 3 machines, m machines. Here we proposed this technique to build the basic understanding in approaching more complicated situation. In near future this new time deviation methods can be extended to solve job sequencing problem to obtained optimal sequence and minimum total elapsed time and idle time without using existing conditions.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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