Application of Laplace Transforms to Solve ODE using MATLAB

Research Article

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Abstract. Laplace transform is a very powerful mathematical tool applied in various areas of engineering and science. With the increasing complexity of engineering problems, Laplace transforms help in solving complex problems with a very simple approach just like the applications of transfer functions to solve ordinary differential equations. In this paper we present a new technological approach to solve Ordinary differential equation. The application of MATLAB to compute and visualize the Laplace transforms is also discussed.

Keywords. Laplace transforms; Ordinary differential equation (ODE); MATLAB

MSC. 44A10; 68N01

1. Introduction

Laplace transforms [1] are frequently used in solving physical problems which involve integral and ordinary differential equations with constant coefficients as those arising in the analysis of electronic circuits. As a consequence of this most of them not only may be quickly computed with the aid of computer algebraic systems but the most commonly used mathematical software have also got subroutines that facilitate its handling and even force the general solving methods to use them. One of such powerful software packages is MATLAB [2], [3], [5], [6], [7] that contains many easy to use tools and built-in functions to solve or simulate differential equations by Laplace transform.

2. General Method to Solve Ordinary Differential Equation by Laplace Transforms

The Laplace transform will allow us to transform an initial-value problem for a linear ordinary differential equation with constant coefficients into a linear algebraic equation that can be easily solved. The solution of an initial-value problem can then be obtained from the solution of the algebraic equation by taking its inverse Laplace transform. The chart below illustrates this process:

(1) Start with differential equation with initial conditions: $y'' + y = e^t$ $y(0) = 0$, $y'(0) = 0$	**(2)** Take Laplace transform and obtain subsidiary equation: $(s^2 + 1)Y(s) = \frac{1}{(s-1)}$
(4) Use inverse Laplace transform to get back to solution: $y(t) = \left(\frac{1}{2}\right)e^t - \left(\frac{1}{2}\right)\cos(t) - \left(\frac{1}{2}\right)\sin(t)$	**(3)** Solve subsidiary equation for Y: $Y(s) = \left(\frac{1}{2(s-1)}\right) + \left(-\frac{1}{2(s^2-1)}\right) + \left(-\frac{1}{2(s^2-1)}\right)$
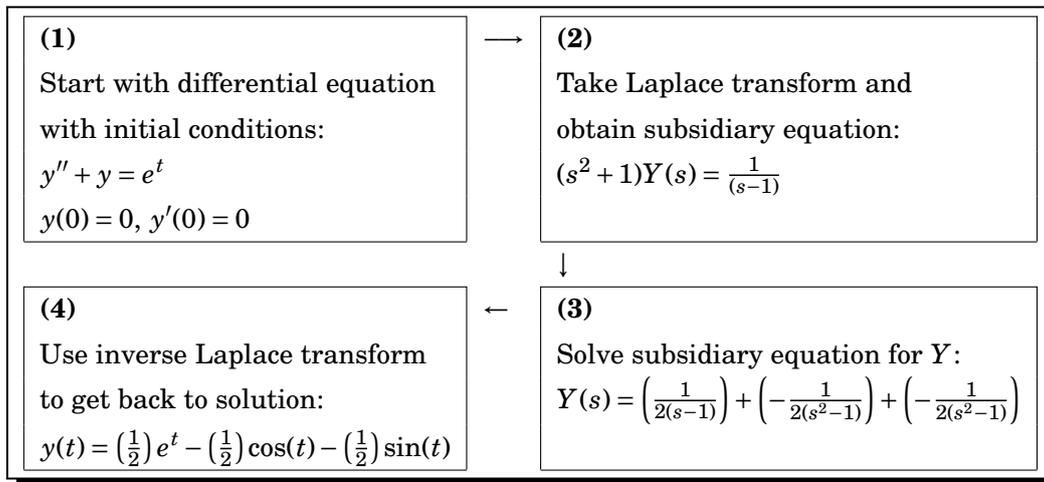
Example. Solve the equation

$$Y'' - 3Y' + 2Y = 4e^{2t}, \quad \text{where } Y(0) = -3, \ Y'(0) = 5. \tag{1}$$

Solution. Let the Laplace transform of $Y(t)$ be $y(s)$.

Taking the Laplace transform of both sides of (1), we obtain

$$L[Y''] - 3L[Y'] + 2L[Y] = 4L[e^{2t}].$$

Using the theorems for the transforms of derivatives, we get

$$\{s^2 y - sY(0) - Y'(0)\} - 3\{sy - Y(0)\} + 2y = \frac{4}{s-2}.$$

Substituting in the values of $Y(0) = -3$, $Y'(0) = 5$ we get

$$\{s^2 y + 3s - 5\} - 3\{sy + 3\} + 2y = \frac{4}{s-2},$$

$$(s^2 - 3s + 2)y + 3s - 14 = \frac{4}{s-2},$$

$$y = \frac{4}{(s^2 - 3s + 2)(s - 2)} + \frac{14 - 3s}{s^2 - 3s + 2}$$

$$= \frac{-3s^2 + 20s - 24}{(s - 1)(s - 2)^2},$$

$$y = \frac{-7}{s - 1} + \frac{4}{s - 2} + \frac{4}{(s - 2)^2}.$$

The solution is then

$$Y = L^{-1}[y]$$

$$= L^{-1}\left[\frac{-7}{s - 1} + \frac{4}{s - 2} + \frac{4}{(s - 2)^2}\right]$$

$$= -7e^t + 4e^{2t} + 4te^{2t}.$$

3. New Technological Approach to Solve Ordinary Differential Equation by Laplace Transforms

MATLAB has some powerful features for solving differential equations of all types. We will explore some of these features using Laplace transforms. The approach here will be that of the Symbolic Math Toolbox. The result will be the form of the function and it may be readily plotted with MATLAB.

MATLAB Programme to solve same Example 1 (ODE)

Example 1. Solve the equation $Y'' - 3Y' + 2Y = 4e^{2t}$ where $Y(0) = -3$, $Y'(0) = 5$

```
syms s t Y
f = 4 * exp(2 * t)
F = laplace(f,t,s)
Y1 = s * Y + 3
Y2 = s * Y1 - 5
Sol = solve(Y2 - 3 * Y1 + 2 * Y - F, Y)
sol = ilaplace(Sol,s,t)
pretty(sol)
ezplot(sol,[0,10])
grid on, title('GRAPHIC DISPLAY OF SOLUTION OF ODE BY LAPLACE TRANSFORM')
xlabel('time'),ylabel('f(t)')
legend(' laplace transform')
```
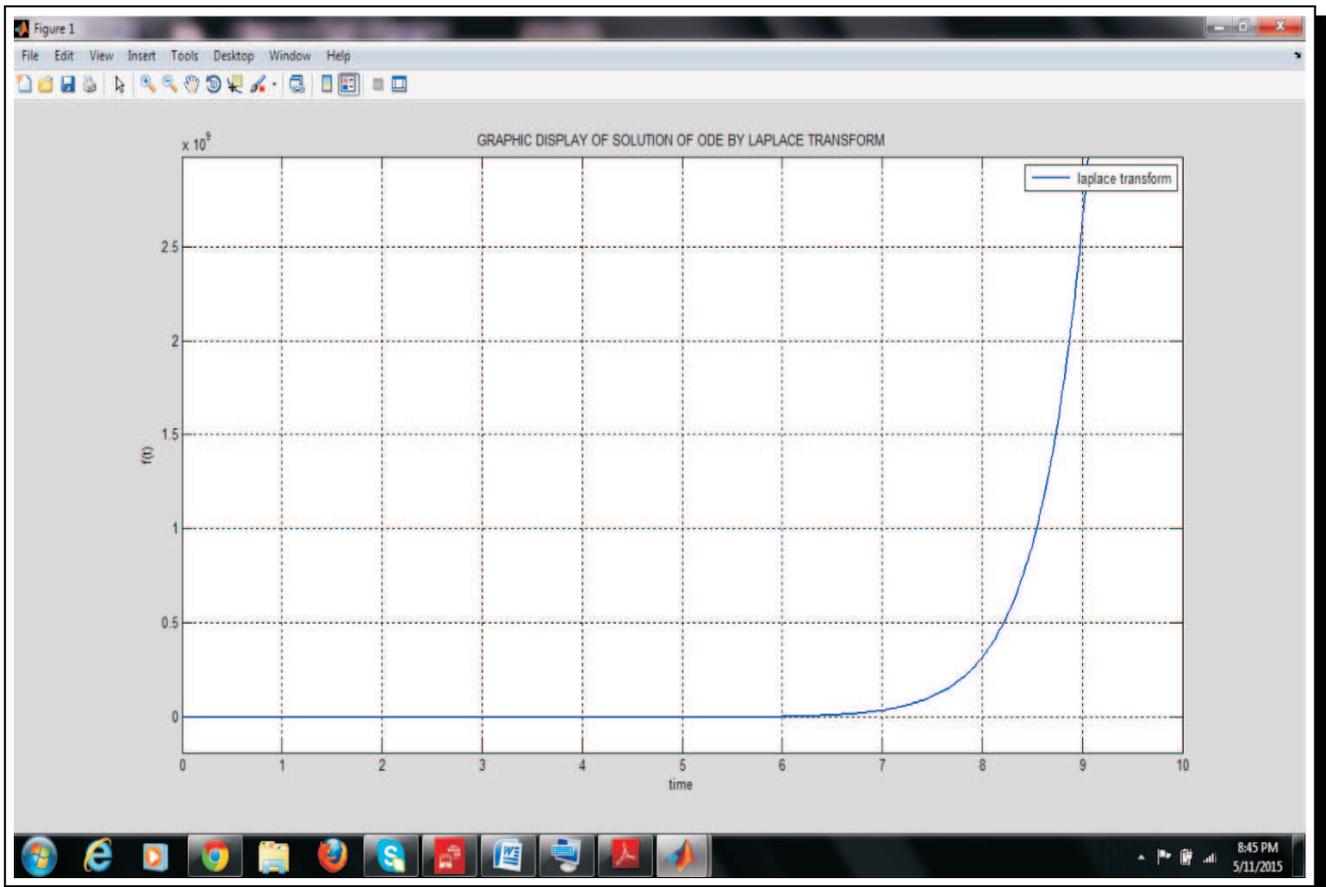
Output:

```
f =4 * exp(2 * t)
```

```
F = 4/(s - 2)
Y1 = Y * s + 3
Y2 = s * (Y * s + 3) - 5
Sol = (4/(s - 2) - 3 * s + 14)/(s^2 - 3 * s + 2)
sol = 4 * exp(2 * t) - 7 * exp(t) + 4 * t * exp(2 * t)
4 exp(2 t) - 7 exp(t) + 4 t exp(2 t)
```


Figure 1. Graphic display of solution of ODE by Laplace transform

Two approaches have been used here to solve an Initial Value problem with the general method and technological method using MATLAB and it has been observed that both the approaches give same solution. Through The technological approach using MATLAB we will be able to model a mathematical problem. It is helpful to develop different computational skills as discussed in [5].

4. Conclusion

Mathematical software saves a lot of time in routine calculations and application of standard techniques and Laplace transform is a very effective mathematical tool to simplify very complex

problems in the area of stability and control. This paper presents an examples of MATLAB applications in mathematical computation including generating the graph of the respective function. From these examples, we can see MATLAB can be a significant tool in mathematical computation and graphical representation of the functions. With the ease of application of Laplace transforms many research softwares have made it possible to simulate the Laplace transformable equations directly which has made a good advancement in the research field.

Competing Interests

The author declare that she has no competing interests.

Authors' Contributions

The author contributed and significantly in writing this article. The author read and approved the final manuscript.

References

[1] J.L. Schiff, *The Laplace Transform: Theory and Applications*, Springer Science and Business Media (1999).

[2] G.A.O. Feng, Applications of Matlab in mathematical analysis, *Journal of Software* **6** (7) (2011), 1225–1229.

[3] M.P. Legua, The heaviside function and Laplace transforms, *Proceedings of the 10th WSEAS International Confenrence on Applied Mathematics*, USA (2006), 445–450.

[4] S.M. Khairnar, Bilateral Laplace-Mellin integral transform and its applications, *International Journal of Pure and Applied Sciences and Technology* **1** (2) (2010), 114–126.

[5] P. Rai, Mathematical modeling with technology, *International Journal of Innovative Research in Computer Science & Technology* **3** (2) (2015), 117–120.

[6] Symbolic Math Toolbox User's Guide (2006), The MathWorks, Inc. (www.mathworks.com).

[7] S. Kadry, Learning basic mathematics using MATLAB, *International Journal of Information Technology & Computer Science* **14** (A202) (2014), 48–56.

[8] S. Wolfram, Computer software in science and mathematics, *Scientific American* **251** (3) (1984), 188–203.

[9] S. Alexander, *Introduction to Numerical Ordinary and Partial Differential Equations Using* MATLAB, John Wiley & Sons, Hoboken, NJ (2005).