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Modern Method to Compute the Determinants of Matrices of Order 3

Research Article

Ali A. M. Ahmed[1] and K. L. Bondar[2]

[1] *Aldarb Moriah Rainah, Sharab Alrona, Taiz, Yemen*
[2] *P.G. Department of Mathematics, NES Science College, Nanded, India*
Corresponding author: ali_ahmed.journal@yahoo.com

Abstract. In this paper we present a modern method to compute the determinants of 3×3 matrices. This method gives an easy schemes to compute the determinants of 3×3 matrices. It also gives fast approximation as described only five elements of the determinant. This modern method creates opportunities to find other modern methods to compute the determinants of higher orders, also.

Keywords. Methods to compute the determinant of 3×3 matrix

MSC. 11C20

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1. Introduction

1.1 Definition

Let A be $n\times n$ matrix, $A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$.

Determinant of n order square matrix is the sum, which has $n!$ different terms $\varepsilon_{j_1,j_2,\ldots,j_n} a_{1j_1}a_{2j_2}\ldots a_{nj_n}$ (see [4], [8], [9], [10])

$$\det A=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \Sigma_{s_n}\varepsilon_{j_1,j_2,\ldots,j_n} a_{1j_1}a_{2j_2}\ldots a_{nj_n}$$

where

$$\varepsilon_{j_1,j_2,\dots,j_n} = \begin{cases} +1 & \text{if } j_1,j_2,\dots,j_n \text{ is even permutation,} \\ -1 & \text{if } j_1,j_2,\dots,j_n \text{ is odd permutation.} \end{cases}$$

2. Some Methods to Compute the Determinants of Matrices of Order 3

There are some methods to calculate the determinants of matrices of third order.

2.1 Definition Method

In base of Definition 1.1, determinant of the third order matrix (for $n = 3$) can be computed as follows (see [4], [8], [9], [10]):

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

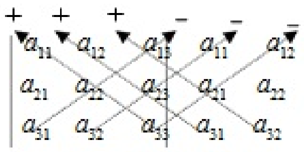
$$= \varepsilon_{123}a_{11}a_{22}a_{33} + \varepsilon_{132}a_{11}a_{23}a_{32} + \varepsilon_{312}a_{13}a_{21}a_{32}$$

$$+ \varepsilon_{321}a_{13}a_{22}a_{31} + \varepsilon_{231}a_{12}a_{23}a_{31} + \varepsilon_{213}a_{12}a_{21}a_{33}$$

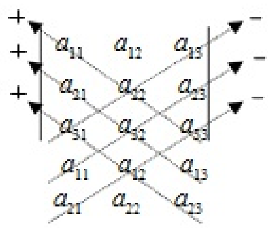
$$= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} + a_{12}a_{23}a_{31} - a_{12}a_{21}a_{33}$$

2.2 Sarrus Method

Using Sarrus rule, we have following scheme (see [4], [9], [11], [12]):



(Scheme 1)



(Scheme 2)

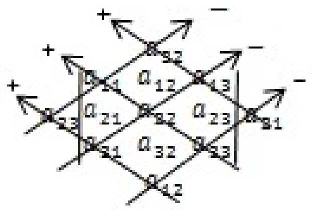
From the description of first two columns of the determinant (first and second columns) will form Scheme 1. Similarly two rows (first and second rows) form Scheme 2. The terms, which will be formed by the products of diagonal elements in the left side in both the schemes 1 and 2 become "+" sign, those in the right side become "−" sign. In this way we get the Sarrus rule, which is valuable just to compute the determinants of the third order matrix. In base of the Sarrus rule we have:

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

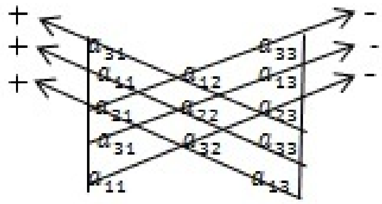
$$= a_{11}a_{22}a_{33} + a_{32}a_{13}a_{21} + a_{23}a_{12}a_{31} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23}.$$

2.3 Hajrizaj Method

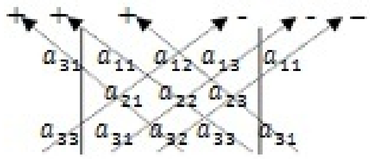
Using Hajrizaj rule, we have following schemes (see [6]):



scheme 3



scheme 4



scheme 5

From the description of the first and last element of the second row and the first and last elements of the second column of the determinants will form Scheme 3. Respectively, first and last elements of first and last columns will form Scheme 4. Respectively, first and last elements of first and last rows will form Scheme 5. The terms, which will be formed by the products of diagonal elements in the right side in three schemes 3, 4 and 5 become "−" sign and those in the left side become "+" sign. In this way we get the Hajrizaj rule, which are valuable just to compute the determinants of the third order. In base of the Hajrizaj rule we have:

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}a_{22}a_{33} + a_{32}a_{13}a_{21} + a_{23}a_{12}a_{31} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23}.$$

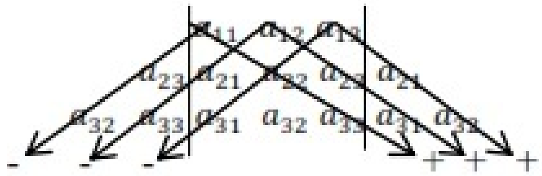
3. Modern Method to Compute the Determinants of Third Order

The modern method to compute the determinants of the third order might be one of the easiest methods to compute the determinants of the third order. Assume a determinant of the third order

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}.$$

Let's start by describing how we order the elements.

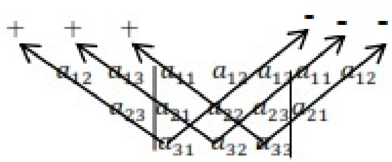
(1) Write the first and second elements of the third row after a_{33} respectively in the third row of the determinant, likewise write last and third elements of the same row before a_{31} respectively in the third row of determinant. Now write the first element of the second row after last element a_{23} in the second row of the determinant and the last element before the first element a_{21} in the second row of the determinant. Thus we obtain following scheme 6.



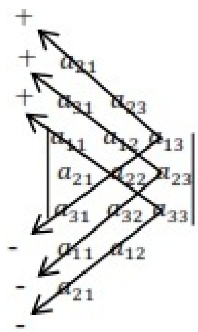
scheme 6

(2) The scheme formed of six diagonals containing three different elements each of the determinant. The product of elements in three diagonals of right side with "+" sing and the product of elements in three different diagonals of the left side with "−" sing.

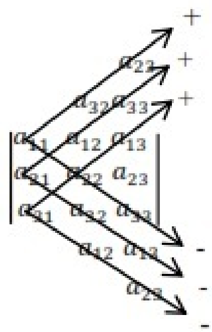
(3) This gives formula to compute the determinant of third order. This new method consist of three other schemes, which will be formed in the same way like the preliminary scheme 6, but these three other different schemes manipulate with elements in other rows and columns from the scheme 6. The other forms of this method are shown in the following schemes (Scheme 7, 8 and 9)



scheme 7



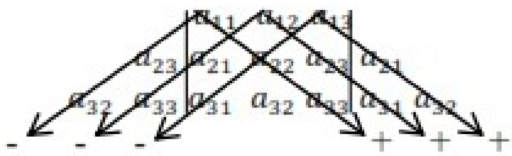
scheme 8



scheme 9

Proof. While applying the modern method with the Schemes 6, 7, 8 and 9 the determinants of the third order, we have:

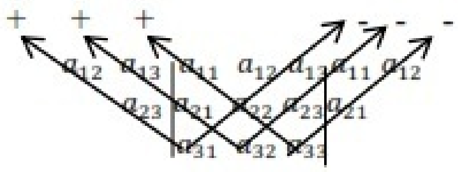
(i) for the scheme 6,



$$a_{11}a_{22}a_{33} + a_{32}a_{13}a_{21} + a_{23}a_{12}a_{31} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

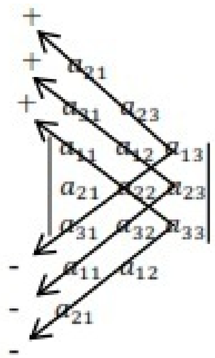
(ii) for the schem 7,



$$a_{11}a_{22}a_{33} + a_{32}a_{13}a_{21} + a_{23}a_{12}a_{31} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

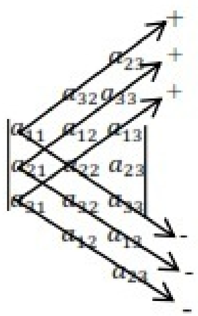
(iii) for the schem 8,



$$a_{11}a_{22}a_{33} + a_{32}a_{13}a_{21} + a_{23}a_{12}a_{31} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(iv) for the schem 9,



$$a_{11}a_{22}a_{33} + a_{32}a_{13}a_{21} + a_{23}a_{12}a_{31} - a_{22}a_{13}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{32}a_{23}$$

$$= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

4. Conclusion

This modern method, comparing with other known methods, is one of the most usable ones, based on quickness and easiness of computing the third order determinant. Furthermore, this modern method enables the further research in computing methods of higher than third order determinants. What is more, a modern method, enabling the sum computation of two third order determinants will be possible by combining the schemes of this method.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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