Journal of Informatics and Mathematical Sciences
Volume 1 (2009), Numbers 2 & 3, pp. 147–155

Stochastic Analysis of A Two-Identical Unit Cold Standby System with Maximum Repair Time and Correlated Busy and Idle Times of the Operator Cum Repairman

Aquil Ahmad and Lakhan Singh

Abstract. This paper is considered with modelling and analysis of a two identical unit standby system with the concept of rest of the operator cum repairman. Here, the concept of correlation between busy time and idle time of the operator cum repairman is considered which is not, generally, seen in the literature. If the repairman is not able to repair a failed unit within a specified time limit (pre fixed) then the failed unit is replaced by a replacement facility. The different measures of economic importance are obtained by using regenerative point technique. The MTSF and profit function have also been studied through graphs in respect of various parameters in a particular case when repair time distribution is taken as exponential.

1. Introduction

Commonly, in the literature of reliability, it is assumed that two persons (operator and repairman) are always available with the system. But, practically there may be some situations where a single person is sufficient for both the jobs of operator and repairman. Further, it is also, commonly, assumed that operator/repairman work continuously without taking rest. But, rest is necessary for proper and efficient working for a person. Very few authors including Goel and Srivastava [2, 3] have considered the provision of rest but they have taken busy time and idle time as uncorrelated random variables. But, naturally, the idle time depends on the busy time of the operator cum repairman. Correlated working and rest time of repairman is recently considered by [5].

Taking above facts in view, in the present study, the busy time and idle time of the operator cum repairman are considered as correlated random variables having their joint distribution as bivariate exponential. It is also assumed that if the repair of a failed unit is not completed within a specified time (pre fixed) then for the efficient and smooth running of the system it will be better that the failed unit is replaced by a replacement facility who is always available with the system.

Explicit expressions for the following system characteristics are obtained in general sense, also, particular cases are studied through graphs.

(i) Mean time to system failure (MTSF).

(ii) Availability of the system and expected up time of the system in (o, t) and in steady state.

(iii) Expected busy period of the repairman / replacement facility in (o, t) and in steady state.

(iv) Expected profit earned by the system in (o, t) and in steady state.

2. Model Description and Assumptions

(i) The system consists of two identical units arranged in standby configuration. Initially one unit is operative and other is kept as cold standby. The operation of one unit is sufficient to do the required job. Each unit has two modes normal (N) and total failure (F).

(ii) An operator cum repairman is always available with the system which can take rest randomly. During his rest, the system becomes down (no function and no repair). Also the busy time and idle time correlated random variables having their joint distribution as bivariate exponential.

(iii) After a maximum repair time (fixed), the failed unit will be replaced by a replacement facility.

(iv) During the replacement process the operator cum repairman can not leave the system. Also when both the units are in F-mode, he can not go for rest until one unit becomes operative.

(v) Each repaired unit works as good as new.

(vi) The discipline of repair as well as replacement is FCFS.

(vii) The switching device is perfect and instantaneous.

(viii) The failure time distribution of each unit is negative exponential while repair time distribution is taken as general.

(ix) The distribution of lead time and replacement time are negative exponential with different parameters.

3. Notations and Symbols for the System States

E : Set of regenerative states, i.e., $E \equiv (S_0, S_2, S_4, \ldots, S_7)$.

X, Y : Random variables representing the busy time and idle time of the operator cum repairman, respectively.

α : Constant failure rate of the operative unit.

β : Constant repair rate of the failed unit.

$\phi_i(u)$: p.d.f. of U_i ($i = 1$ for lead time and $i = 2$ for replacement time), i.e. $\phi_i(u) = \theta_i \exp(-\theta_i u)$

$f(x,y)$: joint p.d.f. of $(X,Y) = \lambda\mu e^{-\lambda x-\mu y} I_0[2\sqrt{(\lambda\mu r x y)}]$, $x, y, \lambda, \mu > 0$, $|r| < 1$, where $I_0[2\sqrt{(\lambda\mu r x y)}] = \sum_{k=0}^{\infty} \frac{(\lambda\mu r x y)^k}{(k!)^2}$

$g(x), G(x)$: marginal p.d.f. and c.d.f. of x $g(x) = \lambda(1-r)e^{-\lambda(1-r)x}$; $G(x) = 1 - e^{-\lambda(1-r)x}$

$h(y), H(y)$: marginal p.d.f. and c.d.f. of y $h(y) = \mu(1-r)e^{-\mu(1-r)y}$; $H(y) = 1 - e^{-\mu(1-r)y}$

$k(y/x), K(y/x)$: conditional p.d.f. and c.d.f. of y given x $k(y/x) = \mu e^{-\mu y-\lambda r x} I_0[2\sqrt{(\lambda\mu r x y)}]$ and $K(y/x) = \int_0^y k(t/x)dt$

Symbols for the states of the system

N_O/N_S : Unit in N mode and operative/standby

N_g : Unit in N mode and in good condition (non functioning)

F_r/F_{wr} : Unit in F mode and under repair/waiting for repair

U_R/U_{WR} : Unit is under replacement/waiting for replacement

With these symbols, the possible states of the system are:

Up states	Down states	Failed states
$S_0 = (N_0, N_s), S_2 = (F_r, N_0)$	$\underline{S}_1 = (N_g, N_g)$,	$S_4 = (F_r, F_{wr}), S_6 = (U_R, F_r)$
$S_5 = (U_R, N_0)$,	$\underline{S}_3 = (F_{wr}, N_g)$	$S_7 = (U_R, U_{wr})$

The underlined states are non-regenerative. The possible transitions among the states are shown in Figure 1.

4. **Transition Probabilities and Sojourn Times**

The non zero elements p_{ij} of the *transition probability matrix* (tpm) for the considered system model are as follows:

$$p_{00}^{(1)} = \frac{\lambda(1-r)}{\alpha + \lambda(1-r)}, \qquad p_{02} = \frac{\alpha}{\alpha + \lambda(1-r)},$$

$$p_{22}^{(3)} = \frac{\lambda(1-r)}{\alpha + \beta + \theta_1 + \lambda(1-r)}, \qquad p_{10} = p_{32} = p_{75} = 1,$$

$$p_{20} = \frac{\beta}{\alpha + \beta + \theta_1 + \lambda(1-r)}, \qquad p_{24} = \frac{\alpha}{\alpha + \beta + \theta_1 + \lambda(1-r)},$$

$$p_{25} = \frac{\theta_1}{\alpha + \beta + \theta_1 + \lambda(1-r)}, \qquad p_{42} = \frac{\beta}{\beta + \theta_1},$$

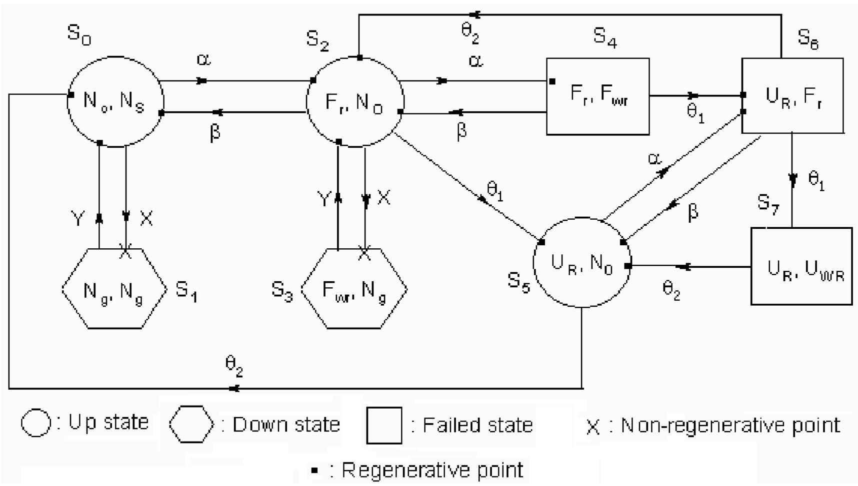

Figure 1. Transition diagram

$$p_{46} = \frac{\theta_1}{\beta+\theta_1}, \qquad p_{50} = \frac{\theta_2}{\alpha+\theta_2},$$
$$p_{56} = \frac{\alpha}{\alpha+\theta_2}, \qquad p_{62} = \frac{\theta_2}{\theta_1+\theta_2+\beta},$$
$$p_{65} = \frac{\beta}{\theta_1+\theta_2+\beta}, \quad p_{67} = \frac{\theta_1}{\theta_1+\theta_2+\beta}.$$

It can easily be verified that

$$p_{00}^{(1)} + p_{02} = 1, \quad p_{20} + p_{22}^{(3)} + p_{24} + p_{25} = 1, \quad p_{42} + p_{46} = 1,$$
$$p_{50} + p_{56} = 1, \quad p_{62} + p_{65} + p_{67} = 1.$$

Using the formula $\psi_i = \int^* P(Ti > t)dt$ for the mean sojourn time in state $S_i \in E$, its value for various states are

$$\psi_0 = \frac{1}{\alpha+\lambda(1-r)}, \qquad \psi_1 = \psi_3 = \frac{1+r}{\mu},$$
$$\psi_2 = \frac{1}{\alpha+\beta+\theta_1+\lambda(1-r)}, \quad \psi_4 = \frac{1}{\beta+\theta_1},$$
$$\psi_5 = \frac{1}{\alpha+\theta_2}, \qquad \psi_6 = \frac{1}{\beta+\theta_1+\theta_2}, \quad \psi_7 = \frac{1}{\theta_2}.$$

5. Reliability Analysis and MTSF

The reliability of the system when it starts operation from $S_i \in E$ is given by

$$R_i(t) = P[T_i > t]$$

*The limit of the integration when 0 to ∞ are not mentioned.

By probabilistic arguments, we have the following relations:

$$R_0(t) = Z_0(t)+q_{00}^{(1)}(t)©R_0(t)+q_{02}(t)©R_2(t)+q_{01}(t)©Z_1(t), \quad (5.1)$$

$$R_2(t) = Z_2(t)+q_{20}(t)©R_0(t)+q_{22}^{(3)}(t)©R_2(t)+q_{23}(t)©Z_3(t) + q_{25}(t)©R_5(t), \quad (5.2)$$

$$R_5(t) = Z_5(t)+q_{50}(t)©R_0(t), \quad (5.3)$$

where

$$Z_0(t) = e^{-[\alpha+\lambda(1-r)]t}, \qquad Z_2(t)=e^{-[\alpha+\beta+\theta_1+\lambda(1-r)]t},$$

$$Z_1(t) = Z_3(t)=\int\left\{\int_t^{\infty} \mu e^{-\mu y-\lambda r x} I_o[2\sqrt{(\lambda\mu r x y)}]dy\right\}g_1(x)dx,$$

$$Z_5(t) = e^{-(\alpha+\theta_2)t}.$$

Taking Laplace transform of relations (5.1)-(5.3) and simplifying for $R_0^*(s)$, we obtain,

$$R_0^*(s)=\frac{(Z_0^*+q_{01}^*Z_1^*)+q_{02}^*(Z_2^*+q_{23}^*Z_3^*)+q_{02}^*q_{25}^*Z_5^*}{(1-q_{00}^{(1)^*})(1-q_{22}^{(3)^*})-q_{02}^*q_{20}^*-q_{02}^*q_{25}^*q_{50}^*}. \quad (5.4)$$

Using the usual formula, the M.T.S.F. is given by

$$E(T_0) = \lim_{s\to 0}R_0^*(s) = \frac{(\psi_0+p_{01}\psi_1)(1-p_{22}^{(3)})+p_{02}(\psi_2+p_{23}\psi_3)+p_{02}p_{25}\psi_5}{(1-p_{00}^{(1)})(1-p_{22}^{(3)})-p_{02}p_{20}-p_{02}p_{25}p_{50}}. \quad (5.5)$$

6. **Availability Analysis**

From the theory of regenerative process, the point wise availabilities $A_i(t)$ of the system are seen to satisfy the following recursion relations:

$$A_0(t) = Z_0(t)+q_{00}^{(1)}(t)©A_0(t)+q_{02}(t)©A_2(t), \quad (6.1)$$

$$A_2(t) = Z_2(t)+q_{20}(t)©A_0(t)+q_{22}^{(3)}(t)©A_2(t)+q_{24}(t)©A_4(t) + q_{25}(t)©A_5(t), \quad (6.2)$$

$$A_4(t) = q_{42}(t)©A_2(t)+q_{46}(t)©A_6(t), \quad (6.3)$$

$$A_5(t) = Z_5(t)+q_{50}(t)©A_0(t)+q_{56}(t)©A_6(t), \quad (6.4)$$

$$A_6(t) = q_{02}(t)©A_2(t)+q_{65}(t)©A_5(t)+q_{67}(t)©A_7(t), \quad (6.5)$$

$$A_7(t) = q_{75}(t)©A_5(t). \quad (6.6)$$

Taking Laplace transform of equations (6.1)-(6.6) and solving for $A_0^*(s)$, we have

$$A_0^*(s)=N_2(s)/D_2(s), \quad (6.7)$$

where

$$\begin{aligned} N_2(s) &= [(1-q_{22}^{(3)*})(1-q_{56}^{(3)*}+q_{56}^{*}q_{62}^{*})-q_{24}^{*}q_{42}^{*}(1-q_{56}^{*}+q_{56}^{*}q_{65}^{*}) \\ &\quad -q_{62}^{*}(q_{24}^{*}q_{46}^{*}+q_{25}^{*}q_{56}^{*})]Z_0^{*}+q_{02}^{*}(1-q_{56}^{*}+q_{56}^{*}q_{62}^{*})Z_2^{*} \\ &\quad +q_{02}^{*}(q_{25}^{*}+q_{24}^{*}q_{46}^{*}q_{67}^{*})Z_5^{*}, \\ D_2(s) &= (1-q_{00}^{(1)*})(1-q_{22}^{(3)*})[1-q_{50}^{*}(1-q_{62}^{*})] \\ &\quad -(1-q_{00}^{(1)*})q_{24}^{*}q_{42}^{*}[1-q_{56}^{*}(1-q_{62}^{*})] \\ &\quad -(1-q_{00}^{(1)*})q_{62}^{*}(q_{24}^{*}q_{46}^{*}+q_{25}^{*}q_{56}^{*})-q_{02}^{*}q_{20}^{*}[1-q_{56}^{*}(1-q_{62}^{*})] \\ &\quad -q_{02}^{*}q_{50}^{*}[q_{25}^{*}+q_{24}^{*}q_{46}^{*}(1-q_{62}^{*})]. \end{aligned}$$

For brevity, the argument 's' is omitted from $q_{ij}^{*}(s)$ and $Z_i^{*}(s)$. Now, the steady state availability is given by,

$$A_0 = N_2/D_2,$$

where

$$\begin{aligned} N_2 &= N_2(0) = [(1-p_{22}^{(3)})(1-p_{56}+p_{56}p_{62})-p_{24}p_{42}(1-p_{56}+p_{56}p_{65}) \\ &\quad -p_{62}(p_{24}p_{46}+p_{25}p_{56})]\psi_0+p_{02}(1-p_{56}+p_{56}p_{62})\psi_2 \\ &\quad +p_{02}(p_{25}+p_{24}p_{46}p_{67})\psi_5, \\ D_2 &= (\psi_1 p_{01}+\psi_0)[p_{24}p_{50}p_{46}(1-p_{62})-p_{56}p_{20}(1-p_{62})+p_{25}p_{50}+p_{20}] \\ &\quad +(\psi_3 p_{23}+\psi_2)[p_{02}(p_{50}+p_{56}p_{62})]+p_{02}p_{24}[1-p_{56}(1-p_{62})]\psi_4 \\ &\quad +p_{02}[p_{25}+p_{24}p_{46}(1-p_{62})]\psi_5+p_{02}[p_{24}p_{46}+p_{25}p_{56}]\psi_6 \\ &\quad +p_{02}p_{67}[p_{24}(p_{46}-1)-p_{25}p_{56}]\psi_7. \end{aligned}$$

7. **Busy Period Analysis**

Let $B_i^R(t)/B_i^{RP}(t)$ be the respected probabilities that the repair facility/ replacement facility being busy at time 't' when system initially starts from state $S_i \in E$. Using simple probabilistic arguments the system of integral equations for $B_i^R(t)/B_i^{RP}(t)$ can easily be developed and by the technique of L.T., the values of $B_0^R(t)/B_0^{RP}(t)$ in terms of their L.T. can be found.

The steady state probabilities B_0^R/B_0^{RP} are given respectively as follows:

$$B_0^R = N_3/D_2 \quad \text{and} \quad B_0^{RP} = N_4/D_2,$$

where

$$\begin{aligned} N_3 &= [\{(1-p_{22}^{(3)})p_{24}p_{42}\}\{p_{56}(p_{65}+p_{67})-1\}-p_{62}p_{24}p_{46}]\psi_0 \\ &\quad +[p_{02}\{1-p_{56}(p_{65}+p_{67})\}]\psi_2+[p_{24}p_{02}\{1-p_{56}(p_{67}+p_{65})\}]\psi_4 \\ &\quad +p_{02}[p_{25}+p_{24}p_{46}(p_{65}+p_{67})]\psi_5+p_{02}[p_{25}p_{56}+p_{24}p_{46}]\psi_6 \\ &\quad +[p_{67}p_{02}(p_{24}p_{46}+p_{25}p_{56})]\psi_7, \\ N_4 &= p_{02}[p_{25}+p_{24}p_{46}(1-p_{62})]\psi_5+p_{02}(p_{24}p_{46}+p_{25}p_{56})(\psi_6+p_{67}\psi_7), \end{aligned}$$

and D_2 is the same as in availability analysis.

8. **Profit Function Analysis**

The net expected profit incurred during (o, t) is given by

$$\begin{aligned} P(t) &= \text{Expected total revenue during } (o, t) \\ &\quad - \text{Expected total expenditure during } (o, t) \\ &= K_0\mu_{up}(t) - K_1\mu_b^R(t) - K_2\mu_b^{RP}(t), \end{aligned}$$

where K_0 be the revenue per unit up time by the system, and K_1/K_2 be the amounts per unit time paid to the repair facility/replacement facility.

Also, $\mu_{up}(t) = \int_o^t A_0(u)du$ s.t. $\mu_{up}^*(s) = A_0^*(s)/s$

In the similar way $\mu_b^R(t)$ and $\mu_b^{RP}(t)$ can be defined.

Now the expected profit per unit time in steady state is given by

$$\begin{aligned} P &= \lim_{t\to\infty} P(t)/t \\ &= \lim_{s\to 0} s^2 P^*(s) \\ &= K_0A_0 - K_1B_0^R - K_2B_0^{RP}. \end{aligned}$$

Graphical Analysis

For a more concrete study of system's behaviour, the curves for MTSF and profit function are plotted in Figure 2 and 3. In Figure 2, curves represent the graph for MTSF with respect to α, for three different values of β as 0.1, 0.2 and 0.3 while the other parameters are fixed as $\lambda = 0.04$, $\mu = 0.06$, $\theta_1 = 0.05$, $\theta_2 = 0.06$ and $r = 0.50$. We observe that MTSF decreases uniformly as the value of (failure rate) increases and the value of MTSF increases as the value of β (repair rate) increases.

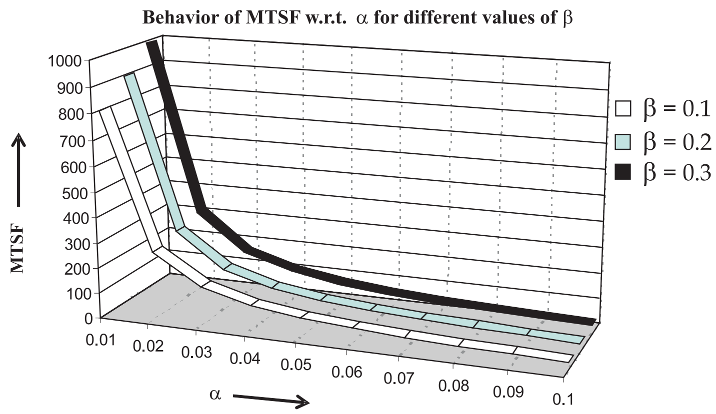

Figure 2

In Figure 3, curves represent the change in profit with respect to α, for three different values of β $(0.1, 0.2, 0.3)$. Two different sets of curves are plotted for two different values of 'r' as 0.25 and 0.50 while other parameters are kept same as in graphical analysis of MTSP with $K_0 = 2800$, $K_1 = 500$, $K_2 = 400$. We observe that the profit decreases uniformly as the value of α (failure rate) increases and the profit increases as the value of β (repair rate) and r increase.

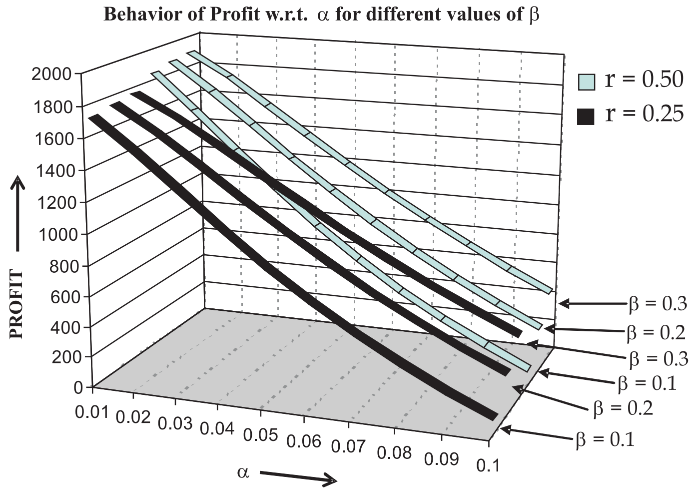

Figure 3

Practical Importance

In the present study the model formulation is based on quite practical assumptions such as passive redundancy, strategic-repair/replacement of the failed unit, amount of the time spent on repair, repair cost and random behaviour of the operator cum repairman. In industry there are so many systems which are fully or partially based on the assumptions taken in the present study and hence the configurational modeling and stochastic analysis of the system behaviour are useful for system managers, engineers, analysts and researchers who are engaged with similar systems. The considered concept of correlation between idle time and buys time of the operator cum repairman provides a new angle for stochastic analysis of system behaviour.

References

[1] R.K. Agnihotri and S.K. Satsangi, Two unit identical system with priority based on repair and inspection, *Microelectron Reliab.* **36**(1996), 279–282.

[2] L.R. Goel and P. Srivastava, Profit analysis of a two unit redundant system with provision for rest and correlated failures and repairs, *Microelectron Reliab.* **31**(5) (1991), 827–833.

[3] L.R. Goel, P. Srivastava and R. Gupta, Two unit cold standby system with correlated failures are repairs, *International Journal of System Science* **23**(3)(1992), 379–391.

[4] R. Gupta, R. Goel and L.R. Goel, Profit analysis of a two multicomponent unit standby system with MRT, *Microelectron Reliab.* **31**(5)(1991), 7–10.

[5] R. Gupta and V. Sharma, A two non-identical unit standby system with correlated working and rest time of repairmam, *Journal of Combinatorics, Information & System Science* **32**(1-4)(2007), 241–255.

Aquil Ahmad, *Department of Statistics, Lucknow University, Lucknow 226 007, India*.
E-mail: `wordsmithasif@gmail.com`

Lakhan Singh, *Department of Statistics, Lucknow University, Lucknow 226 007, India*.
E-mail: `lakhansaha@yahoo.co.in`

Received May 13, 2009
Revised August 3, 2009
Accepted August 8, 2009