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# Absolute Area Approximation in Channel Routing is NP-Hard

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**Abstract.** The computational complexity of minimizing area of routing in two- and three-layer channels is known to be NP-hard [9, 13]. In this paper we establish the result of computing an absolute approximate solution for no-dogleg two-layer *VH* channel routing is NP-hard. This result holds for channels with only two-terminal nets, where we impose a restriction of a partition of nets, such that the nets of the same class in the partition are to be assigned to the same track in any routing solution. We have proved the NP-hardness of the absolute area approximation problem for channels with nets having a bounded number of terminals per net. The later results have also been extended for routing using restricted doglegging. All the problems considered above for the two-layer *VH* routing model remain NP-hard even in the three-layer *HVH* routing model.

## 1. **Introduction**

In VLSI layout design it is required to realize a specified interconnection among different modules using minimum possible area. This is known as the routing problem. There exist several routing strategies for efficient interconnection among different modules. One of the most important types of routing strategies is *channel routing* [5, 6, 7, 10, 11, 15]. A channel has two open ends, the left and right sides of the rectangle. The other two sides (viz., the upper and lower sides of the rectangle) have two rows of terminals. The terminals are aligned vertically in *columns*. A set of terminals that need to be electrically connected together is called a *net*. A *subnet* of a net is a subset of the set of terminals of the net. Typically, the connections required are specified as two equal sized lists of numbers, one for the terminals of the upper row of the channel and the other for the terminals of the lower row of the channel. The size of these lists is the number of columns in the channel. The terminals of the same net are assigned the same number. The unconnected terminals are assigned number zero.

Throughout the paper we consider the *reserved layer Manhattan routing model*, where only horizontal and vertical wire segments are used for interconnecting

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the nets, and the wire segments are assigned to the respective layers [7, 10, 15]. The layer that has only horizontal (vertical) wire segments is called a *horizontal* (*vertical*) layer $H$ ($V$). The connection between a horizontal and a vertical wire segment of the same net in two such adjacent layers is achieved using a *via hole*.

Often in order to obtain a feasible routing solution or a solution with fewer numbers of tracks, the horizontal wire segment of a net is split into two (or more) parts and assigned to different tracks. This kind of routing is known as *dogleg routing* [5]. In *no-dogleg routing* no such splitting of horizontal wire segments is allowed, and it needs less via holes. If the route for a net is allowed to dogleg only in those columns in which it contains a terminal, then it is called a *restricted dogleg route*, otherwise it is known as an *unrestricted dogleg route*. In this paper we study the issues of establishing results on approximate area minimization for two- and three-layer no-dogleg and restricted dogleg routing only.

### 1.1. *Preliminaries and Basic Definitions*

The *channel routing problem* (*CRP*) is the problem of computing a feasible route for the nets so that the number of tracks required (and hence the channel area) is minimized. We say that a routing solution is feasible if all the nets can be assigned without any conflict. We assume that in a feasible routing solution, the routing wires do not extend beyond the left and right ends of the channel. Therefore, in order to minimize the routing area, the horizontal wire segments of the nets need to be distributed amongst a minimum number of tracks. This process of assignment of the horizontal wire segments to tracks is guided by two important constraints viz., the *horizontal constraints* and the *vertical constraints*. Let $L_i$ ($R_i$) be the leftmost (rightmost) column position of net $n_i$, then $I_i = (L_i, R_i)$ is known as the *interval* or *span* of the net. Suppose we use a single horizontal wire segment for routing each net. Then, this wire segment spans the entire interval $I_i$ of net $n_i$. So, routing the nets amounts to assigning intervals to horizontal tracks of the channel. The horizontal constraints determine whether two intervals $I_i$ and $I_j$ of two different nets $n_i$ and $n_j$, respectively, are assignable to the same track.

Vertical constraints determine the order in which the intervals should be assigned from top to bottom across the channel. Suppose we have a column in a channel with a terminal of net $n_i$ on the top and a terminal of net $n_j$ ($n_j \neq n_i$) at the bottom. Then, in order to assign the vertical wire segments of the nets in the column we must keep a gap of at least one track so that these two wire segments do not overlap. In other words we can say that if the horizontal wire segment of net $n_i$ is assigned to track $t_i$ and the horizontal wire segment of net $n_j$ is assigned to track $t_j$, then $t_i$ is a track nearer the top row than track $t_j$.

These two constraints are represented by two important constraint graphs viz., the *horizontal constraint graph* (*HCG*) and the *vertical constraint graph* (*VCG*), respectively [10, 15]. Horizontal constraints can be represented using an *HCG*,

$HC = (V, E)$, where a vertex $v_i \in V$ corresponds to interval $I_i$ of net $n_i$ in the channel. An undirected edge $\{v_i, v_j\} \in E$, if the intervals $I_i$ and $I_j$, corresponding to the nets $n_i$ and $n_j$, intersect at a column. An undirected edge $\{v_i, v_j\}$ in the *HCG* indicates that the corresponding intervals $I_i$ and $I_j$ are *horizontally constrained*, and are not assignable to the same track in the channel.

The *local density* of a column is the maximum number of nets passing through the column. The *channel density* (or only *density* of a channel) is the maximum of all the local densities. We denote channel density by $d_{max}$. A channel of density $d_{max}$ has at least one column spanned by $d_{max}$ nets. Each of these $d_{max}$ nets must be put in distinct tracks of the same horizontal layer.

We often represent horizontal constraints by the complement of the *HCG*. We call this graph the *horizontal non-constraint graph* (*HNCG*) and denote it by $HNC = (V, E')$, where $V$ is the set of vertices corresponding to the intervals, and $E' = \{\{v_i, v_j\} | \{v_i, v_j\} \notin E\}$ [10]. A set of vertices of a graph such that each pair of vertices in the set has an edge between them is called a *clique*. Note that a clique of the HNCG corresponds to a set of non-overlapping intervals that may safely be assigned to the same track in a routing solution.

The *VCG*, $VC = (V, A)$ is constructed to represent the vertical constraints. Here a vertex $v_i \in V$ corresponds to interval $I_i$ of net $n_i$ in the channel. A net whose corresponding vertex in the *VCG* is incident to a directed edge is said to be *vertically constrained*. Suppose we wish to assign two nets, $n_i$ and $n_j$ to tracks of the same horizontal layer. A directed edge $(v_i, v_j)$ in the *VCG* indicates that the net $n_i$ has to connect a top terminal and the net $n_j$ has to connect a bottom terminal at the same column position. Therefore, interval $I_i$ must be assigned to a track above the one to which interval $I_j$ is assigned. For an acyclic *VCG* we denote the *length of the longest path* in the *VCG* by $v_{max}$, where $v_{max}$ is equal to the number of vertices belonging to the path. Thus the minimum number of tracks required to route a two-layer channel is $max(d_{max}, v_{max})$.

We say that a multi-layer routing solution is a *density routing solution*, if it requires $\lceil d_{max}/i \rceil$ tracks where $i$ is the number of horizontal layers. Note that $\lceil d_{max}/i \rceil$ is a lower bound on the number of tracks required to route a channel of density $d_{max}$.

### 1.2. *Existing Results and Our Contribution*

Area minimization is the key objective in channel routing. Since the problem of minimizing area in two-layer *VH* and three-layer *HVH* channel routing is known to be NP-hard [9, 13], several heuristics for area minimization have been proposed, which generate routing solutions for several standard benchmark channels within a small number of tracks more than the optimal number required to route those channels[3, 4, 7, 11, 12, 15]. Even though most of these heuristics run in polynomial time, it is not known whether there exists a polynomial time algorithm that computes a routing solution for any given instance of the *CRP* within a constant number of tracks more than the optimal number required for that instance. Such an algorithm is called an *absolute approximation algorithm*. In this paper we have established that the problem of computing an absolute approximate solution for no-dogleg two-layer *VH* routing is NP-hard. This implies it is unlikely that there is a polynomial time absolute approximation algorithm for area minimization.

There are very few NP-hard optimization problems whose absolute approximations can be computed in polynomial time. One problem is that of determining the minimum number of colours needed to colour a planar graph [8]. Determining if a planar graph is three colourable is NP-hard. However, all planar graphs are four colourable. Another problem is the *maximum programs stored problem* [8]. Assume that we have $n$ programs and two storage devices, say disks. Let $l_i$ be the amount of storage needed to store the $i$th program. Let $L$ be the storage capacity of each disk. Determining the maximum number of these $n$ programs that can be stored on the two disks (without splitting a program over the disks) is NP-hard. However, by considering programs in order of nondecreasing storage requirement $l_i$, we can obtain a polynomial time absolute approximation algorithm. The NP-hardness of computing absolute approximations for a problem depicts the degree of hardness of the problem. The results of NP-hardness of absolute approximation problems derived in this paper depict that channel routing is indeed a very hard computational problem.

Computational problems become easier to solve for simpler inputs and consequently, proving intractability results becomes harder. So far we have considered channels with multi-terminal nets that have virtually unlimited number of terminals per net. A natural question is whether the absolute area approximation problem is NP-hard for channels with *bounded degree nets*. A net with a bounded number of terminals is known as a bounded degree net. We have proved the NP-hardness of the absolute area approximation problem for channels with nets having a maximum of five terminals per net.

We have also proved that computing an area absolute approximate solution is NP-hard in the no-dogleg routing model for channels with two-terminal nets under a certain restriction. The restriction imposed is that nets must be assigned to tracks in pre-specified groups. The motivation for such a restriction of groupings of nets (in their assignment to tracks) stems from practical design issues in VLSI, where it is preferable to group certain nets into the same track provided their spans do not overlap.

All the problems considered above for the two-layer *VH* routing model remain NP-hard even in the three-layer *HVH* routing model. The above results have also been extended for routing using restricted doglegging.

### 1.3. *Organization of the Paper*

The paper is organized as follows. In Section 2, we prove that the problem of computing a $k$-absolute approximate solution for channels with multi-terminal nets in the no-dogleg routing model is NP-hard. In Section 3, we prove that the problem of computing a $k$-absolute approximate solution for channels with multi-terminal nets in the restricted dogleg routing model is NP-hard under a certain restriction. In Section 4, we prove that the problem of computing a $k$-absolute approximate solution for channels with bounded degree nets in the no-dogleg routing model is NP-hard. In Section 5, we conclude the paper with a few remarks.

## 2. **Absolute Approximation for Two-Layer No-Dogleg Routing**

We know that the problem of routing a channel with two-terminal nets using a minimum number of tracks in the reserved two-layer no-dogleg Manhattan routing model has been shown to be NP-hard by LaPaugh [9]. Consequently, the problem of routing a channel with multi-terminal nets using a minimum number of tracks in the reserved two-layer restricted dogleg Manhattan routing model is also NP-hard. The two-layer channel routing problem remains NP-hard even if unrestricted doglegging is permitted [14]. So, it is unlikely that there exist polynomial time algorithms for minimizing channel area. A natural question arises: *Is there a polynomial time absolute approximation algorithm for two-layer no-dogleg channel routing*? In this section, we prove that the problem of computing an absolute area approximate solution in the no-dogleg two-layer *VH* routing model is NP-hard. We use a reduction from the well-known problem *TNVHK* [9], of computing a routing solution of minimum number of tracks for a given instance of two-terminal nets of the two-layer *CRP*. The problem *TNVHK* is as follows.

**Problem.** Two-terminal No-dogleg *VH* channel routing (*TNVHK*).

**Instance.** Two $m$ element vectors TOP and BOTTOM containing the terminals of $n \leq m$ two-terminal nets assigned to the channel, and a number $k$ of tracks between TOP and BOTTOM.

**Question.** Is there a legal wiring of the channel in the no-dogleg *VH* routing model of interconnect using no more than $k$ tracks?

*TNVHK* is a kind of *CRP* for which no known polynomial time algorithm finds the optimal solution for all instances. Consequently, note that the problem of routing a channel with multi-terminal nets using a minimum number of tracks in the reserved two-layer restricted dogleg Manhattan routing model (*MRVHK*) is also NP-hard. It is easy to see that *MRVHK* reduces to *TNVHK* by restricting the number of terminals for each net to two [9]. The problem *MRVHK* is as follows.

**Problem.** Multi-terminal Restricted dogleg *VH* channel routing (*MRVHK*).

**Instance.** Two $m$ element vectors TOP and BOTTOM containing the terminals of $n \leq m$ multi-terminal nets assigned to the channel, and a number $k$ of tracks between TOP and BOTTOM.

**Question.** Is there a legal wiring of the channel in the restricted dogleg *VH* routing model of interconnect using no more than $k$ tracks?

Consider the following approximation problem *MNVHAA1*: *Given a channel specification of multi-terminal nets, compute a two-layer no-dogleg routing solution whose number of tracks is at most one more than the minimum number of tracks required for the given instance*. In other words, we wish to compute a 1-*absolute approximate solution* for two-layer no-dogleg routing. We show that the problem *MNVHAA1* is as hard as the problem *TNVHK* by a polynomial transformation from *TNVHK* to *MNVHAA1*.

**Theorem 1.** *The problem MNVHAA1 of computing a two-layer no-dogleg routing solution, whose number of tracks is at most one more than the minimum number of tracks required for a given channel specification of multi-terminal nets, is NP-hard.*

***Proof.*** To show that *MNVHAA1* is NP-hard, we consider the following reduction from *TNVHK* to *MNVHAA1*. We construct an instance $I'$ of *MNVHAA1* from any instance $I$ of *TNVHK* using a polynomial time transformation. Let $t$ be the minimum number of tracks required for routing the instance $I$. We construct the instance $I'$ in such a manner that the minimum number of tracks required to route $I'$ is $2t+1$. Since $2t+1$ tracks are sufficient for routing $I'$, the 1-absolute approximation question for $I'$ can be stated as computing a routing solution for $I'$ within $2t+2$ tracks. We show that $I$ has a $t$-track solution if and only if $I'$ has a $2t+2$ tracks solution, thereby showing that *MNVHAA1* is as hard as *TNVHK*.

Let the number of nets in $I$ be $n$ and the length of the channel in $I$ be $m$. We construct $I'$ by duplicating the channel specification of $I$ into two groups A and B, each group containing $n$ nets. This gives $2n$ nets spread over $2m$ columns in $I'$. Group A consists of one copy of $I$ having $n$ two-terminal nets in the first $m$ columns of $I'$. Group B also has a copy of $I$ having $n$ two-terminal nets in the last $m$ columns of $I'$. The construction of $I'$ is such that in any routing solution for $I'$, any net of group A is assigned to a track above the track to which any net of group B is assigned. In order to achieve this separation, we add one additional net $s$, called the *separator net*, as follows. For each net $a_i$ of A and $b_i$ of B, $1 \leq i \leq n$, we could have introduced vertical constraints $(a_i, s)$ and $(s, b_i)$, in order to achieve the separation. However, it is sufficient to introduce such vertical constraints for nets $a_i$ of A whose corresponding nets in $I$ are sink vertices in the *VCG* of A and for nets $b_j$ of B whose corresponding nets in $I$ are source vertices in the *VCG* of B. Let $p$ $(q)$ be the number of such sink (source) nets in $I$. Therefore, introducing only $p+q$ new columns in $I'$ we can realize the required separation. So, $I'$ has $2n+1$ nets and $2m+p+q$ columns (see Figure 1). This completes the construction of $I'$.

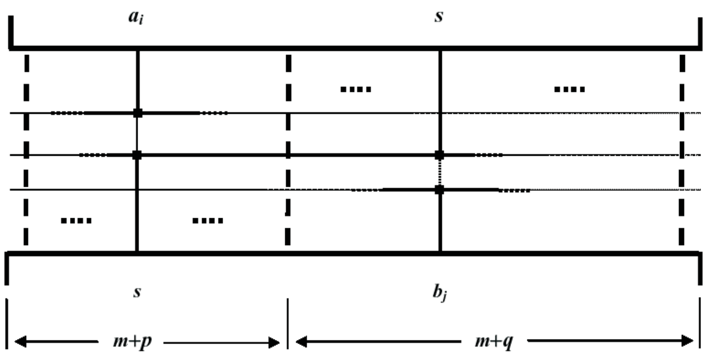


**Figure 1.** The constructed instance $I'$ of multi-terminal nets, where $a_i \in \mathsf{A}$ is a sink net and $b_j \in \mathsf{B}$ is a source net.

Now we show that the instance $I$ has a two-layer no-dogleg routing solution of $t$ tracks if and only if the instance $I'$ has a two-layer no-dogleg routing solution of $2t+2$ tracks.

Suppose, there is a $t$-track two-layer no-dogleg routing solution for the channel specification of $n$ two-terminal nets in $I$. We show that there is a two-layer no-dogleg routing solution $S$ for $I'$ using $2t+2$ tracks. Since $I$ can be routed within $t$ tracks, the nets of group A in $I'$ can be assigned within the topmost $t$ tracks. From the construction of $I'$ we know that the separator net $s$ must be assigned below the nets of group A. So, $s$ can be assigned to the $(t+1)$th track from the top. The nets of group B in $I'$ must be assigned below $s$, within the next $t$ tracks. This gives a $(2t+2)$-track routing solution for $I'$, where the bottommost track in $S$ is an empty track.

Now suppose, there is a $(2t+2)$-track routing solution $S$ for $I'$. We show that there is a $t$-track two-layer no-dogleg routing solution for $I$. Note that according to the construction of $I'$ we have the *VCG* as follows. Vertices corresponding to the sink nets in group A are immediate ancestors of the vertex corresponding to the separator net $s$. Similarly, the vertices corresponding to the source nets in group B are immediate descendants of the vertex corresponding to the separator net $s$. So for any routing solution $S$ for $I'$, no net of group B can be assigned to a track above the track to which a net of group A is assigned. Moreover in $S$, all the nets of group A are separated by the separator net $s$ from all the nets of group B. Therefore, either the nets of group A or the nets of group B use only $t$ tracks. Hence we can compute a two-layer no-dogleg $t$-track routing solution for the channel specification of $n$ two-terminal nets in $I$. □

So far we have proved that the problem *MNVHAA1* is NP-hard. In a similar manner, we can show that the problem *MNVHAAK* is also NP-hard. We pose the problem as follows. *Given a channel specification of multi-terminal nets, compute* *a two-layer no-dogleg routing solution whose number of tracks is at most $k$ (for any fixed $k > 1$) more than the minimum number of tracks required for the given instance.* In other words, we wish to compute a *$k$-absolute approximate solution* for two-layer no-dogleg routing.

Now we show that *MNVHAAK* is NP-hard by reducing the problem *TNVHK* to this problem.

**Theorem 2.** *The problem MNVHAAK of computing a two-layer no-dogleg routing solution, whose number of tracks is at most $k$ (for any fixed $k > 1$) more than the minimum number of tracks required for a given channel specification of multi-terminal nets, is NP-hard.*

***Proof.*** To show that *MNVHAAK* is NP-hard we use a polynomial transformation similar to that in the proof of Theorem 1. We construct an instance $I'$ of *MNVHAAK* for any instance $I$ of *TNVHK* by making $k+1$ copies of the channel specification of $I$ and using $k$ additional separator nets. As in the proof of Theorem 1, we use the separator nets to ensure that all nets of one copy are separated from all the nets of another copy in any routing solution of $I'$. This can be realized by introducing vertical constraints between the sink nets of the $i$th copy and the $i$th separator net, and between the $i$th separator net and the source nets of the $(i+1)$th copy, $1 \le i \le k$, of $I'$. So for any routing solution of $I'$, a net of the $i$th copy must be assigned to a track above the track to which a net of the $(i+1)$th copy is assigned. Assuming that instance $I$ has $p$ sink nets and $q$ source nets, we therefore require a total of $(k+1)m + k(p+q)$ (or $k(m+p+q)+m$) columns and $(k+1)n+k$ (or $k(n+1)+n$) nets in $I'$. This completes the construction of $I'$.

As in the proof of Theorem 1, we can show that the instance $I$ has a two-layer no-dogleg routing solution of $t$ tracks if and only if the instance $I'$ has a routing solution of $(k+1)t+2k$ tracks. $\square$

## 3. Absolute Approximation for Two-Layer Restricted Dogleg Routing

In this section we consider routing with restricted doglegging for instances with multi-terminal nets. Here the horizontal wire segment of a net is permitted to dogleg only at columns where it has a terminal. We propose the following approximation problem. *Given a channel specification of multi-terminal nets, compute a two-layer restricted dogleg routing solution whose number of tracks is at most one more than the minimum number of tracks required for the given instance*. In order to prove this problem NP-hard, it is sufficient to prove that this problem is NP-hard for a restricted class of inputs viz., two-terminal nets [9]. Since the inputs are now restricted to two-terminal nets, restricted dogleg routing of such nets amounts to no-dogleg routing. In other words, all we need to prove is the NP-hardness of the problem of routing a channel of two-terminal nets using no-doglegging with number of tracks at most one more than the minimum number of tracks required for the given instance.

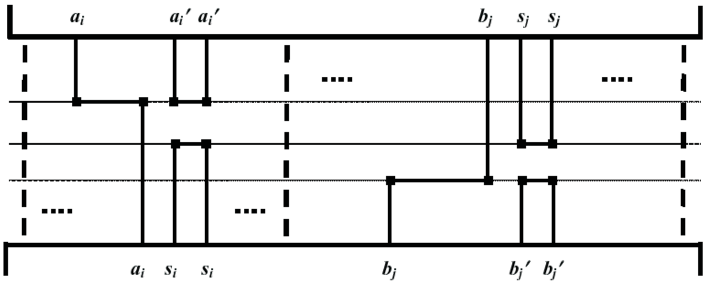


**Figure 2.** The constructed instance $I'$ of two-terminal nets, where (i) $a_i$ and $a_i'$ are to be assigned to the same track, (ii) $b_j$ and $b_j'$ are to be assigned to the same track, and (iii) $s_i$ and $s_j$ are to be assigned to the same track.

In Section 2, we have proved that the problem *MNVHAA1* is NP-hard. There we used a polynomial transformation from the problem *TNVHK*. In that transformation, while constructing an instance of the problem *MNVHAA1*, we had to introduce a separator net and the relevant vertical constraints. Due to these vertical constraints, the constructed instance had a multi-terminal separator net; the other nets in the constructed instance had at most three terminals. So, the technique of proving *MNVHAA1* NP-hard (in Theorem 1) does not appear to be useful in proving the NP-hardness of the following problem. *Given a channel specification of two-terminal nets, compute a two-layer no-dogleg routing solution whose number of tracks is at most one more than the minimum number of tracks required for the given instance*. The computational complexity of this problem remains open. However, for channels with two-terminal nets we can prove NP-hardness for a related approximation problem (*TNVHAA1P*) as stated below. Here we impose a restriction of a partition of two-terminal nets, such that the nets of the same class in the partition are to be assigned to the same track in any routing solution. *Given a channel specification of two-terminal nets and a partition $P$ of nets, compute a two-layer no-dogleg routing solution $S$ so that the number of tracks in $S$ is at most one more than the minimum number of tracks required for the given instance, where any pair of nets in a class of $P$ is assigned to the same track in $S$.*

The proof of NP-hardness of the problem *TNVHAA1P* is identical to that of the problem *MNVHAA1* with the difference that now we use only two-terminal nets and impose a restriction of an a priori partition of nets in the assignment of nets to tracks. We show that *TNVHAA1P* is NP-hard by a polynomial transformation from the problem *TNVHK* to *TNVHAA1P* as follows.

Given an instance $I$ of *TNVHK*, we construct an instance $(I', P)$ of *TNVHAA1P*, such that $I'$ has a two-layer no-dogleg routing solution using $2t + 2$ tracks, where the nets of $I'$ are assigned as per partition $P$, if and only if $I$ has a two-layer no-dogleg routing solution of $t$ tracks.

To prove that *TNVHAA1P* is NP-hard we use a construction similar to that in the proof of Theorem 1. $I$ has $n$ two-terminal nets, $p$ sink nets, and $q$ source nets. In constructing $I'$, we duplicate the instance $I$ into two groups A and B, in a manner similar to that in the proof of Theorem 1. For convenience, we place all the nets of group A to the left of all the nets of group B. Following the proof technique of Theorem 1, all we need to prove is that by using several additional two-terminal nets in $I'$(in addition to the nets of group A and group B), we can ensure that all nets of group A will be assigned to tracks above the tracks to which all nets of group B are assigned in any routing solution of $I'$. These additional nets are used to propagate vertical constraints from each net of group A corresponding to a sink net of $I$ to each net of group B corresponding to a source net of $I$. Let $a_i$ be a net of group A corresponding to any sink net of $I$ and $b_j$ be a net of group B corresponding to any source net of $I$. To propagate a vertical constraint from the net $a_i$ to the net $b_j$ in $I'$, we introduce four two-terminal nets $a_i', b_j', s_i$ and $s_j$, where (i) $(a_i', s_i)$ and $(s_j, b_j')$ are two vertical constraints, (ii) $a_i$ and $a_i'$ are forcibly assigned to the same track, (iii) $b_j$ and $b_j'$ are forcibly assigned to the same track, and (iv) $s_i$ and $s_j$ are forcibly assigned to the same track (see Figure 2). The nets $s_i$ and $s_j$ are called *separator nets*. The conditions (ii), (iii), and (iv) above are ensured by constructing the appropriate partition $P$ of nets that force these three conditions in every routing solution of $I'$.

We now state precisely how the additional nets are introduced. Consider the net $a_i$ of group A corresponding to a sink net of $I$. The net $a_i'$ is introduced by inserting two new columns to the right of the rightmost column of $a_i$. The net $s_i$ is also made to span exactly these two columns. The vertical constraints $(a_i', s_i)$ are realized by introducing two top terminals of the two-terminal net $a_i'$ and two bottom terminals of the two-terminal separator net $s_i$ in these two columns. In a similar manner, the net $b_j'$ is introduced by inserting two new columns to the right of the rightmost column of $b_j$. The net $s_j$ is also made to span exactly these two columns. The vertical constraints $(s_j, b_j')$ are realized by introducing two top terminals of the two-terminal separator net $s_j$ and two bottom terminals of the two-terminal net $b_j'$ in these two columns. Note that the separator nets are introduced by inserting two new columns just to the right of each of the sink nets $a_i$ of group A and each of the source nets $b_j$ of group B. So, no two separator nets overlap each other. Therefore, we can place all the separator nets in the same class of partition $P$ and force them to be assigned to the same track in every routing solution of $I'$. Each net of A (B) that does not correspond to a sink (source) net of $I$, is in a separate singleton class of $P$. This completes the construction of $I'$. It is easy to see that the number of columns and the number of nets in $I'$ are polynomial in the size of the instance $I$. We summarize the result in the following theorem.

**Theorem 3.** *The problem TNVHAA1P of computing a two-layer no-dogleg routing solution S, whose number of tracks is at most one more than the minimum number of* *tracks required for a given channel specification of two-terminal nets, where any pair of nets in a class of P is assigned to the same track in S for a given partition P of nets, is NP-hard.*

Now we state the general problem *TNVHAAKP* of computing a $k$-absolute approximate solution for two-terminal no-dogleg two-layer channel routing, and show that this problem is NP-hard. We pose the problem as follows. *Given a channel specification of two-terminal nets and a partition $P$ of nets, compute a two-layer no-dogleg routing solution $S$ so that* (i) *number of tracks in $S$ is at most $k$* (*for any fixed $k > 1$*) *more than the minimum number of tracks required for the given instance, and* (ii) *any pair of nets in a class of $P$ is assigned to the same track in $S$*.

Note that the problem *TNVHAAKP* is similar to the problem *MNVHAAK*. We can show that the problem *TNVHAAKP* is NP-hard by a polynomial transformation from the problem *TNVHK* to *TNVHAAKP* in a manner similar to the proof of NP-hardness of *MNVHAAK*. The construction is as in the proof of Theorem 3. We state the result in the following theorem.

**Theorem 4.** *The problem TNVHAAKP of computing a two-layer no-dogleg routing solution $S$, whose number of tracks is at most $k$ (for any fixed $k > 1$) more than the minimum number of tracks required for a given channel specification of two-terminal nets, where any pair of nets in a class of $P$ is assigned to the same track in $S$ for a given partition $P$ of nets, is NP-hard.*

It follows from the above result that allowing for multi-terminal nets and restricted doglegging does not improve the possibility of finding a $k$-absolute approximate solution for two-layer channel routing. So, we can conclude that the following problem is also NP-hard. *Given a channel specification of multi-terminal nets and a partition $P$ of subnets, compute a two-layer restricted dogleg routing solution $S$ so that* (i) *number of tracks in $S$ is at most $k$* (*for any fixed $k > 1$*) *more than the minimum number of tracks required for the given instance, and* (ii) *any pair of subnets in a class of $P$ is assigned to the same track in $S$*.

## 4. Absolute Approximation for Two-Layer No-Dogleg Routing with Bounded Degree Nets

In this section we consider the problem of two-layer no-dogleg routing for instances with nets having a bounded number of terminals. We define a net with a bounded number of terminals as a *bounded degree net*. The problem of finding a $k$-absolute approximate solution for a channel specification of two-terminal nets in two-layer no-dogleg routing is an open problem. So, the following question naturally arises. *Is there a polynomial time $k$-absolute approximation algorithm for a channel specification of bounded degree nets*? In this section we prove that the problem of computing a $k$-absolute approximate solution is as hard as the problem *MNVHAAK*; we show that for channels with nets having as low as an upper bound of five on the number of terminals per net. In this regard we wish to address the following approximation problem *BNVHAA1*: *Given a channel specification of bounded degree nets, compute a two-layer no-dogleg routing solution whose number of tracks is at most one more than the minimum number of tracks required for the given instance.* Now we prove that this problem is NP-hard.

The proof of NP-hardness of the problem *BNVHAA1* is identical to that of the problem *MNVHAA1* with the difference that now we use only bounded degree nets. We show that the problem *BNVHAA1* is NP-hard by a polynomial transformation from the problem *TNVHK* to *BNVHAA1* as follows. Given an instance $I$ of *TNVHK*, we construct an instance $I'$ of *BNVHAA1* such that the constructed channel specification $I'$ has a two-layer no-dogleg routing solution using $2t + p + q - 1$ tracks if and only if $I$ has a two-layer no-dogleg routing solution of $t$ tracks, where $p$ is the number of sink nets and $q$ is the number of source nets in $I$.

The initial part of the construction is same as in the proof of Theorem 1. So, we have two groups A and B of nets, each having $n$ two-terminal nets. Here the objective is to use only bounded degree separator nets so that a net of group A is assigned to a track above the track to which a net of group B is assigned. This is achieved by (i) propagating vertical constraints from the $p$ nets of group A corresponding to the $p$ sink nets of $I$, to a single separator net $s_1$, (ii) propagating a vertical constraint from the separator net $s_1$ to another separator net $s_2$, and (iii) propagating vertical constraints from $s_2$ to the $q$ nets of group B corresponding to the $q$ source nets of $I$ (see Figure 3). The vertical constraint $(s_1, s_2)$ is introduced to realize the propagation of the vertical constraint from $s_1$ to $s_2$, mentioned in (ii) above. The propagation of vertical constraints as mentioned in (i) and (iii) above requires more elaboration. Since (i) and (iii) are similar and symmetric, we discuss the realization of (i) only. We use a tree of separator nets, where each net is of bounded degree. The leaf nodes of the tree are the $p$ nets of group A corresponding to the $p$ sink nets of $I$. The root of the tree is the separator net $s_1$. The leaf or lowest level of the tree consists of these $p$ nets. Every next higher level in the tree has $\lceil x/2 \rceil$ nodes, each node being the parent of at most two nodes of the next lower level, where the number of nodes in the lower level is $x$. For each child-parent pair $(c, p)$, we introduce a vertical constraint $(c, p)$ in the *VCG* of $I'$. This completes the construction of the tree realizing the propagation of vertical constraints in (i) above. Note that each separator net $s_1, s_2$, or any net in the tree above, requires five terminals: one to propagate a vertical constraint further, two to receive two vertical constraints from two other nets, and two more to tie up the net at the left and right ends so that all such nets overlap each other (see Figure 4).

In order to ensure that no two separator nets are assigned to the same track in any routing solution of $I'$, we construct the separator nets so that all of them overlap each other. We tie up each separator net introducing a *left tie up block*

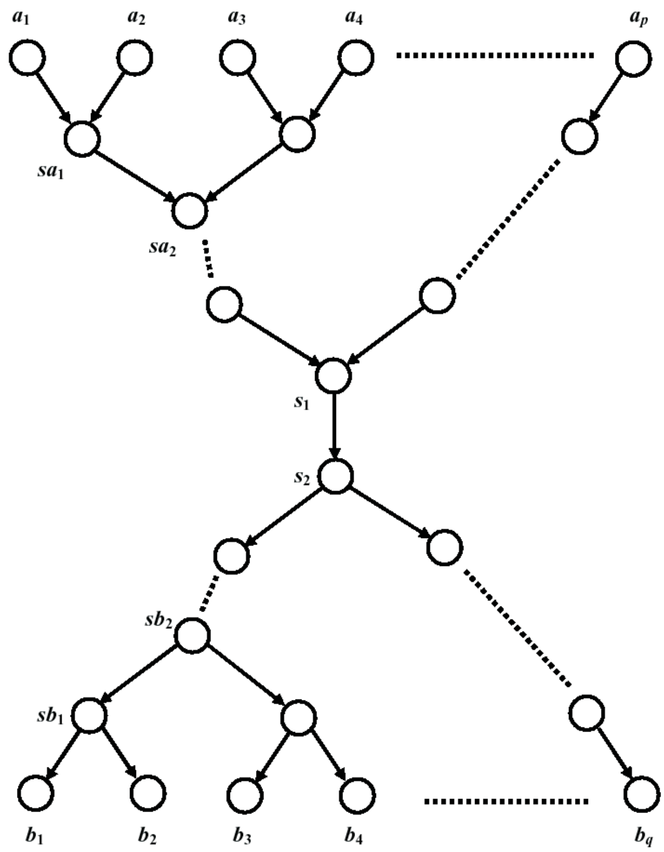


**Figure 3.** Propagation of vertical constraints from $p$ sink nets of group A to $q$ source nets of group B through bounded degree separator nets.

to the left of the constructed channel and a *right tie up block* to the right of the constructed channel. In each tie up block there is a column with a terminal of a separator net at the bottom and the top terminal unconnected (see Figure 4). This completes the construction of $I'$.

It is easy to see that the number of columns and the number of nets in $I'$ are polynomial in the size of the instance $I$. In $I'$ we have $2n + p + q - 2$ bounded degree nets, where $p$ ($q$) is the number of sink (source) nets in $I$. If the length of the channel specification of $I$ be $m$ of $n$ two-terminal nets, then the length of the constructed channel specification in $I'$ is $O(m + n)$.

As in the proof of Theorem 1, it is easy to see that the instance $I$ has a two-layer no-dogleg routing solution of $t$ tracks if and only if the instance $I'$ has a two-layer no-dogleg routing solution of $2t + p + q - 1$ tracks. Note that in any feasible routing solution each of the separator nets is assigned to a separate track. Moreover, according to the construction no net of group B is assigned to a track

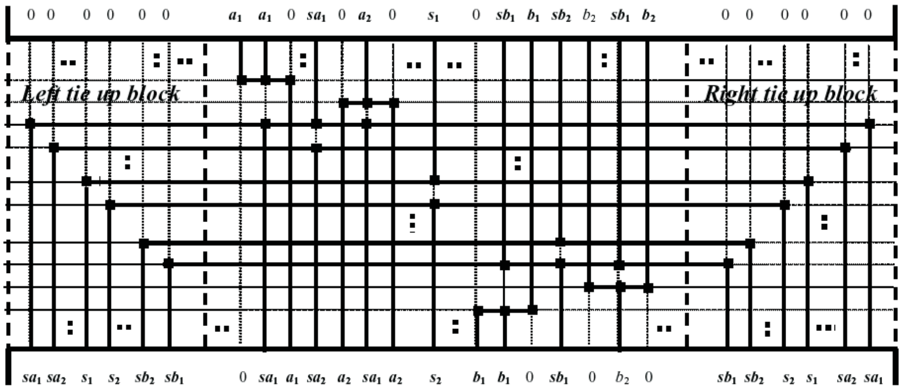


**Figure 4.** The constructed instance $I'$ with bounded degree nets.

above the track to which a net of group A is assigned. The remaining part of the proof is similar to the proof of Theorem 1. We summarize the result in the following theorem.

**Theorem 5.** *The problem BNVHAA1 of computing a two-layer no-dogleg routing solution, whose number of tracks is at most one more than the minimum number of tracks required for a given channel specification of bounded degree nets, is NP-hard.*

Now we propose the general problem *BNVHAAK* of computing a $k$-absolute approximate solution for a channel specification of bounded degree nets in the two-layer no-dogleg routing model and show that this problem is also NP-hard. We pose the problem as follows. *Given a channel specification of bounded degree nets, compute a two-layer no-dogleg routing solution whose number of tracks is at most $k$ (for any fixed $k > 1$) more than the minimum number of tracks required for the given instance.* We can prove that this problem is also NP-hard.

Note that the problem *BNVHAAK* is similar to the problem *MNVHAAK*; here the only difference is that we use $k(p+q-2)$ bounded degree separator nets instead of using $k$ multi-terminal separator nets as in the proof of *MNVHAAK*. We can show that the problem *BNVHAAK* is NP-hard by a polynomial time transformation from the problem *TNVHK* to *BNVHAAK* in a manner similar to the proof of NP-hardness of *MNVHAAK*. The construction of the instance of problem *BNVHAAK* is similar to that of the instance of problem *BNVHAA1* as in the proof of Theorem 5.

It is easy to verify that the number of nets and the length of the channel specification in $I'$ are polynomial in the size of the instance $I$. It is not a difficult task to prove that the instance $I'$ is such that $I$ has a two-layer no-dogleg routing solution of $t$ tracks if and only if $I'$ has a two-layer no-dogleg routing solution of $(k+1)t + k(p+q-2) + k$ tracks, where $p$ ($q$) is the number of sink (source) nets in $I$. We summarize the result in the following theorem.

**Theorem 6.** *The problem BNVHAAK of computing a two-layer no-dogleg routing solution, whose number of tracks is at most $k$ (for any fixed $k > 1$) more than the minimum number of tracks required for a given channel specification of bounded degree nets, is NP-hard.*

We have proved that the problem *BNVHAAK* of computing a $k$-absolute approximate solution in the two-layer no-dogleg routing is NP-hard for channels with bounded degree nets (for any fixed $k \geq 1$); an open question is whether this problem remains NP-hard for channels with two-terminal nets. Since we cannot answer this question, we cannot show that the problem of computing a $k$-absolute approximate solution in the two-layer restricted dogleg routing model is NP-hard for channels with unbounded degree nets. Nevertheless, the bounded degree result implies that the problem of computing a $k$-absolute approximate solution is NP-hard for routing using a limited form of restricted doglegging, where instead of permitting doglegging at every terminal column of each net, we permit doglegging only at terminal columns separated by a constant number of terminals for each net. This constant is same as the bound on the number of terminals of each net for which the problem *BNVHAAK* has been proved NP-hard.

All the results proved so far for the two-layer *VH* routing model hold even in the three-layer *HVH* routing model.

## 5. Conclusion

In this paper we have proved that several problems of computing absolute approximate solutions in two-layer and three-layer channel routing are NP-hard. Since bounded degree nets are simpler than nets with an unbounded number of terminals, it is relatively harder to prove NP-hardness results for channels with bounded degree nets. We have shown that the NP-hardness results also hold for channels with bounded degree nets.

Interesting questions concerning the computational complexity of channel routing problems that remain open are those of computing absolute approximate solutions under the multi-layer $V_iH_i$, $2 \leq i < d_{max}$, and $V_iH_{i+1}$, $3 \leq i+1 < d_{max}$, routing models, where horizontal and vertical layers of interconnect alternate. The computational complexity of computing relative approximate solutions under the two-layer and three-layer routing models is also open. These problems are likely to be polynomial time solvable. A polynomial time algorithm for the relative approximation question for two-layer channel routing has been designed for the case where routing of wires is allowed beyond the left and right ends of the channel (see [1, 2]). A density routing solution can be computed in polynomial time for the $V_iH_{i-1}$, $1 \leq i-1 < d_{max}$, routing model. Such a density solution with $\lceil d_{max}/(i-1) \rceil$ tracks is related by a constant factor to the minimum number of tracks $\lceil d_{max}/i \rceil$ ($\lceil d_{max}/(i+1) \rceil$) in the $V_iH_i$ ($V_iH_{i+1}$) routing model. So, the relative approximation question remains open only for the two-layer *VH* routing model and the three-layer *HVH* routing model.

The problems of computing absolute approximate solutions are also open for the cases of two-terminal no-dogleg two-layer *VH* routing model and two-terminal no-dogleg three-layer *HVH* routing model.

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## References

[1] B.S. Baker, S.N. Bhatt and F.T. Leighton, An approximate algorithm for Manhattan routing, in *VLSI Theory*, F.P. Preparata (editor), JAI Press Inc., Greenwich, pp. 205–229, (*Advances in Computing Research* **2**), 1984.

[2] M. Burstein, Channel routing, in *Layout Design and Verification*, T. Ohtsuki (editor), Elsevier Science Publisher, Amsterdam, The Netherlands, 1986, pp. 133–167.

[3] M. Burstein and R. Pelavin, Hierarchical channel router, *INTEGRATION: the VLSI Jour.* **1** (1983), 21–38.

[4] M. Burstein and R. Pelavin, Hierarchical wire routing, *IEEE Trans. on CAD of Integrated Circuits and Systems* **2** (1983), 223–234.

[5] D.N. Deutsch, A dogleg channel router, *Proc. of 13th ACM/IEEE Design Automation Conf.*, 1976, pp. 425–433.

[6] A. Hashimoto and J. Stevens, Wire routing by optimizing channel assignment within large apertures, *Proc. of 8th ACM Design Automation Workshop*, 1971, pp. 155–169.

[7] T.-T. Ho, S.S. Iyengar and S.-Q. Zheng, A general greedy channel routing algorithm, *IEEE Trans. on CAD of Integrated Circuits and Systems* **10** (1991), 204–211.

[8] E. Horowitz and S. Sahni, *Fundamentals of Computer Algorithms*, Reading, Computer Science Press, Rockville, Maryland, 1978.

[9] A.S. LaPaugh, *Algorithms for Integrated Circuit Layout: An Analytic Approach*, Ph.D. thesis, Lab. for Computer Sc., MIT, Cambridge, November 1980.

[10] R.K. Pal, A.K. Datta, S.P. Pal, M.M. Das and A. Pal, A general graph theoretic framework for multi-layer channel routing, *Proc. of 8th VSI/IEEE Int. Conf. on VLSI Design*, 1995, pp. 202–207.

[11] J. Reed, A. Sangiovanni-Vincentelli and M. Santomauro, A new symbolic channel router: YACR2, *IEEE Trans. on CAD of Integrated Circuits and Systems* **4** (1985), pp. 208–219.

[12] R.L. Rivest and C.M. Fiduccia, A 'Greedy' channel router, *Proc. of 19th ACM/IEEE Design Automation Conf.*, 1982, pp. 418–424.

[13] G.A. Schaper, *Multi-Layer Channel Routing*, Ph.D. thesis, Department of Computer Science, University of Central Florida, Orlando, August 1989.

[14] T.G. Szymanski, Dogleg channel routing is NP-complete, *IEEE Trans. on CAD of Integrated Circuits and Systems* **4** (1985), 31–41.

[15] T. Yoshimura and E.S. Kuh, Efficient algorithms for channel routing, *IEEE Trans. on CAD of Integrated Circuits and Systems* **1** (1982), 25–35.

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