



# Multiplicative Connectivity Indices of Tri-Hexagonal Boron Nanotube and Nanotori

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**Abstract.** A topological index is a numeric value that can be used to characterize some property of the graph representing a molecule. In this article, we compute multiplicative connectivity indices namely, multiplicative version of first Zagreb index ( $\Pi_1^*$ ), second multiplicative Zagreb index ( $\Pi_2$ ), first and second multiplicative hyper-Zagreb index ( $H\Pi_1, H\Pi_2$ ), general first and second multiplicative Zagreb index ( $MZ_1^a, MZ_2^a$ ), multiplicative sum-connectivity index ( $X\Pi$ ), multiplicative product-connectivity index ( $\chi\Pi$ ), multiplicative atom-bond connectivity index ( $ABC\Pi$ ) and multiplicative geometric-arithmetic index ( $GA\Pi$ ) for tri-hexagonal boron nanotube, tri-hexagonal boron nanotorus and tri-hexagonal boron- $\alpha$  nanotorus.

**Keywords.** Multiplicative connectivity indices; Tri-hexagonal boron nanotube; Tri-hexagonal boron nanotorus; Tri-hexagonal boron- $\alpha$  nanotorus

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## 1. Introduction

Mathematical chemistry is a branch of theoretical chemistry in which chemical structure can be predicted by using different mathematical tools. Chemical graph theory is one of the tool, which implements graph theory to study mathematical modeling of chemical aspects. A topological index in the chemical graph theory is used to predict bioactivity of the molecular graphs of chemical compounds. Molecular graphs models the chemical structures of molecules and

molecular compounds by considering atoms as vertices and the chemical bonds between the atoms as edges.

We consider  $G(V, E)$  to be finite, undirected, simple graph where  $V$  and  $E$  be the sets containing vertices and edges of  $G$ , respectively. Number of elements in  $V$  is called the order of  $G$ , denoted as  $|V|$  and the number of elements in  $E$  is called the size of  $G$ , denoted as  $|E|$ . The degree of a vertex  $v$  is denoted by  $d_G v$ . Let  $uv$  represent an edge between the two vertices  $u$  and  $v$ . For undefined terminologies we refer to [1].

A topological index is a numeric value mathematically derived from the graph representing a molecule. Topological indices are of two main categories, one depends on vertex distance and the other depends on vertex degree.

In 2010, Todeshine and Consonni [7] gave two new vertex invariants which are

$$\Pi_1(G) = \prod_{v \in V} d_G v^2 \quad \text{and} \quad \Pi_2(G) = \prod_{uv \in E} (d_G u \times d_G v).$$

**Definition 1.1** ([7]). Multiplicative version of first and Second Zagreb index of  $G$  is

$$\Pi_1^*(G) = \prod_{uv \in E} (d_G u + d_G v), \quad \Pi_2(G) = \prod_{uv \in E} (d_G u \times d_G v).$$

**Definition 1.2** ([3]). First and second multiplicative hyper-Zagreb index of  $G$  is

$$H\Pi_1(G) = \prod_{uv \in E} (d_G u + d_G v)^2, \quad H\Pi_2(G) = \prod_{uv \in E} (d_G u \times d_G v)^2.$$

**Definition 1.3** ([4]). General first and second multiplicative Zagreb index of  $G$  is

$$MZ_1^a(G) = \prod_{uv \in E} (d_G u + d_G v)^a, \quad MZ_2^a(G) = \prod_{uv \in E} (d_G u \times d_G v)^a.$$

**Definition 1.4** ([2]). Multiplicative sum-connectivity index of  $G$  is

$$X\Pi(G) = \prod_{uv \in E} \frac{1}{\sqrt{d_G u + d_G v}}.$$

**Definition 1.5** ([2]). Multiplicative product-connectivity index of  $G$  is

$$\chi\Pi(G) = \prod_{uv \in E} \frac{1}{\sqrt{d_G u \times d_G v}}.$$

**Definition 1.6** ([2]). Multiplicative atom-bond connectivity index  $G$  is

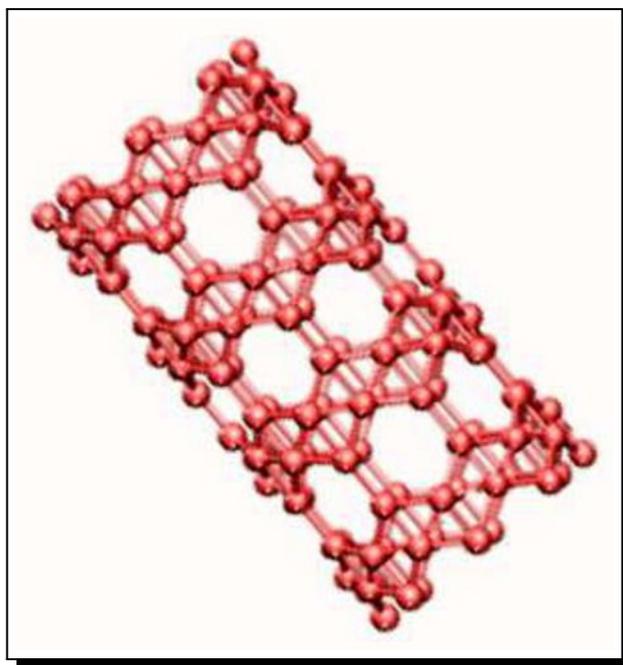
$$ABC\Pi(G) = \prod_{uv \in E} \sqrt{\frac{d_G u + d_G v - 2}{d_G u \times d_G v}}.$$

**Definition 1.7** ([2]). Multiplicative geometric-arithmetic index of  $G$  is

$$GA\Pi(G) = \prod_{uv \in E} \frac{2\sqrt{d_G u \times d_G v}}{d_G u + d_G v}.$$

Boron nanotubes has remarkable qualities like, at high temperatures it has high resistance to oxidation, high chemical stability and are a stable wide band-gap semiconductor. Because of their special properties it can be used for applications at high temperatures and also in corrosive environments such as batteries, high speed machines as solid lubricant, super capacitors and fuel cells. The stability, mechanical and electronic properties has been discussed in [5, 9].

In 2009, Wang et al. [8] gave a type of boron nanotube contains triangles, hexagons and was called as tri-hexagonal boron nanotube. The three dimensional image of this is given in Figure 1.



**Figure 1.** Three-dimensional perception of Tri-Hexagonal boron nanotube —  $C_3C_6(H)[p, q]$ .

In this paper, we have computed all the above defined multiplicative topological indices for three different nanotubes: tri-hexagonal boron nanotube, tri-hexagonal boron nanotorus and tri-hexagonal boron- $\alpha$  nanotorus.

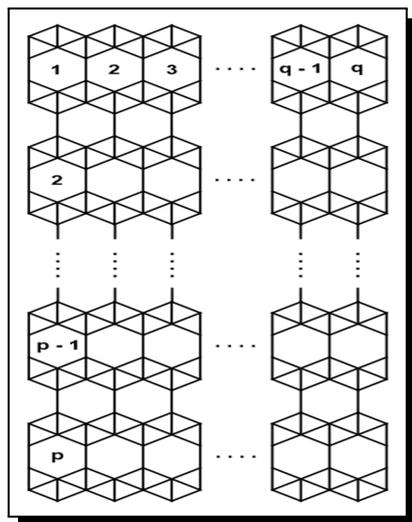
## 2. Main Results

In this section, we consider Tri-Hexagonal boron nanotube represented as  $C_3C_6(H)[p, q]$ , Tri-Hexagonal boron nanotorus represented as  $THBC_3C_6[p, q]$  and Tri-Hexagonal boron- $\alpha$  nanotube represented as  $THBAC_3C_6[p, q]$ . Recently,  $ABC$ ,  $ABC_4$ ,  $GA$  and  $GA_5$  indices are studied for tri-hexagonal boron nanotori [6], motivated by which we will compute multiplicative connectivity indices of  $C_3C_6(H)[p, q]$ ,  $THBC_3C_6[p, q]$  and  $THBAC_3C_6[p, q]$ .

In each case edge set of the corresponding graph is partitioned according to unordered degree pairs of the end vertices of edges in the graph are used to find multiplicative connectivity indices.

### 2.1 Tri-Hexagonal boron nanotube — $C_3C_6(H)[p, q]$

In this section, we calculate multiplicative connectivity indices of  $C_3C_6(H)[p, q]$ , here  $p$  indicates the count of hexagons in a column and  $q$  indicates the count of hexagons in a row of the two-dimensional molecular graph of  $G = C_3C_6(H)[p, q]$  nanotube as given in Figure 2. Here  $|V| = 8pq$  and  $|E| = q(18p - 1)$ .



**Figure 2.** A two-dimensional molecular graph of Tri-Hexagonal boron nanotube —  $C_3C_6(H)[p, q]$ .

**Theorem 2.1.** For the graph  $G = C_3C_6(H)[p, q]$ ,

- i.  $\Pi_1^*(G) = (2^5 9^6 5^2)^{2pq} \left(\frac{8^5}{9^6}\right)^q$
- ii.  $\Pi_2(G) = (2^{16} 5^{10})^{2pq} \left(\frac{3^6}{4^8}\right)^q$
- iii.  $H\Pi_1(G) = (2^5 9^6 5^2)^{4pq} \left(\frac{8^5}{9^6}\right)^{2q}$
- iv.  $H\Pi_2(G) = (2^{16} 5^{10})^{4pq} \left(\frac{3^6}{4^8}\right)^{2q}$
- v.  $MZ_1^a(G) = (2^5 9^6 5^2)^{2apq} \left(\frac{8^5}{9^6}\right)^{aq}$
- vi.  $MZ_2^a(G) = (2^{16} 5^{10})^{2apq} \left(\frac{3^6}{4^8}\right)^{aq}$
- vii.  $X\Pi(G) = (2^5 9^6 5^2)^{-pq} \left(\frac{8^5}{9^6}\right)^{-\frac{q}{2}}$
- viii.  $\chi\Pi(G) = (2^{16} 5^{10})^{-pq} \left(\frac{3^6}{4^8}\right)^{-\frac{q}{2}}$
- ix.  $ABC\Pi(G) = \left(\frac{7^3 \sqrt{6}}{10^5}\right)^{2pq} \left(\frac{\sqrt{2}^{21}}{7^3 \sqrt{3}}\right)^q$
- x.  $GA\Pi(G) = \left(\frac{4\sqrt{5}}{9}\right)^{12pq} \left(\frac{9\sqrt{3}}{16}\right)^{6q}$

*Proof.* Let  $G = C_3C_6(H)[p, q]$ . Edge set is partitioned into four types, based on degrees of end vertices of edges:

- $E_1 = E_{(3,5)} = \{uv \in E \mid d_G u = 3 \text{ and } d_G v = 5\},$
- $E_2 = E_{(4,4)} = \{uv \in E \mid d_G u = d_G v = 4\},$
- $E_3 = E_{(4,5)} = \{uv \in E \mid d_G u = 4 \text{ and } d_G v = 5\}$
- $E_4 = E_{(5,5)} = \{uv \in E \mid d_G u = d_G v = 5\}.$

The number of edges in  $E_1$ ;  $E_2$ ;  $E_3$  and  $E_4$  are  $6q$ ;  $q(2p - 1)$ ;  $6q(2p - 1)$  and  $4pq$ , respectively. Now  $\Pi_1^*$ ,  $\Pi_2$ ,  $H\Pi_1$ ,  $H\Pi_2$ ,  $MZ_1^a$ ,  $MZ_2^a$ ,  $X\Pi$ ,  $\chi\Pi$ ,  $ABC\Pi$  and  $GAPi$  of  $G$  is computed.

**i.** 
$$\begin{aligned} \Pi_1^*(G) &= \prod_{uv \in E} (d_G u + d_G v) \\ &= \prod_{uv \in E_1} (d_G u + d_G v) \times \prod_{uv \in E_2} (d_G u + d_G v) \times \prod_{uv \in E_3} (d_G u + d_G v) \\ &\quad \times \prod_{uv \in E_4} (d_G u + d_G v) \\ &= \prod_{uv \in E_1} (8) \times \prod_{uv \in E_2} (8) \times \prod_{uv \in E_3} (9) \times \prod_{uv \in E_4} (10) \\ &= 8^{6q} \times 8^{q(2p-1)} \times 9^{6q(2p-1)} \times 10^{4pq} \end{aligned}$$

$$\therefore \Pi_1^*(G) = (2^5 9^6 5^2)^{2pq} \left(\frac{8^5}{9^6}\right)^q$$

**ii.** 
$$\begin{aligned} \Pi_2(G) &= \prod_{uv \in E} (d_G u \times d_G v) \\ &= \prod_{uv \in E_1} (d_G u \times d_G v) \times \prod_{uv \in E_2} (d_G u \times d_G v) \times \prod_{uv \in E_3} (d_G u \times d_G v) \\ &\quad \times \prod_{uv \in E_4} (d_G u \times d_G v) \\ &= \prod_{uv \in E_1} (15) \times \prod_{uv \in E_2} (16) \times \prod_{uv \in E_3} (20) \times \prod_{uv \in E_4} (25) \\ &= 15^{6q} \times 16^{q(2p-1)} \times 20^{6q(2p-1)} \times 25^{4pq} \end{aligned}$$

$$\therefore \Pi_2(G) = (2^{16} 5^{10})^{2pq} \left(\frac{3^6}{4^8}\right)^q$$

**iii.** 
$$\begin{aligned} H\Pi_1(G) &= \prod_{uv \in E} (d_G u + d_G v)^2 \\ &= \prod_{uv \in E_1} (d_G u + d_G v)^2 \times \prod_{uv \in E_2} (d_G u + d_G v)^2 \times \prod_{uv \in E_3} (d_G u + d_G v)^2 \\ &\quad \times \prod_{uv \in E_4} (d_G u + d_G v)^2 \\ &= \prod_{uv \in E_1} (8)^2 \times \prod_{uv \in E_2} (8)^2 \times \prod_{uv \in E_3} (9)^2 \times \prod_{uv \in E_4} (10)^2 \\ &= 8^{2(6q)} \times 8^{2q(2p-1)} \times 9^{2(6q(2p-1))} \times 10^{2(4pq)} \end{aligned}$$

$$\therefore H\Pi_1(G) = (2^5 9^6 5^2)^{4pq} \left(\frac{8^5}{9^6}\right)^{2q}$$

**iv.** 
$$\begin{aligned} H\Pi_2(G) &= \prod_{uv \in E} (d_G u \times d_G v)^2 \\ &= \prod_{uv \in E_1} (d_G u \times d_G v)^2 \times \prod_{uv \in E_2} (d_G u \times d_G v)^2 \times \prod_{uv \in E_3} (d_G u \times d_G v)^2 \\ &\quad \times \prod_{uv \in E_4} (d_G u \times d_G v)^2 \\ &= \prod_{uv \in E_1} (15)^2 \times \prod_{uv \in E_2} (16)^2 \times \prod_{uv \in E_3} (20)^2 \times \prod_{uv \in E_4} (25)^2 \\ &= 15^{2(6q)} \times 16^{2q(2p-1)} \times 20^{2(6q(2p-1))} \times 25^{2(4pq)} \end{aligned}$$

$$\therefore H\Pi_2(G) = (2^{16} 5^{10})^{4pq} \left(\frac{3^6}{4^8}\right)^{2q}$$

**v.** 
$$\begin{aligned} MZ_1^a(G) &= \prod_{uv \in E} (d_G u + d_G v)^a \\ &= \prod_{uv \in E_1} (d_G u + d_G v)^a \times \prod_{uv \in E_2} (d_G u + d_G v)^a \times \prod_{uv \in E_3} (d_G u + d_G v)^a \\ &\quad \times \prod_{uv \in E_4} (d_G u + d_G v)^a \\ &= \prod_{uv \in E_1} (8)^a \times \prod_{uv \in E_2} (8)^a \times \prod_{uv \in E_3} (9)^a \times \prod_{uv \in E_4} (10)^a \\ &= 8^{a(6q)} \times 8^{aq(2p-1)} \times 9^{a(6q(2p-1))} \times 10^{a(4pq)} \end{aligned}$$

$$\therefore MZ_1^a(G) = (2^5 9^6 5^2)^{2apq} \left(\frac{8^5}{96}\right)^{aq}$$

**vi.** 
$$\begin{aligned} MZ_2^a(G) &= \prod_{uv \in E} (d_G u \times d_G v)^a \\ &= \prod_{uv \in E_1} (d_G u \times d_G v)^a \times \prod_{uv \in E_2} (d_G u \times d_G v)^a \times \prod_{uv \in E_3} (d_G u \times d_G v)^a \\ &\quad \times \prod_{uv \in E_4} (d_G u \times d_G v)^a \\ &= \prod_{uv \in E_1} (15)^a \times \prod_{uv \in E_2} (16)^a \times \prod_{uv \in E_3} (20)^a \times \prod_{uv \in E_4} (25)^a \\ &= 15^{a(6q)} \times 16^{aq(2p-1)} \times 20^{a(6q(2p-1))} \times 25^{a(4pq)} \end{aligned}$$

$$\therefore MZ_2^a(G) = (2^{16} 5^{10})^{2apq} \left(\frac{3^6}{48}\right)^{aq}$$

**vii.** 
$$\begin{aligned} X\Pi(G) &= \prod_{uv \in E} \frac{1}{\sqrt{d_G u + d_G v}} \\ &= \prod_{uv \in E_1} (d_G u + d_G v)^{-\frac{1}{2}} \times \prod_{uv \in E_2} (d_G u + d_G v)^{-\frac{1}{2}} \\ &\quad \times \prod_{uv \in E_3} (d_G u + d_G v)^{-\frac{1}{2}} \times \prod_{uv \in E_4} (d_G u + d_G v)^{-\frac{1}{2}} \\ &= \prod_{uv \in E_1} (8)^{-\frac{1}{2}} \times \prod_{uv \in E_2} (8)^{-\frac{1}{2}} \times \prod_{uv \in E_3} (9)^{-\frac{1}{2}} \times \prod_{uv \in E_4} (10)^{-\frac{1}{2}} \\ &= 8^{-\frac{1}{2}(6q)} \times 8^{-\frac{q}{2}(2p-1)} \times 9^{-\frac{1}{2}(6q(2p-1))} \times 10^{-\frac{1}{2}(4pq)} \end{aligned}$$

$$\therefore X\Pi(G) = (2^5 9^6 5^2)^{-pq} \left(\frac{8^5}{96}\right)^{-\frac{q}{2}}$$

**viii.** 
$$\begin{aligned} \chi\Pi(G) &= \prod_{uv \in E} \frac{1}{\sqrt{d_G u \times d_G v}} \\ &= \prod_{uv \in E_1} (d_G u \times d_G v)^{-\frac{1}{2}} \times \prod_{uv \in E_2} (d_G u \times d_G v)^{-\frac{1}{2}} \\ &\quad \times \prod_{uv \in E_3} (d_G u \times d_G v)^{-\frac{1}{2}} \times \prod_{uv \in E_4} (d_G u \times d_G v)^{-\frac{1}{2}} \\ &= \prod_{uv \in E_1} (15)^{-\frac{1}{2}} \times \prod_{uv \in E_2} (16)^{-\frac{1}{2}} \times \prod_{uv \in E_3} (20)^{-\frac{1}{2}} \times \prod_{uv \in E_4} (25)^{-\frac{1}{2}} \\ &= 15^{-\frac{1}{2}(6q)} \times 16^{-\frac{q}{2}(2p-1)} \times 20^{-\frac{1}{2}(6q(2p-1))} \times 25^{-\frac{1}{2}(4pq)} \end{aligned}$$

$$\therefore \chi\Pi(G) = (2^{16} 5^{10})^{-pq} \left(\frac{3^6}{48}\right)^{-\frac{q}{2}}$$

**ix.** 
$$\begin{aligned} ABC\Pi(G) &= \prod_{uv \in E} \sqrt{\frac{d_G u + d_G v - 2}{d_G u \times d_G v}} \\ &= \prod_{uv \in E_1} \sqrt{\frac{d_G u + d_G v - 2}{d_G u \times d_G v}} \times \prod_{uv \in E_2} \sqrt{\frac{d_G u + d_G v - 2}{d_G u \times d_G v}} \\ &\quad \times \prod_{uv \in E_3} \sqrt{\frac{d_G u + d_G v - 2}{d_G u \times d_G v}} \times \prod_{uv \in E_4} \sqrt{\frac{d_G u + d_G v - 2}{d_G u \times d_G v}} \\ &= \prod_{uv \in E_1} \sqrt{\frac{6}{15}} \times \prod_{uv \in E_2} \sqrt{\frac{6}{16}} \times \prod_{uv \in E_3} \sqrt{\frac{7}{20}} \times \prod_{uv \in E_4} \sqrt{\frac{8}{25}} \\ &= \left(\sqrt{\frac{6}{15}}\right)^{6q} \times \left(\sqrt{\frac{6}{16}}\right)^{q(2p-1)} \times \left(\sqrt{\frac{7}{20}}\right)^{6q(2p-1)} \times \left(\sqrt{\frac{8}{25}}\right)^{4pq} \end{aligned}$$

$$\therefore ABC\Pi(G) = \left(\frac{7^3\sqrt{6}}{10^5}\right)^{2pq} \left(\frac{\sqrt{2}^{21}}{7^3\sqrt{3}}\right)^q$$

**x.**

$$GA\Pi(G) = \prod_{uv \in E} \frac{2\sqrt{d_{Gu} \times d_{Gv}}}{d_{Gu} + d_{Gv}}$$

$$= \prod_{uv \in E_1} \frac{2\sqrt{d_{Gu} \times d_{Gv}}}{d_{Gu} + d_{Gv}} \times \prod_{uv \in E_2} \frac{2\sqrt{d_{Gu} \times d_{Gv}}}{d_{Gu} + d_{Gv}}$$

$$\times \prod_{uv \in E_3} \frac{2\sqrt{d_{Gu} \times d_{Gv}}}{d_{Gu} + d_{Gv}} \times \prod_{uv \in E_4} \frac{2\sqrt{d_{Gu} \times d_{Gv}}}{d_{Gu} + d_{Gv}}$$

$$GA\Pi(G) = \prod_{uv \in E_1} \frac{2\sqrt{15}}{8} \times \prod_{uv \in E_2} (1) \times \prod_{uv \in E_3} \frac{2\sqrt{20}}{9} \times \prod_{uv \in E_4} (1)$$

$$= \left(\frac{2\sqrt{15}}{8}\right)^{6q} \times (1)^{q(2p-1)} \times \left(\frac{2\sqrt{20}}{9}\right)^{6q(2p-1)} \times (1)^{4pq}$$

$$\therefore GA\Pi(G) = \left(\frac{4\sqrt{5}}{9}\right)^{12pq} \left(\frac{9\sqrt{3}}{16}\right)^{6q}$$

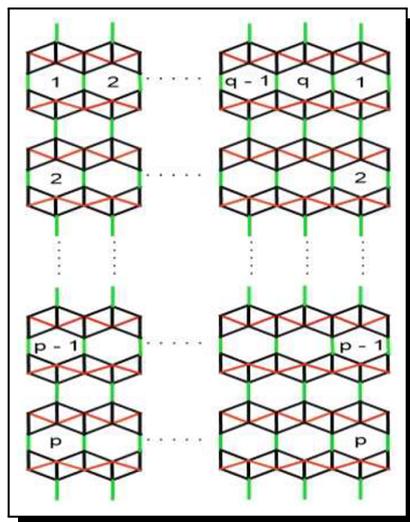
which is the required result. □

### 2.2 Tri-Hexagonal boron nanotorus — $THBC_3C_6[p, q]$

In this section, we calculate multiplicative connectivity indices of  $THBC_3C_6[p, q]$ , here  $p$  indicates the count of hexagons in a column and  $q$  indicates the count of hexagons in a row of the two-dimensional molecular graph of  $G = THBC_3C_6[p, q]$  nanotorus as given in Figure 3. Here  $|V| = 8pq$  and  $|E| = 18pq$ .

**Theorem 2.2.** For the graph  $G = THBC_3C_6[p, q]$ ,

- i.  $\Pi_1^*(G) = (2^5 9^6 5^2)^{2pq}$
- ii.  $\Pi_2(G) = (4^4 5^5)^{4pq}$
- iii.  $H\Pi_1(G) = (2^5 9^6 5^2)^{4pq}$
- iv.  $H\Pi_2(G) = (4^4 5^5)^{8pq}$
- v.  $MZ_1^a(G) = (2^5 9^6 5^2)^{2apq}$
- vi.  $MZ_2^a(G) = (4^4 5^5)^{4apq}$
- vii.  $X\Pi(G) = (2^5 9^6 5^2)^{-2pq}$
- viii.  $\chi\Pi(G) = (4^4 5^5)^{-4pq}$
- ix.  $ABC\Pi(G) = \left(\frac{7^3\sqrt{6}}{10^5}\right)^{2pq}$
- x.  $GA\Pi(G) = \left(\frac{4\sqrt{5}}{9}\right)^{12pq}$



**Figure 3.** A two-dimensional molecular graph of Tri-Hexagonal boron nanotorus —  $THBC_3C_6[p, q]$ .

*Proof.* Let  $G = THBC_3C_6[p, q]$ . Edge set is partitioned into three types based on degrees of end vertices of edges:

$$E_1 = E_{(4,4)} = \{uv \in E \mid d_{Gu} = d_{Gv} = 4\}, \quad E_2 = E_{(4,5)} = \{uv \in E \mid d_{Gu} = 4 \text{ and } d_{Gv} = 5\},$$

$$E_3 = E_{(5,5)} = \{uv \in E \mid d_{Gu} = d_{Gv} = 5\}.$$

The number of edges in  $E_1$ ;  $E_2$  and  $E_3$  are  $2pq$ ;  $12pq$  and  $4pq$ , respectively.

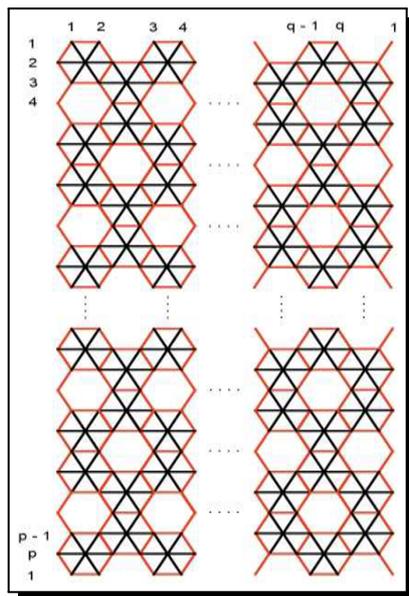
Similar to the above theorem we can obtain the required results.  $\square$

### 2.3 Tri-Hexagonal boron- $\alpha$ nanotorus — $THBAC_3C_6[p, q]$

In this section, we calculate multiplicative connectivity indices of  $THBAC_3C_6[p, q]$ , here  $p$  indicates the count of rows and  $q$  indicates the count of columns of the 2D molecular graph of  $G = THBAC_3C_6[p, q]$  nanotorus as given in Figure 4. Here  $|V| = 4pq/3$  and  $|E| = 7pq/2$ .

**Theorem 2.3.** For the graph  $G = THBAC_3C_6[p, q]$ ,

- i.  $\Pi_1^*(G) = (\sqrt{10}^3 11^2)^{pq}$
- ii.  $\Pi_2(G) = (5^5 6^2)^{pq}$
- iii.  $H\Pi_1(G) = (\sqrt{10}^3 11^2)^{2pq}$
- iv.  $H\Pi_2(G) = (5^5 6^2)^{2pq}$
- v.  $MZ_1^a(G) = (\sqrt{10}^3 11^2)^{apq}$
- vi.  $MZ_2^a(G) = (5^5 6^2)^{apq}$
- vii.  $X\Pi(G) = (\sqrt{10}^3 11^2)^{-pq}$
- viii.  $\chi\Pi(G) = (5^5 6^2)^{-pq}$
- ix.  $ABC\Pi(G) = \left[\left(\frac{\sqrt{2}}{5}\right)^{\frac{5}{2}} \times 3\right]^{pq}$
- x.  $GA\Pi(G) = \left(\frac{2\sqrt{30}}{11}\right)^{2pq}$



**Figure 4.** A two-dimensional molecular graph of Tri-Hexagonal boron- $\alpha$  nanotorus —  $THBAC_3C_6[p, q]$ .

*Proof.* Let  $G = THBAC_3C_6[p, q]$ . Edge set is partitioned into two types based on degrees of end vertices of edges:

$$E_1 = E_{(5,5)} = \{uv \in E \mid d_G u = d_G v = 5\}, \quad E_2 = E_{(5,6)} = \{uv \in E \mid d_G u = 5 \text{ and } d_G v = 6\}.$$

The number of edges in  $E_1$  and  $E_2$  are  $3pq/2$  and  $2pq$ , respectively.

Similar to the first theorem we can obtain the required results. □

### 3. Conclusion

Topological indices have their own significance in the study of chemical graph theory. In this study, we compute various degree based multiplicative topological indices of tri-hexagonal boron nanotube and tri-hexagonal boron nanotori.

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### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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