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Research Article

Central Positions in R&D Networks

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Abstract. In this paper, we determine the ideal characteristics of highly connected firms in terms of the market structure and the network connectivity. The outcomes of the paper indicate that the performance of such firms is a function of each of the competition toughness and the network density. These findings emphasize the role of central firms in developing the R&D network structure in many empirical papers.

Keywords. Network game; Equilibria; Outcomes maximization

MSC. 91A13; 91A20; 91A43

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1. Introduction

The network framework has appeared as an alternative approach to visualize the interaction of individuals in many scientific fields. For example, the Internet, the World Wide Web, social interactions of intermarriages between individuals, organizational systems, knowledge diffusion and business relations between firms. The contributions of the networks in those areas are based on applied social network analysis and statistical tools in order to understand the changes accompanying with developing the organizations. In addition to categorizing profitable structures that ensure high outcomes.

In this paper, we apply a network game by Goyal and Moraga-Gonzalez (2001) to study R&D cooperation between firms [6]. The model is constructed on three stages as follows. In the first stage, the cooperation network is formed through establishing bilateral links between cooperating firms. Firms with many links are defined as centers in the cooperation network where their partners in R&D or most of them (peripheral firms) are not linked to each other. In the second stage, firms choose the amount of investment in R&D in order to reduce the production cost. In terms of non-cooperating firms, R&D spillovers are applied between them to ensure partial benefits taken from their investments. In the third stage, firms compete in the market by setting the quantities of production in order to maximize their profits.

In the theoretical R&D literature, many papers have tended to focus on characteristics of overall R&D networks to explicitly present features of the beneficial networks. The contribution of this paper mainly concerns results that reflect the importance of firms' positions in the R&D network to improve the outcomes. The robustness of the results appears when the investigation involves different structures of the market that also represent the competition toughness.

The outcomes of this paper can be summarized as follows. Firstly, the central positions in the R&D network are always profitable. This reflects the positive relationship between the profits of companies and the number of new R&D agreements. Nevertheless, the competition toughness plays a role in determining the benefit rates. In general, we can describe this role as an inverse relationship between the competition toughness and the outcomes of the centers. Thus the outcomes are maximized in a weakly competitive market.

Secondly, developing the central firms in the network is affected by the competition rate. As the competition increases, the outcomes of the peripheral firms decrease which in turn may limit the growth of these firms. Thirdly, the outcomes of the central firms are affected by increasing the R&D spillover. The effect of the spillover is consistent with the competition toughness. In a sense that the negative impact of the spillover occurs in a competitive market.

Fourthly, the emergence of new central firms has a positive effect on the profit of existing central firms. In addition to this, cooperation of the central firms always improves their profits. These results indicate that a cooperation network that is dominated by interlinked centers is a profitable structure for these centers. This result may consist with the empirical outcomes that confirm the contribution of the central agents in developing the characteristic features of the organization (e.g., [1, 12, 14–16]). The main conclusion of those studies points out that the coherence of the network depends on those central agents. Others have investigated the role of the central agents in controlling knowledge flow between individuals [3, 4, 10, 13, 16]. They found that the central agents reduce the average distances between the other agents which in turn improve the knowledge flow.[1]

Finally, the growth of the cooperation between the peripheral firms has an impact on the equilibrium outcomes of the central firms. In a weakly competitive market, the central firms prefer a complex R&D organization between peripheral firms. However, in a competitive market, the central firms prefer isolated peripheral firms from each other.

The paper is structured as follows. In the second section, we review in the social network and microeconomics. In the third section, we present our outcomes. In the fourth section, we conclude our study.

[1] The reduction of the graph theoretic distances between agents allows the network to exhibit the properties of a small world network [17].

2. Background

2.1 Networks

A **network** is formed by a set of vertices (nodes) and a set of edges (links) connecting these vertices [11]. We define N as a set of all vertices labeled by letters or numbers and E as a set of all edges in the network. Let $\mathscr{G}^n$ be a set of all distinct networks generated from n firms. Then $G(N,E) \in \mathscr{G}^n$ denotes a network with nodes N and links E, and for simplicity the network is denoted by G. For the purpose of this article, we focus on undirected networks; meaning that each link between any two vertices runs in both directions (i.e., each two links ij and ji in G are the same). We also focus on simple networks that have neither parallel edges (edges that have the same end vertices) nor loops (edges where their start and end vertices are the same).

Nodes linked to node $i \in N$ is a set of **neighbors** of that node: $N_i = \{j \in N : ij \in E\}$. The length of the neighbors' set of node i is a **degree** of that node i.e., the degree of node $i \in N$ is $\deg(i) = |N_i|$ where $0 \le deg(i) \le n-1$. If $|N| = n$ is the number of nodes and $|E| = m$ is the number of links, the density of the network G is $D = 2m/n(n-1)$ where $0 \le D \le 1$. The **clustering coefficient** for each node $i \in N$ measures the proportion of neighbors of node i that link to each other:

$$C_i = \frac{2\,|\,jk : j,k \in N_i \text{ and } jk \in E\,|}{\deg(i)\big(\deg(i)-1\big)}, \tag{2.1}$$

where $0 \le C_i \le 1$ [9]. The average clustering coefficient for network G is the total of clustering coefficients for all nodes in the network divided by the number of nodes: $C = \frac{\sum_{i=1}^{n} C_i}{n}$.

A **walk** between any two nodes i and j is a finite sequence of links between them. A **path** is a walk such that each node appears once. Therefore, any two nodes are connected if there is a path between them; otherwise they are disconnected. The length of the path is the number of links constructed that path. The **distance** between any two nodes i and j is the length of the shortest path (d_{ij}). The average path length (average distance) in the network G is $AD = \sum_{i \neq j} d_{ij}/\big(n(n-1)\big)$.

A **star network** with n nodes (denoted by S_n) is a graph consists of a central node linked to other nodes in the network (called periphery) such that the latter nodes are not linked to each other. From this, the clustering coefficient of the center of a star network is zero, but with each new link formed between any two nodes in the periphery, the clustering coefficient of the center increases. Alternatively, the centralization of a node i in a star network can be measured by the probability $P(i) = 1 - C_i$. As the peripheral nodes form links between them, the central node starts lose its position in the network and verse versa. The **interlinked stars** is a graph consists of at least two star networks linked by their centers. Figure 1 displays an example of the star and the interlinked stars.

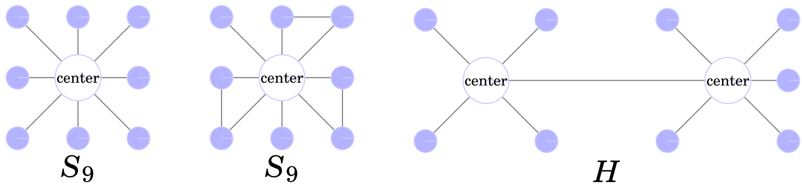

Figure 1. An example of the star and the interlinked stars. The first network (on the left) is a star network with nine firms, $P(\text{center}) = 1$. The second network (middle) is the same star network with links between some peripheral nodes, $P(\text{center}) = 25/28$. The third network (H) contains two stars interlinked by the centers

2.2 The Model

The emphasis in this paper is on the linear-quadratic function of consumers given by Hackner [8]:

$$U = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\left(\sum_{i=1}^{n} q_i^2 + 2\beta \sum_{j\neq i} q_i q_j\right) + I\,. \tag{2.2}$$

Here the demand parameters $\alpha > 0$ denotes the willingness of consumers to pay and q_i is the quantity consumed of product (good) i, while I measures the consumer's consumption of all other products. The parameter β such that $-1 \le \beta \le 1$ captures the marginal rate of differentiation between different products. If $\beta < 0$, the products are complements and if $\beta > 0$, the products are substitutes. For integer values, if $\beta = -1$, $\beta = 0$ or $\beta = 1$, the products are perfect complements, independent, homogeneous, respectively. The differentiation degree also measures the competition toughness. Meaning that if the degree $\beta < 0$, firms are in a weakly competitive market and with increasing the differentiation degree, the competition increases.

Payoffs. If the consumer buys q_i of good i where m is a consumer's income and p_i is the price of good i, the money spent is $p_i q_i$ and the balance is $I = m - p_i q_i$. By substituting into equation (2.2), we determine the optimal consumption of good i by calculating

$$\frac{\partial U}{\partial q_i} = 0 \Rightarrow \alpha - q_i - \beta \sum_{j\neq i} q_j - p_i = 0\,.$$

This implies the inverse demand function for each good i

$$D_i^{-1} = p_i = \alpha - q_i - \beta \sum_{j\neq i} q_j, \quad i = 1,\dots,n, \tag{2.3}$$

where p_i is the price to produce a unit of good i. We assume that each firm i produces good i and faces cost c_i of production $c_i q_i$. Thus the profit for firm i is

$$\pi_i = (p_i - c_i)q_i = \left(\alpha - q_i - \beta \sum_{j\neq i}^{n} q_j - c_i\right) q_i\,. \tag{2.4}$$

The total welfare is the consumer surplus plus the industry profit

$$TW = \underbrace{\frac{(1-\lambda)}{2} \sum_{i=1}^{n} q_i^2 + \frac{\lambda}{2}\left(\sum_{i=1}^{n} q_i\right)^2}_{\text{consumers surplus}} + \underbrace{\sum_{i=1}^{n} \pi_i}_{\text{industry profit}}\,. \tag{2.5}$$

R&D Network Model. The R&D network is a new technique to represent the R&D partnerships between firms. Firms in the network are represented by vertices or nodes and the partnerships between them are represented by links. We assume that the R&D agreement between any two firms requires the consent and full participation of both firms. This in turn means each link between any two firms serve both sides. Goyal and Moraga-Gonzalez (2001) set a network game for firms cooperate in R&D and compete in the industry [6]. The game is constructed on three stages as follows:

The first stage is a network formation where each firm freely choose their R&D partners. At the end of this stage, firms know their locations in the network G through the number of their cooperative links. Each network $G \in \mathcal{G}^n$ is captured by a symmetric $n \times n$ adjacency matrix $A = (a_{ij})$ where $a_{ij} \in \{0,1\}$. If $a_{ij} = 1$, firms i and j are linked (firms i and j cooperate in R&D), and $a_{ij} = 0$ otherwise.

The second stage is an R&D investment where firms choose the amounts of investment in R&D simultaneously and independently. At the end of this stage, we have the effective investment in R&D for each firm.

The effective amount of investment for each firm is sum of the own expenditure on R&D with a part of investments of other firms in the network [6]. The partial benefit of investments of others is determined by an external parameter ϕ called R&D spillover. The spillover captures the idea of knowledge flow between non-cooperating firms in R&D.

$$X_i = x_i + \sum_{j \in N_i} x_j + \phi \sum_{k \notin N_i} x_k, \quad i = 1, \ldots, n, \tag{2.6}$$

where N_i is the set of firms participating in R&D with firm i and $\phi \in [0,1)$ is an exogenous parameter that captures knowledge spillovers acquired from firms not cooperated in R&D with firm i. The effective investment for each firm i (2.6) varies with the network structure and the amount invested by all firms. Under the Goyal and Moraga-Gonzalez model, the cost of each good i is

$$c_i = \bar{c} - x_i - \sum_{j \in N_i} x_j - \phi \sum_{k \notin N_i} x_k . \tag{2.7}$$

In a star network with n firms, there are two types of firms: the first type is the central firm and the second type is the peripheral firms. Based on equation (2.6), the effective investment function of the central and peripheral firms are

$$X_c = x_c + \sum_{i \in N_c} x_i , \tag{2.8}$$

$$X_i = x_i + x_c + \beta \sum_{j \in N_c} x_j , \forall i \in N_c . \tag{2.9}$$

The third stage is a competition model where firms compete in the product market by setting quantities (Cournot competition). In this stage, firms choose their levels of production in order to maximize their profits.

The investment of firms is assumed to be costly and the function of the cost is quadratic, so that the cost of R&D is $C(x_i) = \mu x_i^2$, where $\mu > 0$ indicates the effectiveness of R&D expenditure [5].

From the cost of the R&D, the profit function (2.4) becomes

$$\pi_i = \Big(\alpha - q_i - \beta \sum_{j \neq i}^{n} q_j - c_i\Big) q_i - C(x_i), \tag{2.10}$$

where the marginal cost satisfies $a > \overline{c}$.

2.3 Nash Equilibria

We identify the sub-game perfect Nash equilibrium by using backwards induction. We assume that the marginal cost ($\overline{c}$) is constant and equal for all firms. From the profit function (2.10), we calculate $\frac{\partial \pi_i}{\partial q_i} = 0$ to find the best response function of quantity for each good i. By substituting the best response functions into each other, the symmetric equilibrium output for each good i is

$$q_i^* = \frac{\alpha - n c_i + \sum_{j \neq i} c_j}{n+1}. \tag{2.11}$$

To find the equilibrium profit, we substitute the equilibrium output (2.11) into the profit function which gives

$$\pi_i^* = \left[\frac{\alpha - n c_i + \sum_{j \neq i} c_j}{n+1}\right]^2 - C(x_i). \tag{2.12}$$

From the last two equations, the profit function can be expressed in the following form

$$\pi_i^* = q_i^{*2} - C(x_i). \tag{2.13}$$

The final result of the previous equilibrium outcomes depends on the star network structure. Thus, if the network is set, we can calculate the effective investment and the production cost for each firm (equations (2.8) and (2.9)). We substitute the result into the profit function (2.12) and calculate $\frac{\partial \pi_i^*}{\partial x_i} = 0$ to have the best response function of the R&D investment for each firm i. Then, by using backwards induction, we calculate the others equilibria.

3. The Results

In this section, we discuss a series of factors that have effects on the equilibrium outcomes of the central firms in the star networks. Our discussion throughout this section is based on experimental examples.[2]

3.1 Central Position versus Competition Toughness

In this section, we examine the impact of the competition on the R&D investment and the profit of the central firms. To do this, we assume six firms form a star network as shown in the Figure 2. Figure 3 shows the R&D investment and the profit of the central firm in the star network S_6 with respect to the competition degree.

As observed in Figure 3, there is an inverse relationship between the competition toughness and the equilibrium outcomes of the central firm. This indicates that the expenditure and the profit of the central firm is maximized when firms are in a weakly competitive market. The opposite occurs in a competitive market, where the central firm spends low in R&D and obtain

[2]Dealing with the endogenous R&D network model by Goyal and Moraga-Gonzalez (2001) is quite challenging to generate general equations to describe the equilibrium outcomes in the star networks [6].

a low profit. Also, this relationship is sensitive to the competition degree; meaning that as the degree decreases (increases), the outcomes increases (decreases).

Moreover, the figure also displays the effect of the R&D spillover on the investment and the profit of the central firm. The effect of the spillover depends on the competition toughness. In a weakly competitive market, the spillover is a positive factor contributing to improving the investment and the profit. However, with increasing the competition toughness, the spillover becomes a negative factor plays a role in declining the outcomes.

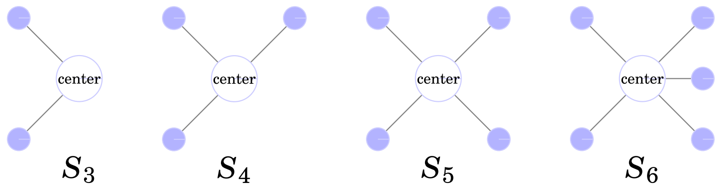

Figure 2. Star networks of sizes from three to six

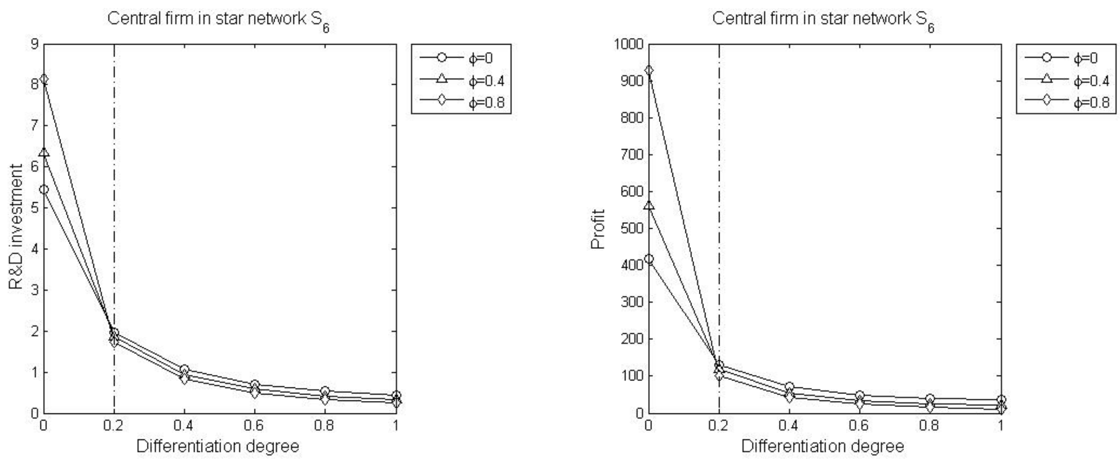

Figure 3. The R&D investment and the profit of the central firm in the star network S_6. The parameters used to plot the results are $a = 120$, $\overline{c} = 100$ and $\mu = 2$

3.2 The Network Density versus Competition Toughness

In this section, we combine impact of the density of the star network and the competition toughness on the equilibrium outcomes of the central firm. For the density, we assume that the number of firms in the periphery increases. We start with the smallest star network size that contains three firms (S_3), including the center firm. Then, we increase the size of the periphery by adding one firm each time to have the star networks S_4, S_5 and S_6 as given in Figure 2.

To examine the impact of the density, we let the size of the peripheral firms increases. For example, we start with the smallest star network that only contains on three firms (S_3), including the center firm. Then, we increase the size of the periphery by adding firms and this forms the star networks S_4, S_5 and S_6 as given in Figure 2. By increasing the periphery, the density and the equilibrium outcomes of the star network change. Table 1 displays the density of the star networks and Figure 4 shows the R&D investment and the profit of the central firms in those networks.

Table 1. The density of the star networks given in Figure 2

Network	S_3	S_4	S_5	S_6
D	2/3	1/2	2/5	1/3

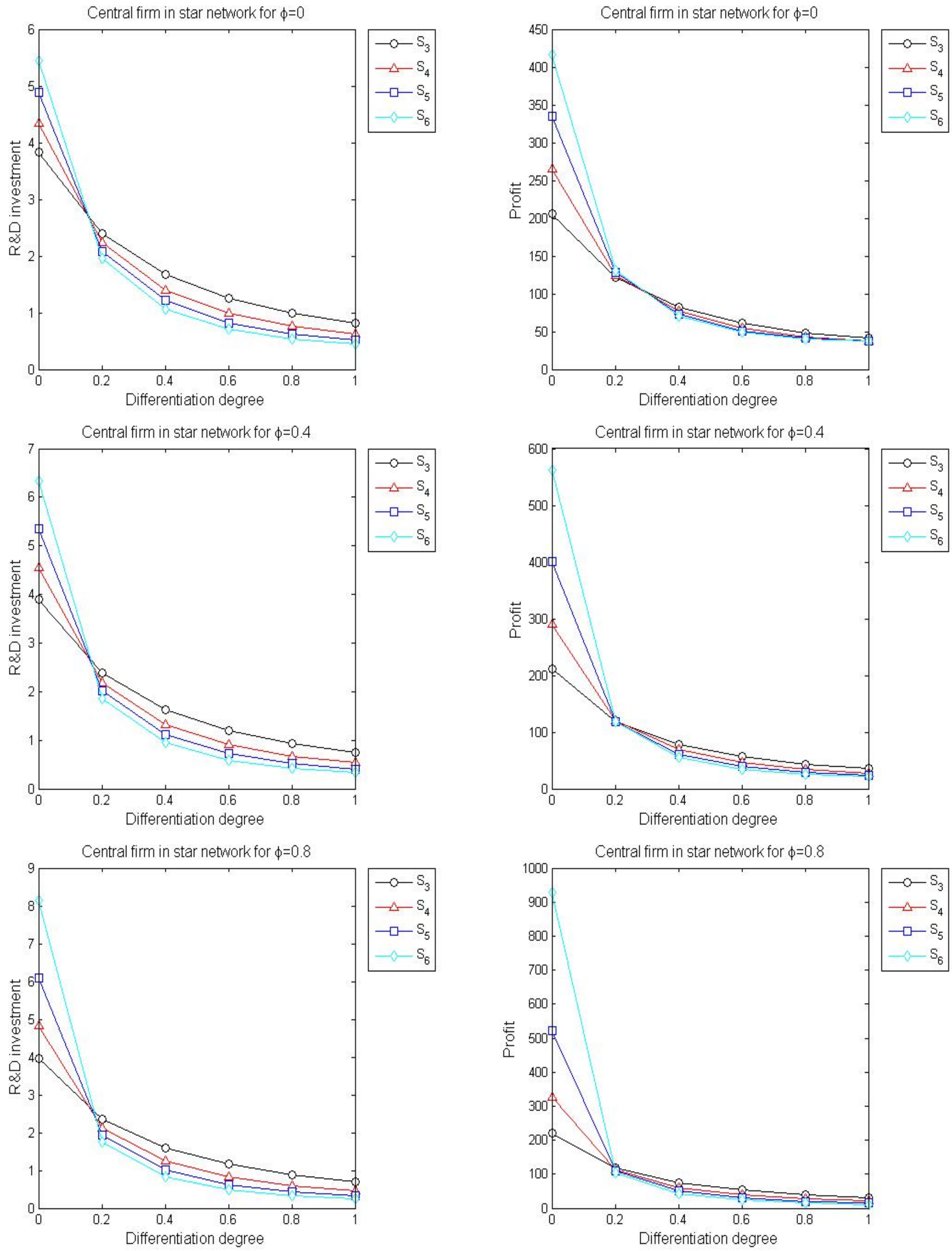

Figure 4. The R&D investment and the profit of the central firm in the star networks given in Figure 2. The parameters used to plot the results are $a = 120$, $\overline{c} = 100$ and $\mu = 2$

It can be observed that the effect of the density of the star network depends on the toughness of the competition. In a weakly competitive market, the increase of the periphery size always improves the investment and profit of the central firm. In a competitive market, the opposite is observed. This result indicates that with increasing the competition, the incentive of the central firm to invest in R&D decreases with increasing the periphery size. Also, the central firm in a such market prefers a small periphery to have a high profit.

3.3 Increase of Central Positions

Under different competition degrees, we examine the impact of the emergence of centers on the outcomes of existing one. To do this examination, we compare the outcomes of the central firm in three different networks given in Figure 5. In the first network G_1, we assume that cooperation of firms in R&D forms one central firm. In the second network G_2, we assume that the cooperation forms two disconnected star networks, each of size three firms. In the third network G_3, we assume that the cooperation generates interlinked star networks. Figure 6 displays the R&D expenditure and the profit of the central firm in the three networks.

The equilibrium investment and profit of the central firm in the three networks can be summarized as follows. Firstly, the R&D expenditure of the central firm is managed by the competition toughness. If the competition degree is not large, the investment in R&D by the center increases with growing the central firms and the cooperation between them. The opposite is realized if the competition degree rises. Also, it is worth to mention that the R&D spillover has a role in determining the difference in the outcomes of the central firm in the networks G_1 and G_2. As the spillover increases, the central firm in both networks produces almost similar outcomes. Secondly, the emergence of central firms is profitable for existing ones, regardless of the competition degree. Moreover, the cooperation of the central firms always improves the profit. This result may consist with the empirical findings. Many authors have concluded that the central firms have a role in developing the network (e.g., [1, 12, 14–16]).

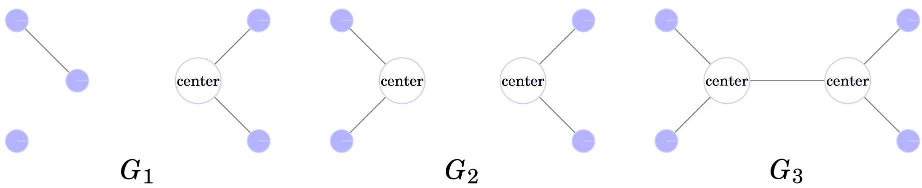

Figure 5. The central firm in three different networks. The network G_1 has one central firm. The network G_2 has two disconnected star networks, each of size three firms. The network G_3 has interlinked star networks.

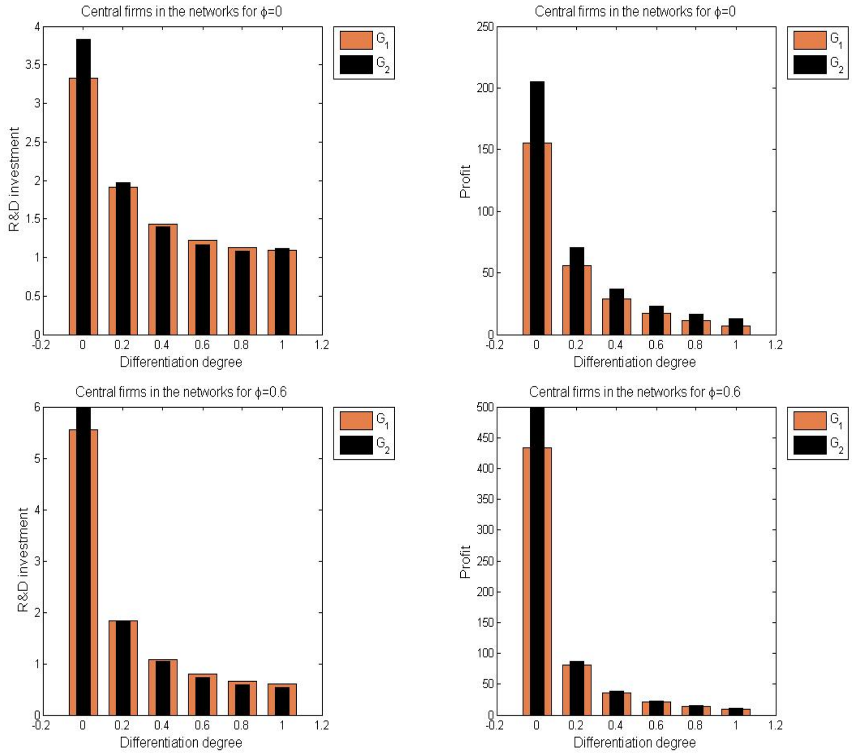

Figure 6. The R&D investment and the profit of the central firm in the networks G_1 and G_2. The parameters used to plot the results are $a = 120$, $\overline{c} = 100$ and $\mu = 2$

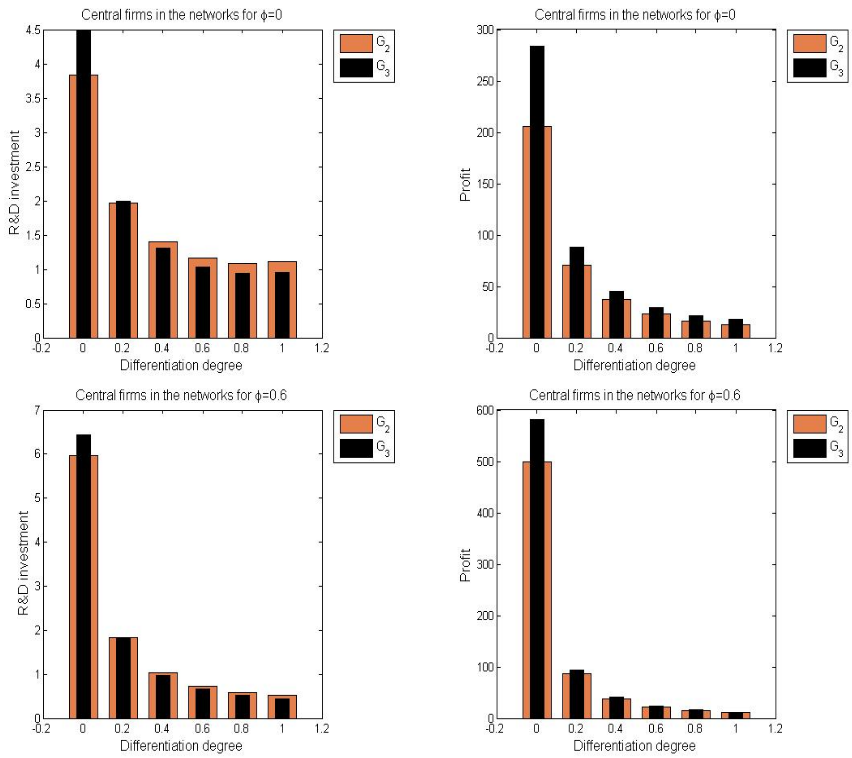

Figure 7. The R&D investment and the profit of the central firm in the networks G_2 and G_3. The parameters used to plot the results are $a = 120, \overline{c} = 100$ and $\mu = 2$

3.4 Growing the Cooperation in the Periphery

In this section, we are interested in understanding how the outcomes of the central firm are influenced by the behavior of the periphery. To cover this interest, we consider the star network S_5 given in Figure 2 and assume that the cooperation between the peripheral firms generate the networks H_1, H_2 and H_3 given in Figure 8. In terms of the central firms, the main difference is the clustering coefficient (measures the rate of the linkages between neighbors). By increasing the links between the peripheral firms, the clustering coefficient of the center increases as in Table 2.

Table 2. The clustering coefficient of the central firm in the networks given in Figure 8

Network	S_5	H_1	H_2	H_3
C_c	0	1/6	1/3	2/3

Figure 9 shows the investment and the profit of the central firm in the star network S_5 and in the generated networks given in Figure 8. It can be observed that the investment of the central firm in R&D decreases as the clustering coefficient increases. This indicates that the cooperation of the peripheral firms always reduces the central firm's expenditure on R&D. In terms of the profit of the center, the impact depends on the market structure. If the competition is not tough, the profit of the center increases with growing the cooperation of the peripheral firms. The opposite occurs with increasing the toughness of the market. This means that with growing the competition between firms, the central firm does not prefer cooperation between the peripheral firms.

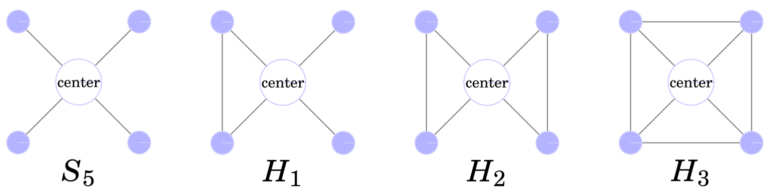

Figure 8. The central firm with growing the cooperation of the peripheral firms. The networks H_1, H_2 and H_3 are generated by adding links between the peripheral firms in the star network S_5

3.5 Termination of the Cooperation

Many empirical studies have shown the role of the highly connected players in constructing the network. One approach to verify this role is by removing those players from the network and examine the consequences (e.g., [2, 7]). They found that removing highly connected players causes significant changes in the statistical features of the network that may modify the identity of the structure.

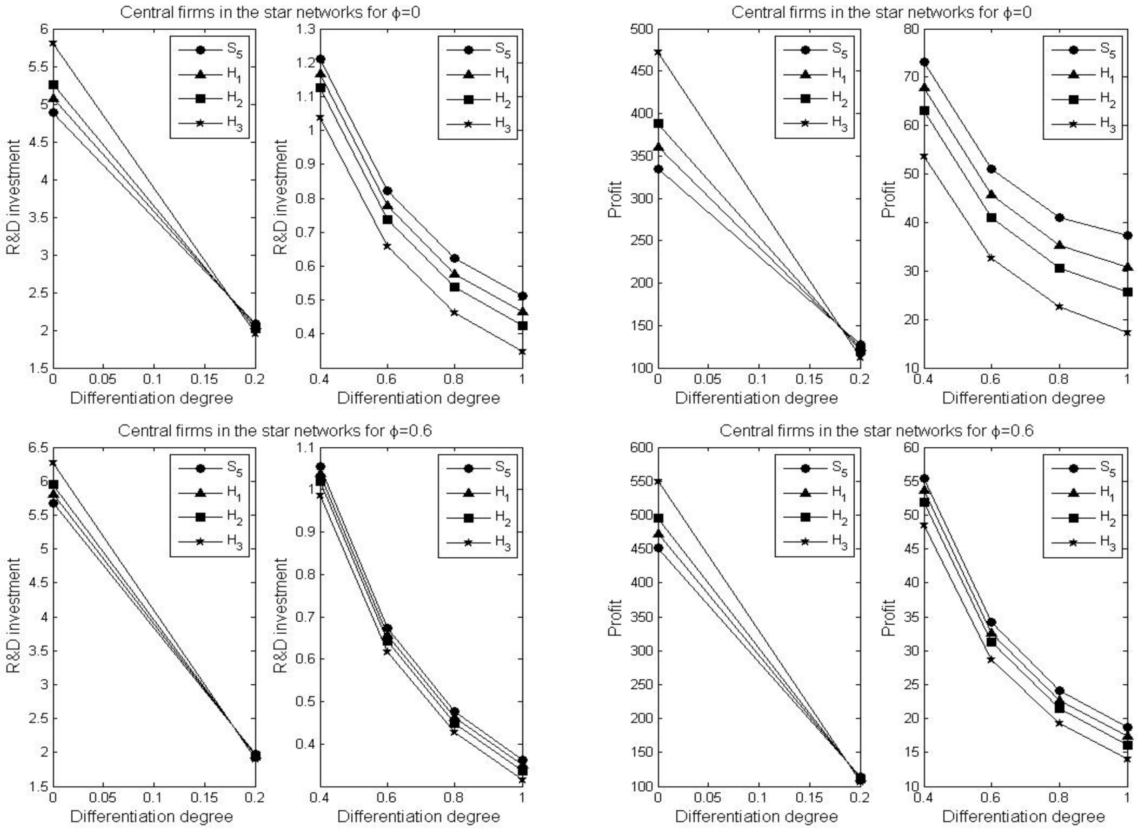

Figure 9. The R&D investment and the profit of the central firm in the networks S_5, H_1, H_2 and H_3. The parameters used to plot the results are $a = 120$, $\bar{c} = 100$ and $\mu = 2$

This approach is used in this section, but without removing the central firm. Meaning that we delete the cooperative links of the central firm in a star network and compare its outcomes in the two different networks. Figure 10 displays the star network S_5 and the resulting network after removing the links of the central firm, the empty network E_5 (a graph without links between firms).

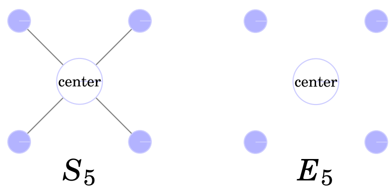

Figure 10. The star network S_5 and the empty network E_5. In the network E_5, there is one type of firms

Figure 11 compares the R&D investment and the profit of the central firm in the networks S_5 and E_5. It can be observed that the incentive of the central firm to invest in R&D is acquired in the star network if the competition is not tough. However, with increasing the competition between firms, the investment of the central firm is maximized in a low level of R&D cooperation. In terms of the profit, the star network is a preferable structure to have high profits, regardless of the competition toughness.

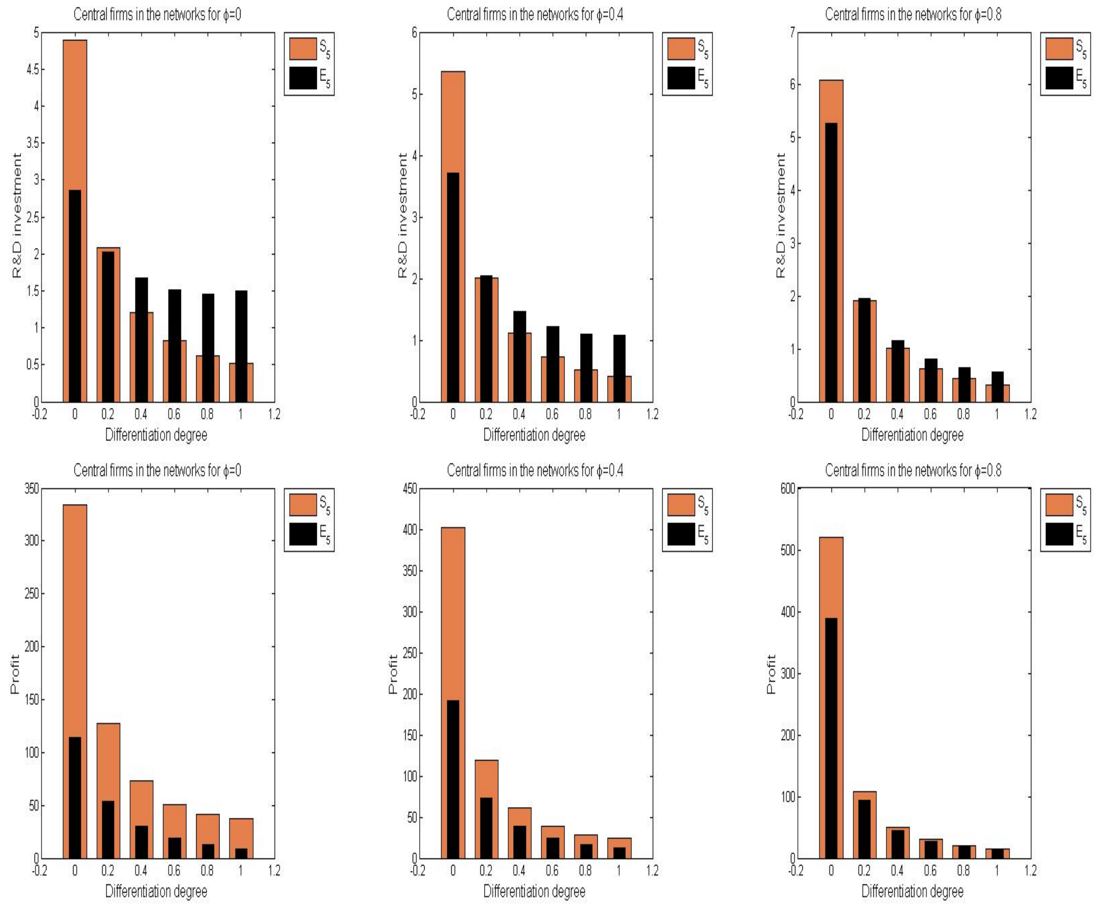

Figure 11. The investment and the profit of the central firms in the networks S_5 and E_5. The parameters used to plot the results are $a = 120, \bar{c} = 100$ and $\mu = 2$.

4. Conclusion

This paper discussed a series of factors that affect the results of the central firms in the R&D networks. The results suggest that the competition toughness plays an important role in determining the strength and the positivity of these factors. Firstly, the outcomes of the central firm vary with the growth of the cooperation in the network. In a weakly competitive market, the R&D agreements always enhance the equilibrium investments and the profits of the central firm. In a competitive market, the benefit behind establishing and developing the central firm is limited to this firm. This result indicates that the central firm always seeks to dominate the R&D network to have high profits.

Secondly, the outcomes of the central firm are affected by the R&D spillover and the cooperation of the peripheral firms. In a weakly competitive market, the impacts of the two factors are positive, in a sense that the investment and the profit of the central firm increase with growing the spillover and the cooperation of firms in the periphery. This indicates that the profit of the central firm is maximized if firms are in a weakly competitive market and the R&D spillover and the cooperation level between the peripherals are high.

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