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Research Article

Rheodynamic Lubrication of an Externally Pressurized Converging Circular Thrust Bearing using Bingham Lubricant

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Abstract. In this research paper we have theoretically analyzed the effects of the angle of convergence and the non-Newtonian characteristics on the performance of an externally pressurized converging circular thrust bearing using Bingham fluid as the lubricant. Since Bingham lubricants are known to have a characterized yield value, there will be a formation of unyielding core in the region between the plates of the bearing. The solutions are obtained for the pressure and load carrying capacity of the bearing for various values of Bingham number and the angle of convergence. The effects of the Non-Newtonian characteristics of the lubricant and the angle of convergence on the performance of the bearing are discussed.

Keywords. Bingham lubricant; Varying film thickness; Externally pressurized bearing; Yield surface; Converging circular thrust bearing

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1. Introduction

By principle, in externally pressurized thrust bearings, the bearing surfaces are separated by a fluid film which is created and maintained by external means. As there are certain operational advantages such as low friction, high load carrying capacity, high stiffness etc., these bearings are widely used in mechanical industries. Further, lubrication of modern machines has been a challenge and emerging interest for the tribologists. The pioneering researchers have laid out a foundation of hydrodynamic lubrication. However, modern researchers intend to use non-Newtonian fluids as lubricants. Also, the interest has been increasing to use non-Newtonian lubricants characterized by an yield value. Using externally pressurized thrust bearing Khalil *et al*. [8] analyzed theoretically the combined effects of centrifugal and convective inertia forces on the performance of externally pressurized circular bearing by assuming turbulent flow conditions. Lubrication of externally pressurized circular plates and conical bearings, was studied by Dinesh Kumar *et al*. [9] using ferrofluids as lubricants under the presence of transversely applied magnetic field. Elsharkawy *et al*. [5] investigated the effect of porous layer on the hydrodynamic lubrication performance of an externally pressurized circular porous bearing. The static and dynamic characteristics of externally pressurized circular step thrust bearing was studied by Jaw Ren Lin [10] using couple stress fluid as the lubricant. The bearing performance of an externally pressurized thrust bearing is analyzed by kandasamy [7] using non-Newtonian lubricants.

The modern aspect of tribology immensely focuses on the application of the lubricants with variable viscosity namely non-Newtonian lubricants. Hence researchers have been keen on using Non-Newtonian fluids with yield stress as lubricants such as Bingham, Casson, Herschel-Bulkely fluids. A study on Bingham fluid filling of a 2-D cavity was presented by Alexandroum *et al*. [4]. Shi-Pu Yang *et al*. [13] discussed the effects of Bingham fluids in squeeze flow between parallel discs. A fractal model for the initial pressure gradient for Bingham Fluid was investigated by Meijuan Yun *et al*. [14] in a porous medium. By the means of 3-D computational fluid dynamics Gertzos et al. [6] analysed the performance characteristics of a hydrodynamic journal bearing lubricated with Bingham fluid. Jayakaran Amalraj *et al*. [2] studied the effect of Bingham fluid in an externally pressurized Bingham thrust bearing.

A very few researchers studied the performance of converging or diverging bearing. Roy *et al*. [11] have discussed the effect of inertia forces in an externally pressurized bearing with converging, uniform and diverging film thickness using visco-elastic lubricant. Vishwanath *et al*. [12] have analyzed the problem of a squeeze film bearing with converging and diverging squeeze films. Further, Jayakaran Amalraj *et al*. [1] worked on the effects of the angle of convergence on the thickness of the core in a converging externally pressurized thrust bearing theoretically. Although a lot of study has been done on externally pressurized thrust bearings with uniform film thickness, only little work has been done with variable film thickness.

In this research work we have studied the effects of non-Newtonian characteristics and the angle of convergence on the performance of Externally Pressurized Converging Thrust Bearing. Numerical solutions have been obtained for the bearing performances such as pressure distributions and load carrying capacity for different Bingham number and angle of convergence.

2. Mathematical Formulation of the Problem

The geometry of the bearing is as shown in Figure 1.

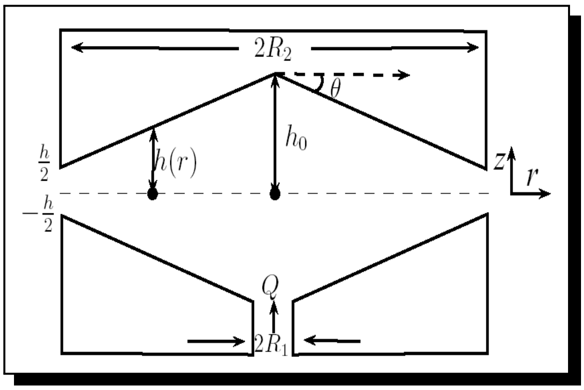

Figure 1. Geometry of an externally pressurized convergent thrust bearing

We consider an isothermal incompressible steady flow of the time independent Bingham fluid between two converging circular plates, separated by a varying distance 'h'. Let R_1 and R_2 be the radius of the film inlet and outlet, respectively, p the pressure of the film, p_a the atmospheric pressure and ρ the density of the fluid. The origin is fixed at the center of the plate and the cylindrical polar co-ordinates with axial symmetry is considered. Let r and z be the distance along the radial and axial direction, respectively. Let v_r and v_z be velocity components along r and z directions, respectively. The geometry of the problem is shown in Figure 1.

The constitutive equation of a Bingham fluid is given by

$$\tau_{ij} = 2\left[\eta_1 + \frac{\eta_2}{I^{1/2}}\right] e_{ij}, \quad \frac{1}{2}\tau_{ij}\tau_{ij} \geq \eta_2^2, \tag{1}$$

where τ_{ij} are the deviatoric stress components, η_1 and η_2 are constants namely the plastic viscosity and yield value respectively, e_{ij} represents the rate of deformation components and $I = 2e_{ij}e_{ij}$ is strain invariant. However, for all practical purposes we consider the one dimensional form of eq. (1) which is given in eq. (5). There will be a region called "core region" where shear stress is less than the yield stress that moves with the constant velocity v_c. Let the boundaries of the core be $z = \frac{-\delta(r)h}{2}$ and $z = \frac{\delta(r)h}{2}$ as shown in Figure 2.

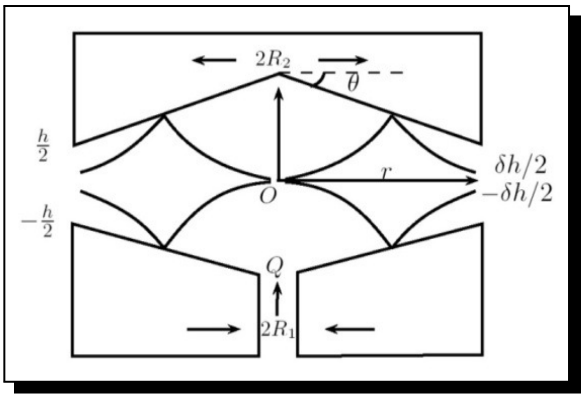

Figure 2. Shape of the core in an externally pressurized converging thrust bearing

Applying the basic assumptions of lubrication theory for thin films and ignoring inertia effects and the body forces, the equations of continuity and momentum for the externally pressurized circular thrust bearing will be as follows.

Equation of continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0. \tag{2}$$

Equations of momentum:

$$-\frac{\partial p}{\partial r} = \frac{\partial \tau_{rZ}}{\partial z}, \tag{3}$$

$$\frac{\partial p}{\partial z} = 0. \tag{4}$$

The constitutive equation given in (1) becomes

$$\tau_{rz} = \eta_2 \pm \eta_1 \left(\frac{\partial v_r}{\partial z}\right). \tag{5}$$

The boundary conditions are:

$$v_r = 0 \ \text{ at } \ z = \pm\frac{h}{2}, \tag{6}$$

$$v_r = v_c \ \text{ at } \ z = \pm\frac{\delta h}{2}, \tag{7}$$

$$p = p_a \ \text{ at } \ r = R_2, \tag{8}$$

$$\frac{\partial v_r}{\partial z} \ \text{ is continuous, at } \tau = \eta_2, \tag{9}$$

where p_a is atmospheric pressure.

3. Solution

The velocity distributions for the flow region and core region can be obtained by solving governing equations (2)-(5) and using the boundary conditions (6) and (7).

It is given by

$$v_r = \frac{1}{2}\left(\frac{1}{\eta_1}\right)\left(\frac{dp}{dr}\right)\left[\left(\frac{h}{2} - \frac{\delta h}{2}\right)^2 - \left(z - \frac{\delta h}{2}\right)^2\right], \quad \text{where } \frac{\delta h}{2} \le z \le \frac{h}{2} \tag{10}$$

and the velocity of the core region,

$$v_c = \frac{1}{2}\left(\frac{1}{\eta_1}\right)\left(\frac{dp}{dr}\right)\left(\frac{h}{2}\right)^2 (1-\delta)^2, \quad 0 \le z \le \frac{\delta h}{2}. \tag{11}$$

The conservation of mass for externally pressurized bearing in integral form is

$$Q = 4\pi r \int_0^{h/2} v_r dz, \tag{12}$$

where Q is the flow rate per unit width.

Using velocity distribution v_r and v_c in (12), we get

$$Q = \frac{\pi r h^3 \left(\frac{dp}{dr}\right)}{12\eta_1}(1-\delta)^2(2+\delta). \tag{13}$$

Considering the equilibrium of an element in the yield surface $-\frac{\delta h}{2} \le z \le \frac{\delta h}{2}$, it is found that

$$\frac{dp}{dr} = \frac{2\eta_2}{\delta(r)h}. \tag{14}$$

Hence Eq. (13) becomes

$$\left(\frac{dp}{dr}\right) = \frac{12\eta_1 Q}{\pi r h^3 (1-\delta)^2 (2+\delta)}. \tag{15}$$

Equating (14) and (15), we get

$$\frac{6\eta_1 Q}{\pi r h^2 \tau_0} = \frac{(1-\delta)^2(2+\delta)}{\delta}. \tag{16}$$

The variation in film thickness of the lubricant in the converging bearing can be defined as

$$h(r) = h_0 - h_0\left(\frac{r}{R_2}\right)\tan\theta, \tag{17}$$

where $h(r)$ represents the varying film thickness between the plates, h_0 is the maximum film thickness at the center of the bearing and θ is the angle of convergence. The following non-dimensional parameters are introduced.

$$r* = \frac{r}{R_2}\ ; \quad \delta* = \delta(r*)\ ; \quad p* = \frac{p}{\left(\frac{Q\eta_1}{\pi h_0^3}\right)}\ ; \quad h^* = \frac{h}{h_0}\ ; \quad z^* = \frac{z}{h};\ B = \frac{\pi R_2 h_0^2 \eta_2}{2Q\eta_1},$$

where B is the Bingham number.

The non-dimensional form of velocity profiles, pressure gradient and core thickness will be

$$v_r^* = \frac{12}{r^*(1-r^*\tan\theta)(1-\delta^*)^2(2+\delta^*)}\left\{\frac{(1-\delta^*)^2}{8} - \frac{\left(z^* - \frac{\delta^*}{2}\right)^2}{2}\right\}, \quad \frac{\delta^* h^*}{2} \le z^* \le \frac{h^*}{2}, \tag{18}$$

$$v_c^* = \frac{3}{2}\left[\frac{1}{r^*(1-r^*\tan\theta)(2+\delta^*)}\right], \quad 0 \le z^* \le \frac{\delta^* h^*}{2}, \tag{19}$$

$$\frac{dp^*}{dr^*} = \frac{4B(1-r^*\tan\theta)^2}{\delta^*}, \tag{20}$$

$$\frac{3}{Br^*(1-r^*\tan\theta)} = \frac{(1-\delta^*)^2(2+\delta^*)}{\delta^*}. \tag{21}$$

The core thickness $\delta(r^*)$ is obtained by solving eq. (21) for different values of θ and B by using Newton Raphson Method and reported elsewhere [1].

The pressure distribution can be obtained by integrating (20) and using boundary condition (8) and it is given by

$$P^* - P_a^* = \int_{r^*}^{1}\left(\frac{dp^*}{dr^*}\right)dr^*. \tag{22}$$

The load carrying capacity W for the externally pressurized thrust bearing can be obtained from

$$W = \int_{R^*}^{1}\left(P^* - P_a^*\right) r^* dr^*, \tag{23}$$

where $R^* = \frac{R_1}{R_2}$ is the ratio of inside to outside radius of the bearing. The integration is performed numerically for various values of the Bingham numbers and angle of convergence and vlaues are tabulated.

Table 1. Load Capacity for different Bingham value and the angle of convergence

	$\theta=0$	$\theta=5$	$\theta=10$	$\theta=15$	$\theta=20$	$\theta=25$	$\theta=30$	$\theta=35$	$\theta=40$
$B=1$	2.933	3.294	3.769	4.428	5.409	7.021	10.094	17.665	50.268
$B=2$	3.823	4.254	4.812	5.573	6.683	8.468	11.793	19.770	53.213
$B=3$	4.678	5.177	5.817	6.678	7.918	9.877	13.455	21.846	56.139
$B=4$	5.508	6.075	6.794	7.754	9.120	11.253	15.084	23.892	59.043
$B=5$	6.322	6.954	7.751	8.807	10.299	12.602	16.685	25.911	61.926
$B=6$	7.123	7.818	8.692	9.843	11.457	13.929	18.262	27.905	64.788

4. Result and Discussion

The pressure distribution in the region of the fluid film and the load carrying capacity of the bearing for various values of Bingham numbers and the angle of convergence have been determined and are shown in Figure 3, 4 and 5. It is evident from the pressure profiles that the pressure decreases gradually from the centre to the periphery of the bearing. It has been observed that the pressure is found to increase as the Bingham number increases for a given angle of convergence. For a particular Bingham number, a similar trend is observed for the pressure with an increase in the angle of convergence.

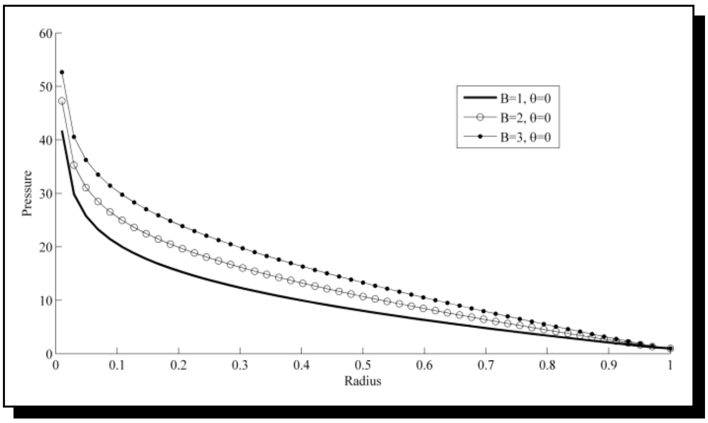

Figure 3. Pressure distribution for different angales of convergence

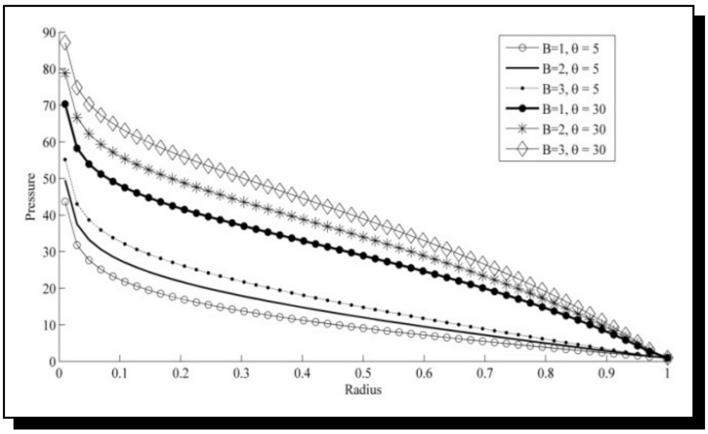

Figure 4. Pressure distribution along radius for zero angle of convergence

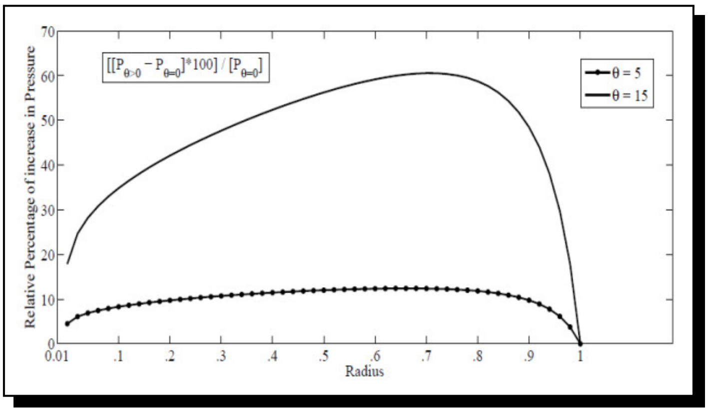

Figure 5. Relative percentage of pressure difference for $B = 1$

By keeping $\theta = 0$ as a reference angle, the relative percentage of increase in pressure distribution for $\theta > 0$ have been computed and depicted in Figure 5. It is evident that the increase in relative percentage of pressure is quite significant with respect to θ.

The numerically computed values of load carrying capacity of the bearing for various values of Bingham number and the angle of convergence are tabulated in the Table 1. From the numerical solutions obtained we infer that the load carrying capacity increases as the Bingham number increases corresponding to any particular angle of convergence. Also the increase in the angle of convergence leads to an increase in the load carrying capacity of the bearing for a particular Bingham number. We further observe that if the angle of convergence is marginal the increase in the load carrying capacity is also marginal whereas if the angle of convergence is appreciable then relatively the performance of the bearing is also appreciable

It is known that in general the non-Newtoninan characteristics enhances the performance of the bearing and it can be concluded that the similar trend continuous for converging bearing also. The load carrying capacity for different value of Bingham numbers corresponding to $\theta = 0$ are in good agreement with Jayakaran Amalraj *et al*. [2] and depicted in Figure 6.

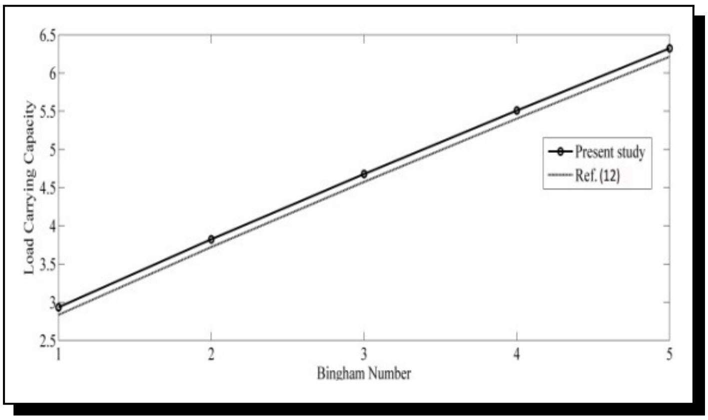

Figure 6. Comparative study on Load capacity

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

The authors wrote, read and approved the final manuscript.

References

[1] I.J. Amalraj and G.A. Raymand, Core variation in an externally pressurized converging thrust bearing with Bingham lubricant, *Applied Mechanics and Materials* **852** (2016), 428 – 434.

[2] I.J. Amalraj, S. Narasimman and A. Kandasamy, Inertia effects in rheodynamic lubrication of an externally pressurized thrust bearing using bingham fluids, *8th Asian Computational Fluid Dymanics Conference*, Hong Kong SAR, China (2010).

[3] I.J. Amalraj, S. Narasimman and A. Kandasamy, Rheodynamic Lubrication of an Externally pressurized Thrust Bearing using Herschel-Bulkley fluid with sinusoidal Injection, *Journal of Applied Fluid Mechanics* **5** (4) (2012), 71 – 79.

[4] A.N. Alexandrou, E. Ducand V. Entov, Inertial, viscous and yield stress effects in Bingham fluid filling of a 2-d cavity, *Journal of Non-Newtonian Fluid Mechanics* **96** (2001), 383 – 403.

[5] A.A. Elsharkawy, S.Z. Kassab and M.M. Nassar, Lubrication analysis of externally pressurized circular porous bearing, *Appl. Math. Modelling* **20** (1996), 870 – 876.

[6] K.P. Gertzos, P.G. Nikolakopoulos and C.A. Papadopoulos, CFD analysis of journal bearing hydrodynamic lubrication by Bingham lubricant, *Tribology International* **41** (2008), 1190 – 1204.

[7] A. Kandasamy, Lubrication of an externally pressurized thrust bearing with non-Newtonian lubricants, in *Proceedings of International Conference on Advances in Mechanical Engineering*, Bangalore, India, 1175 – 1183 (1995).

[8] M.F. Khalil, S.Z. Kassab and A.S. Ismail, Influence of inertia forces on the performance of turbulent externally pressurized bearing, *Tribology International, Butterworth-Heinemann Ltd.* **25** (1) (1992), 17 – 25.

[9] D. Kumar, P. Chandra and P. Sinha, Ferrofluid lubrication of externally pressurized circular plates and conical bearings, *Int. J. Engng Sci.* **31** (4) (1993), 593 – 604.

[10] J.-R. Lin, Static and Dynamic characteristic of externally pressurized circular step thrust bearing lubricated with couple stress fluids, *Tribology International* **32** (1999), 207 – 216.

[11] J.S. Roy, S. Padhy and L.K. Bhopa, Inertia effect in externally pressurized thrust bearing with converging and diverging film using visco-elastic lubricant, *Acta Mechanica* **96** (1993), 1 – 12.

[12] K.P. Vishwanath and A. Kandasamy, Rheodynamic lubrication of a rectangular squeeze film bearing with an exponential curvature variation using Bingham lubricants, *International Journal of Mathematical, Computational, Physics, Electrical and Computer Engineering* **2** (7) (2008), 380 – 384.

[13] S.-P. Yang and K.-Q. Zhu, Analytical solutions for squeeze flow of Bingham fluid with Navier slip condition, *J. Non-Newtonian Fluid Mech.* **138** (2006), 173 – 180.

[14] M. Yun, B. Yu and J. Cai, A fractal model for the starting pressure gradient for Bingham fluids in porous media, *International Journal of Heat and mass Transfer* **51** (1402) (2008), 408.