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Research Article

Analysis of Heat Transfer of Cu-Water Nanofluid Flow Past a Moving Wedge

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Abstract. In this paper, heat transfer of a steady, two-dimensional, incompressible Cu-water nanofluid flow over a moving wedge in the presence of thermal radiation effect are investigated. Gyarmati's variational principle developed on the thermodynamic theory of irreversible processes is employed to solve the problem numerically. The governing boundary layer equations are approximated as simple polynomial functions, and the functional of the variational principle is constructed. The Euler-Langrange equations are reduced to simple polynomial equations in terms of boundary layer thicknesses. The velocity and temperature profiles as well as skin friction and heat transfer are analyzed for various parameters. The obtained numerical solutions are compared with the previously published results and are found to be in good agreement.

Keywords. Nanofluid; Dual solution; Thermal radiation; Gyarmati's variational principle; Boundary layer flow

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1. Introduction

Nanotechnology has been widely used in many industrial applications. Nanofluids are engineered colloids made of a base fluid and nanoparticles. Nanofluids have higher thermal conductivity and single-phase heat transfer coefficients than the base fluids. The term nanofluid was coined by Choi [1]. The boundary layer flow over a static or moving wedge in nanofluid has been considered by Yacob *et al.* [2], which is an extension of the flow over a static wedge considered by Falkner and Skan [3].

Kameswaran *et al*. [4] investigated heat and mass transfer from an isothermal wedge in nanofluids with soret effect. Shanmugapriya and Chandrasekar [5], analyzed the problem of free and forced convection with suction and injection over a non-isothermal wedge. The present paper will study the boundary-layer and heat transfer for a moving wedge immersed in Cu-water nanofluid in the presence of thermal radiation.

The object of the present paper is to study the boundary layer flow and heat transfer for a moving wedge immersed in Cu-water nanofluid in the presence of thermal radiation by using Gyarmati's variational technique. This technique is one of the most general and exact variational technique in solving flow and heat transfer problems. Shanmugapriya [6], Chandrasekar and Kasiviswanathan [7] already applied this technique for steady and unsteady, heat and mass transfer and boundary layer flow problems.

Section 2 presents the mathematical model of the problem. The numerical procedure is obtained in Section 3 and 4. Results and discuss are presents in Section 5. Section 6 presents some useful conclusion.

2. Mathematical Formulation

Consider a steady two-dimensional laminar boundary layer flow of an incompressible viscous nanofluid (Cu-water) of density ρ_{nf} and temperature T_∞ moving over a wedge moving with the velocity $u_w(x)$. Choose the co-ordinate system such that x-axis is along the surface of the wedge and y-axis normal to the surface of the wedge. Further it is assumed that the velocity of ambient fluid is $u_e(x) = U_0 x^m$ and the velocity of the moving wedge is $u_w(x) = U_w x^m$, where U_0, U_w and m are all constant with $0 \le m \le 1$. Here $m = \beta/(2-\beta)$, where β is the Hartree pressure gradient parameter that corresponds to $\beta = \Omega/\pi$ for the total wedge angle Ω. Thermal radiation is included in the energy equation. The governing equations for this case can be written as (Tiwari and Das [8])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e(x)\frac{\partial u_e(x)}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho c_p)_{nf}}\frac{\partial q_r}{\partial y} \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} &y = 0; \quad u = u_w(x) = U_w x^m, \quad v = 0, \quad T = T_w \\ &y \to \infty; \quad u = u_e(x) = U_0 x^m, \quad T \to T_\infty \end{aligned} \tag{4}$$

Here u, v are the velocity components along x and y axes, respectively, T is the temperature of the nanofluid in the boundary layers, μ_{nf} is the viscosity of the nanofluid, ρ_{nf} is the density of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid, which are given by Oztap and Abu-Nada [9].

The effective dynamic viscosity of the nanofluid is given as

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \tag{5}$$

where φ is the solid volume fraction of nanoparticles.

The effective density of the nanofluids is given as

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s. \tag{6}$$

The thermal diffusivity of the nanofluid is

$$\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}, \tag{7}$$

where the heat capacitance of the nanofluid is given by

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s. \tag{8}$$

The thermal conductivity of nanofluids restricted to spherical nanoparticles is

$$\frac{k_{nf}}{k_f} = \frac{\left(k_s + 2k_f\right) - 2\varphi(k_f - k_s)}{\left(k_s + 2k_f\right) + 2\varphi(k_f - k_s)}. \tag{9}$$

Here, the subscript nf, f and s represent the thermophysical properties of the nanofluid, base fluid and nano solid particles, respectively.

Making use of the Rosseland approximation for radiation for an optically thick layer (Brewster [10]), we have

$$q_r = \frac{-4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{10}$$

where σ is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. If temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, then the Taylor series for T^4 about T_∞ after neglecting higher order terms, is given by

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{11}$$

In view of equations (10) and (11), equations (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{16\sigma T_\infty^3}{3k^*(\rho c_p)_{nf}}\frac{\partial^2 T}{\partial y^2}. \tag{12}$$

3. Gyarmati's Variational Principle

Gyarmati introduced a genuine variational principle called the "*Governing Principle of Dissipative Processes*" (GPDP) which is given in its energy picture

$$\delta\int_V [T\sigma - T\Psi - T\Phi]dV = 0. \tag{13}$$

Here the energy dissipation $T\sigma$ and dissipation potentials $T\Psi$, $T\Phi$ are given by

$$T\sigma = -P_{12}\frac{\partial u}{\partial y} - J_q\frac{\partial \ln T}{\partial y},$$

$$T\Psi = \frac{1}{2}\left[L_s\left(\frac{\partial u}{\partial y}\right)^2 + L_\lambda\left(\frac{\partial LnT}{\partial y}\right)^2\right]$$

and

$$T\Phi = \frac{1}{2}\left[R_s P_{12}^2 + R_\lambda J_q^2\right],$$

where $P_{12} = \left(-L_s\frac{\partial u}{\partial y}\right)$ and $J_q = \left(-L_\lambda\frac{\partial T}{\partial y}\right)$ are heat and momentum fluxes, respectively. The constants L's and R's represent conductivities and resistances. It is well known that '$\ln T$' is the proper state variable instead of T when the governing principle assumes energy picture.

The variational principle (13) for the present problem takes the form

$$\delta\int_0^l\int_0^\infty\left\{-J_q\frac{\partial \ln T}{\partial y} - P_{12}\frac{\partial u}{\partial y} - \frac{1}{2}\left[L_\lambda\left(\frac{\partial \ln T}{\partial y}\right)^2 + L_s\left(\frac{\partial u}{\partial y}\right)^2\right] - \frac{1}{2}\left[R_\lambda J_q^2 + R_s P_{12}^2\right]\right\}dy\,dx = 0. \quad (14)$$

In which 'l' is the representative length of the surface.

4. Method of Solution

The velocity and temperature fields inside the respective boundary layers are approximated as a fourth degree polynomial function.

$$\left.\begin{aligned}
&\frac{(u-u_w)}{(u_e-u_w)} = \frac{3y}{d_1} - \frac{3y^2}{d_1^2} + \frac{y^3}{d_1^3}, && (y<d_1)\\
&u = u_e, && (y\ge d_1)\\
&\frac{(T-T_\infty)}{(T_w-T_\infty)} = 1 - \frac{3y}{2d_2} + \frac{y^3}{2d_2^3}, && (y<d_2)\\
&T = T_\infty, && (y\ge d_2)
\end{aligned}\right\} \quad (15)$$

where d_1 and d_2 are hydrodynamical and thermal boundary layer thicknesses, respectively. The velocity and thermal profiles (15) satisfy the following compatibility conditions:

$$\left.\begin{aligned}
&y=0; && u = u_w(x) = U_w x^m, && v=0, && T=T_w\\
&y=d_1; && u = u_e(x) = U_0 x^m, && \frac{\partial u}{\partial y}=0, && \frac{\partial^2 u}{\partial y^2}=0\\
&y=d_2; && T = T_\infty, && \frac{\partial T}{\partial y}=0, && \frac{\partial^2 T}{\partial y^2}=0.
\end{aligned}\right\} \quad (16)$$

Using the boundary conditions (16), the transverse velocity component v is obtained from the mass balance eq. (1) as

$$v = \left[m(U_0-U_w)x^m/x\right]\left[\frac{-3y^2}{2d_1} + \frac{y^3}{d_1^2} - \frac{y^4}{4d_1^3}\right] + (U_0-U_w)x^m\left[\frac{3y^2}{2d_1^2} - \frac{2y^3}{d_1^3} + \frac{3y^4}{4d_1^4}\right]. \quad (17)$$

To formulate Gyarmati's variational principle the velocity and temperature functions (15) are substituted in the momentum and energy balance eqs. (2) and (12), and on direct integration with respect to y with the help of smooth fit boundary conditions $\frac{\partial u}{\partial y}=0$ and $\frac{\partial T}{\partial y}=0$ the fluxes

P_{12} and J_q are obtained respectively as given below.

$$
\begin{aligned}
-\frac{P_{12}}{L_s} &= \frac{\rho_{nf}}{\mu_{nf}}\left[\frac{m(U_0-U_w)^2x^{2m}}{x}\right]\left[-0.5357d_1+\frac{4.5y^3}{3d_1^2}-\frac{3y^4}{2d_1^3}+\frac{3.75y^5}{5d_1^4}-\frac{1.5y^6}{6d_1^5}+\frac{0.25y^7}{7d_1^6}\right]\\
&+\frac{\rho_{nf}}{\mu_{nf}}\left[(U_0-U_w)^2x^{2m}d_1'\right]\left[0.10714-\frac{4.5y^3}{3d_1^3}+\frac{3y^4}{d_1^4}-\frac{11.25y^5}{5d_1^4}+\frac{4.5y^6}{6d_1^6}-\frac{0.75y^7}{7d_1^7}\right]\\
&+\frac{\rho_{nf}}{\mu_{nf}}\left[\frac{mU_w(U_0-U_w)x^{2m}}{x}\right]\left[-1.5+\frac{3y^2}{d_1}-\frac{2y^3}{d_1^2}+\frac{y^4}{2d_1^3}\right]\\
&+\frac{\rho_{nf}}{\mu_{nf}}\left[U_w(U_0-U_w)x^{2m}d_1'\right]\left[0.25-\frac{3y^2}{2d_1^2}+\frac{2y^3}{d_1^3}-\frac{3y^4}{4d_1^4}\right]\\
&+\frac{\rho_{nf}}{\mu_{nf}}\left[\frac{m\left(U_0^2-U_w^2\right)x^{2m}}{x}\right][d_1-y]
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
-\frac{J_q}{L_\lambda} &= \left[\frac{P_r(U_0-U_w)(T_w-T_\infty)x^m}{\vartheta_f\left(1+\frac{4}{3R}\right)}d_1'\right]\left[\frac{0.3d_2^2}{d_1^2}-\frac{0.25d_2^3}{d_1^3}+\frac{0.06429d_2^4}{d_1^4}-\frac{9y^3}{12d_1^2d_2}\right.\\
&\left.+\frac{6y^4}{8d_1^3d_2}-\frac{9y^5}{40d_1^4d_2}+\frac{9y^5}{20d_1^2d_2^3}-\frac{6y^6}{12d_1^3d_2^3}+\frac{9y^7}{56d_1^4d_2^3}\right]\\
&+\left[\frac{P_r(U_0-U_w)(T_w-T_\infty)x^m}{\vartheta_f\left(1+\frac{4}{3R}\right)}d_2'\right]\left[\frac{-0.6d_2}{d_1}-\frac{1.875d_2^2}{d_1^2}-\frac{0.08571d_2^3}{d_1^3}\right.\\
&\left.+\frac{9y^3}{6d_1d_2^2}+\frac{9y^4}{8d_1^2d_2^2}+\frac{3y^5}{10d_1^3d_2^2}-\frac{9y^5}{10d_1d_2^4}+\frac{9y^6}{12d_1^2d_2^4}-\frac{3y^7}{14d_1^3d_2^4}\right]\\
&+\left[\frac{P_rU_w(T_w-T_\infty)x^m}{\vartheta_f\left(1+\frac{4}{3R}\right)}d_2'\right]\left[-0.375+\frac{3y^2}{4d_2^2}-\frac{3y^4}{8d_2^4}\right]\\
&+\left[\frac{P_rm(U_0-U_w)(T_w-T_\infty)x^m}{\vartheta_fx\left(1+\frac{4}{3R}\right)}\right]\left[\frac{-0.3d_2^2}{d_1}+\frac{0.125d_2^3}{d_1^2}-\frac{0.02143d_2^4}{d_1^4}\right.\\
&\left.+\frac{9y^3}{12d_1d_2}-\frac{3y^4}{8d_1^2d_2}+\frac{3y^5}{40d_1^3d_2}-\frac{9y^5}{20d_1d_2^3}+\frac{3y^6}{12d_1^2d_2^3}-\frac{3y^7}{56d_1^3d_2^3}\right]
\end{aligned}
\tag{19}
$$

Using the expressions P_{12} and J_q along with trial functions (15), the variational principle (14) is formulated. On integration with respect to y, the variational principle becomes us

$$\delta\int_0^l L_2\left[d_1,d_2,d_1',d_2'\right]dx=0;\quad P_r\geq 1 \tag{20}$$

where L_1 and L_2 are the Lagrangian densities of the principle.

The boundary layer thicknesses d_1 and d_2 are the independent parameters to be calculated and the Euler-Lagrange equations corresponding to these variational principles are

$$\left(\partial L_{1,2}/\partial d_1\right)-(d/dx)\left(\partial L_{1,2}/\partial d_1'\right)=0\quad\text{and}\quad\left(\partial L_{1,2}/\partial d_2\right)-(d/dx)\left(\partial L_{1,2}/\partial d_2'\right)=0, \tag{21}$$

where $L_{1,2}$ represents the Lagrangian densities L_1 and L_2, respectively. The equations (20) and (21) are second order ordinary differential equations in terms of d_1 and d_2, respectively.

We now introducing the non-dimensional boundary layer thicknesses d_1^* and d_2^* for solving these equations and are given by

$$d_1 = d_1^* \sqrt{\vartheta_f x/u_e(x)} \quad \text{and} \quad d_2 = d_2^* \sqrt{\vartheta_f x/u_e(x)}\,. \tag{22}$$

The Euler-Lagrange equations of the transformed principle assume the simple forms

$$\left(\partial L_{1,2}/\partial d_1^*\right) = 0 \quad \text{and} \quad \left(\partial L_{1,2}/\partial d_2^*\right) = 0\,. \quad (P_r \geq 1) \tag{23}$$

The coefficients of the equations (22) depend on the independent parameters P_r, R, λ and φ, where $P_r = \nu_f/\alpha_f$ (Prandtl number) $R = k_{nf}k^*/4\sigma T_\infty^3$ (Radiation parameter), $\lambda = U_w/U_0$ (ratio of the wall velocity to the free stream fluid velocity), $\lambda(>0)$ corresponds to the situation when the wedge moves in the same direction to the free stream and λ (<0) when the wedge moves in the opposite direction to the free stream, while $\lambda = 0$ corresponds to a static wedge and φ (Solid volume fraction).

After obtaining the values of d_1^* and d_2^* for the given values of P_r, R, λ and φ the velocity and temperature profiles, velocity and temperature gradients, skin friction and heat transfer values are calculated with the help of the following relations, respectively.

$$\eta = y\sqrt{(m+1)u_e(x)/2\vartheta_f x}\,, \tag{24}$$

$$C_f = \mu_{nf}\left[(-P_{12}/L_s)_{y=0}/\rho_f(u_e(x))^2\right], \tag{25}$$

$$Nu_x = -\left[xk_{nf}\left(-\frac{J_q}{L_\lambda} - q_r\right)_{y=0} \Big/ k_f(T_w - T_\infty)\right]. \tag{26}$$

Following Oztop and Abu-Nada [9], the value of the Prandtl number P_r is taken as 6.2 (for water) and the volume fraction of nanoparticles is from 0 to 0.2. The thermophysical properties of the fluid and nanoparticles are given in Table 1.

Table 1. Thermophysical properties of the base fluid and the nanoparticles

Physical properties	Fluid phase (water)	Cu
C_p (J/kg K)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/m K)	0.613	400
$\alpha \times 10^{-7}$(m^2/s)	1.47	1163.1

5. Results and Discussion

Figure 1 and 2 shows the effect of the velocity ratio parameters λ on velocity and temperature profiles for $R = 1$ and $\varphi = 0.1$, respectively. These figures show that there are regions of unique solutions for $\lambda > -1$ and dual solutions for $\lambda_c < \lambda = -1$. The velocity profiles for unique solution increases with increasing value of λ. The first solution of velocity profiles exhibit the identical characters as that of the velocity profiles for unique solution and reverse nature is noticed for the case of the second solution. From Figure 2, it is noticed that the temperature profiles for first solution decreases for an increase of λ and it decreases for the second solution also the unique solution of temperature profiles is similar to the profiles of the first solution.

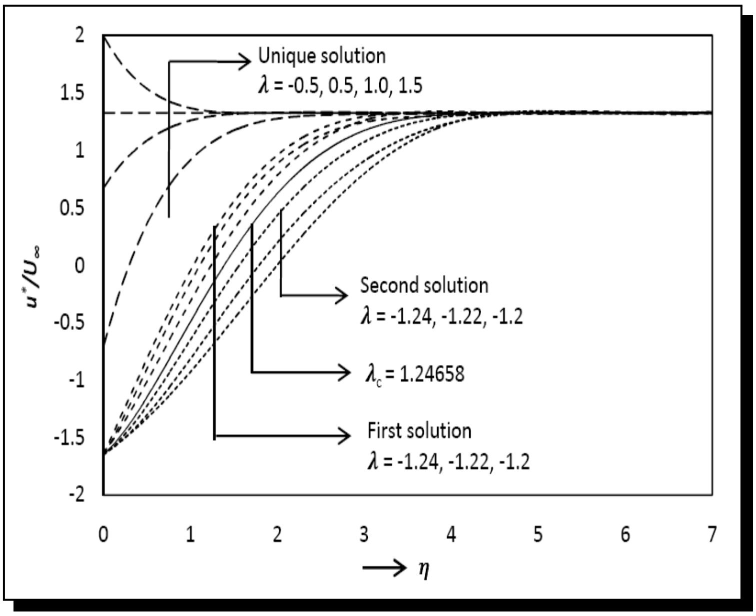

Figure 1. Velocity profile for different values of λ when $R = 1$, $\varphi = 0.1$ and $m = 1$

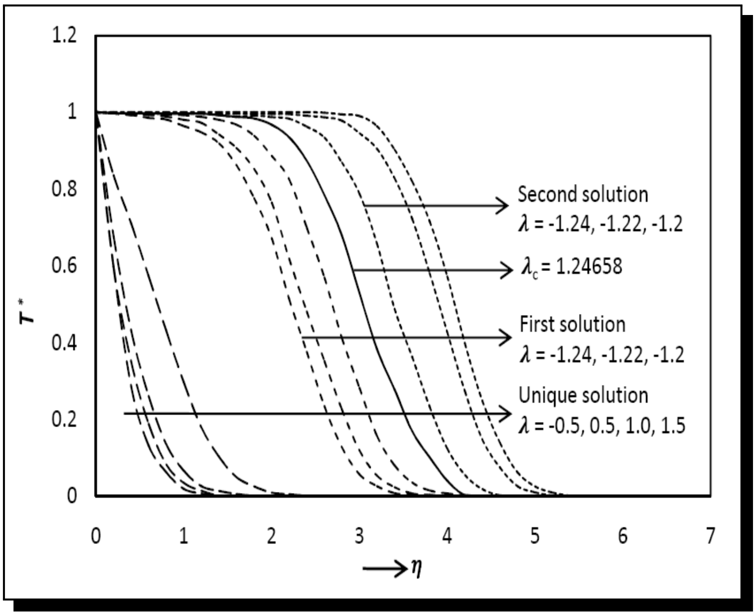

Figure 2. Temperature profile for different values of λ when $R = 1$, $\varphi = 0.1$ and $m = 1$

Figures 3 and 4 represent the velocity and temperature profiles at $\lambda = 1.2$ and $\lambda = -1.2$ for different values of radiation parameters R. From Figure 3, it is observed that the radiation parameter has a negligible effect on the velocity profiles. When $\lambda = 1.2$ there is only a unique solution and the temperature profiles are decreasing with an increase of radiation parameter, the different behavior is appears when $\lambda = -1.2$. The temperature profile of the first solution increases with an increase in R within the thermal boundary layer and the reverse is seen away from the surface. Also, it is observed that, far away from the surface, the temperature profile for the second solution exhibit the identical characters as that of the first solution. For $\lambda = -1.2$, the temperature inside the boundary layer for the first solution is high for large value of R, while outside the boundary layer, the temperature is low with large value of R. For the second solution the behavior is similar, far away from the surface.

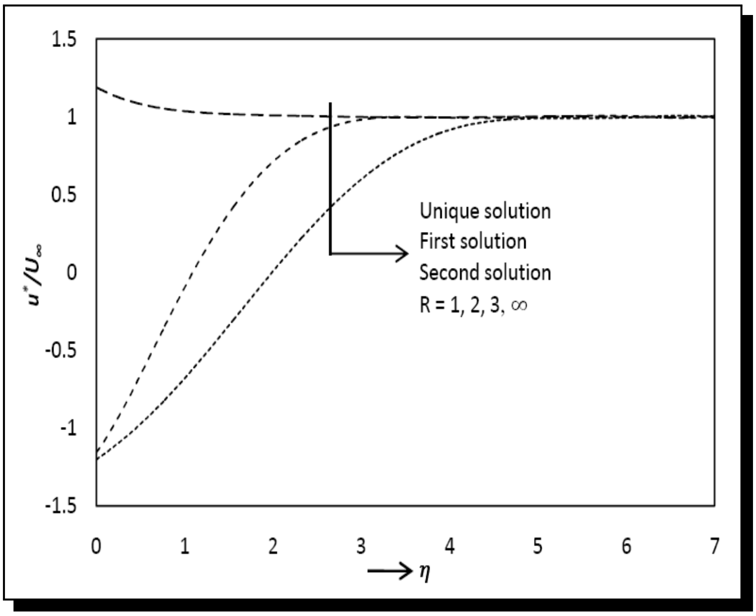

Figure 3. Velocity profile for different values of R when $\varphi = 0.1$ and $m = 1$.

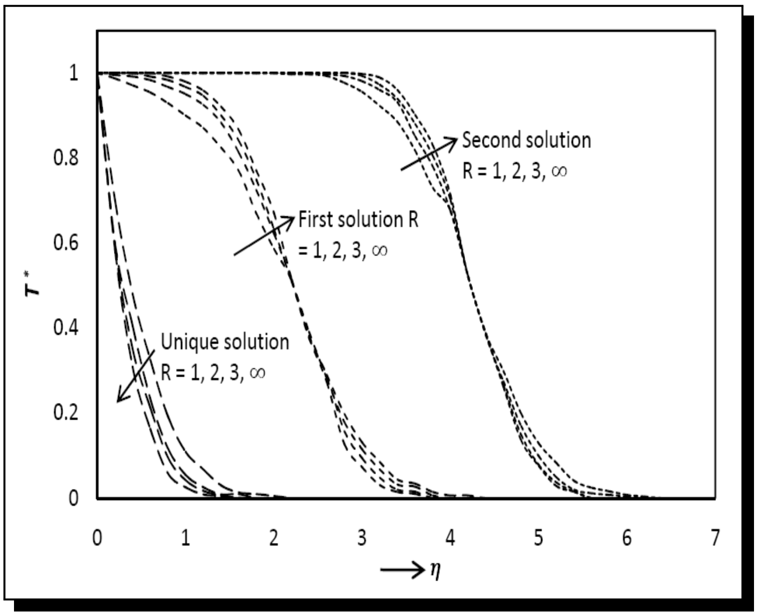

Figure 4. Temperature profile for different values R when $\varphi = 0.1$ and $m = 1$.

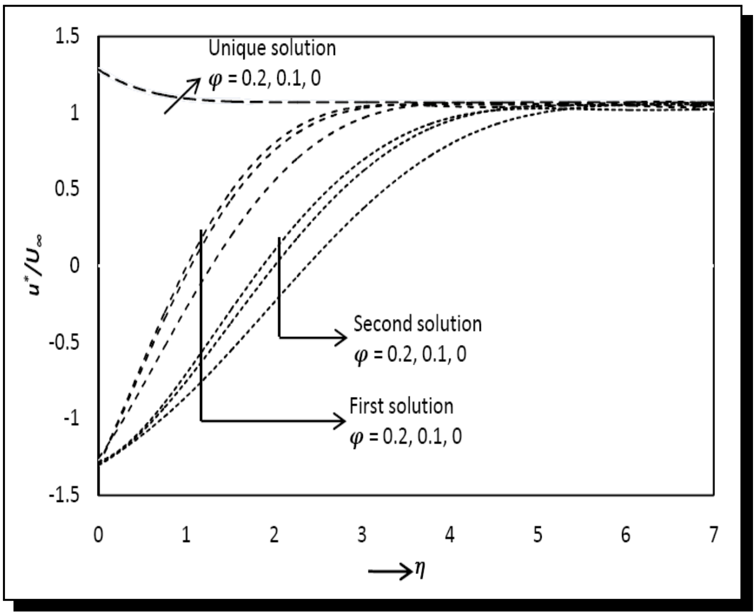

Figure 5. Velocity profile for different values of φ when $R = 1$ and $m = 1$.

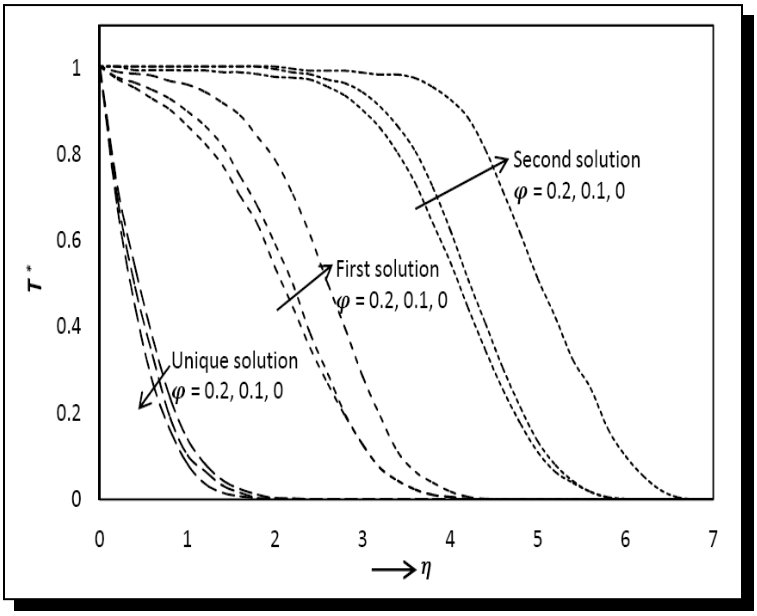

Figure 6. Temperature profile for different values of φ when $R = 1$ and $m = 1$.

6. Conclusion

Numerical analysis is carried out to study the problem of steady, two dimensional boundary layer flow past a moving wedge in a copper-water nanofluid taking into account the effect of thermal radiation. By GPDP, governing partial differential equations are simplified as polynomial equations whose coefficients are of independent parameters P_r, R, λ and φ. This variational technique offers a practicing engineer a rapid way of obtaining heat transfer rates for any combination of these parameters. The advantage involved in this technique is that the results are obtained with the high order of accuracy and the time taken to solve the problem is certainly less when compared with more conventional methods. Hence the practicing engineers and scientists can apply this unique approximate technique as a powerful tool for solving boundary layer flow and heat transfer problems.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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