



Free Convective Unsteady MHD Flow in Slip-Flow Regime Past A Vertical Plate with A Convective Surface Boundary Condition

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Abstract. This paper examines the unsteady free convective viscous incompressible MHD flow past a vertical porous flat plate with convective surface boundary condition in slip flow regime under the influence of uniform magnetic field acting perpendicular to the porous surface. Assuming time dependent variable suction velocity at the porous plate, analytical expressions for the flow characteristics are obtained by using perturbation technique which converts the non-linear partial differential equations into ordinary differential equations. The effect of various parameters such as Prandtl number, Grashoff number, the Magnetic field parameter, Suction parameter and Convective heat change parameter on the transient velocity, transient temperature, skin friction coefficient and the rate of heat transfer are discussed with the help of graphs.

Keywords. Free convection; Magneto-hydrodynamics; Boundary layer; Vertical porous plate; Suction velocity; Convective boundary condition; Slip-flow regime

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1. Introduction

The phenomenon of free convection arises when buoyancy forces are induced due to density gradients caused by temperature variations. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. Flow problems for steady and unsteady free convection past flat surfaces with constant temperatures or with oscillating temperature have been studied by a number of researchers: Martynenko et al. [12], Das et al. [5] and Hossain et al. [9] to mention a few.

Magneto hydrodynamic (MHD) is the field of fluid mechanics which deals with the dynamics of an electrically conducting fluid under the influence of magnetic field and has gained considerable importance because of its wide ranging applications in geophysics and engineering problems such as geothermal energy extraction, oil exploration, MHD power generators, ion propulsion, MHD bearings, MHD pumps etc. In view of its diverse applications, the topic has been studied extensively by several researchers. Sahoo et al. [15] studied MHD unsteady free convection flow with heat source/sink. Heat and mass transfer in MHD free convection from a moving permeable vertical surface using perturbation technique has been investigated by Abdelkhalek [1]. Attia [2] studied unsteady MHD Couette flow of a viscoelastic fluid with heat transfer.

At the macroscopic level, it is well accepted that the boundary condition for a viscous fluid at a solid wall is one of no-slip. However in this modern era of industrialization there are many practical applications, where the fluid particles adjacent to a solid surface no longer acquire the velocity of the surface but have a finite tangential velocity. The flow regime is called slip-flow regime and this effect cannot be neglected. In view of the practical applications of the slip-flow regime it remained of paramount interest for several scholars. Sharma and Chaudhary [16], Khaled and Vafai [10], Mehmood and Ali [13], Chaudhary and Jha [4], Hayat et al. [8] analyzed heat transfer problems with various physical aspects in slip-flow regime.

The objective of the present study is to examine the effect of suction and other parameters on MHD free convection viscous incompressible flow past a vertical porous plate with a convective surface boundary condition in slip-flow regime. The idea of convective heat exchange at the surface with the surrounding was first introduced by Aziz [3]. Subsequently other authors, for instance Makinde [11], Gangadhar et al. [6], Olanjrewaju et al. [14] and Garg et al. [7] also considered problems of fluid flows set up by the convective heating of the surface.

In this paper, a two dimensional unsteady free convective viscous incompressible MHD flow past a vertical porous flat plate with convective surface boundary condition in slip-flow regime has been discussed. A uniform magnetic field acts perpendicular to the porous surface and the suction velocity at the porous plate is assumed to be variable, varying with time. Analytical expressions for the flow characteristics are obtained. The effects of various parameters such as Prandtl number, Grashoff number, the Magnetic Field Parameter, Suction parameter and Convective heat change parameter on the transient velocity, transient temperature, skin friction coefficient and the rate of heat transfer are discussed with the help of graphs.

2. Formulation of the Problem

Consider an unsteady free convective MHD laminar two-dimensional flow of a viscous incompressible fluid past an infinite vertical porous flat plate with convective surface boundary condition in slip flow regime. The variable suction velocity $V = V_0(1 + \varepsilon A e^{i\bar{\omega}\bar{t}})$ and the magnetic field are applied perpendicularly to the plate. We take \bar{x} axis along the plate and \bar{y} -axis along the normal to the plate as shown in Figure 1.

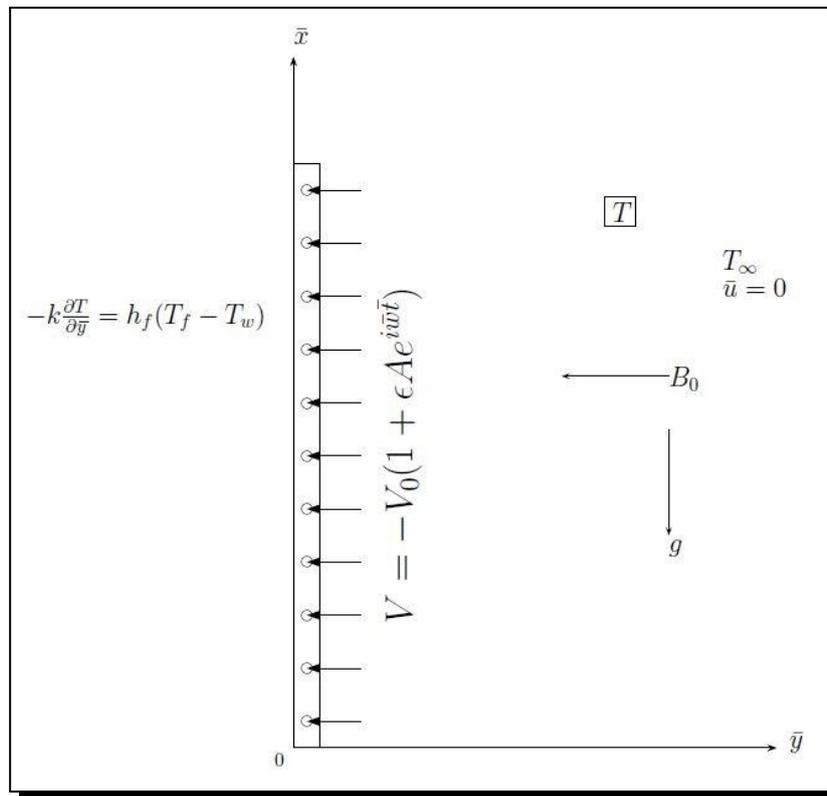


Figure 1. Physical model

Since the plate is considered to be infinite in the \bar{x} -direction, therefore all the flow variables are the functions of \bar{y} and \bar{t} only. Assuming negligible viscous and magnetic dissipation along with Boussinesq approximations, the boundary layer equations for the problem can be written as

$$\frac{\partial \bar{u}}{\partial \bar{t}} - V_0(1 + \varepsilon A e^{i\bar{\omega}\bar{t}}) \frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T - T_\infty) - \frac{\sigma}{\rho} B_0^2 \bar{u} \quad (2.1)$$

$$\frac{\partial T}{\partial \bar{t}} - V_0(1 + \varepsilon A e^{i\bar{\omega}\bar{t}}) \frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} \quad (2.2)$$

where \bar{u} and \bar{v} are the \bar{x} (along the plate) and the \bar{y} (normal to the plate) components of the velocities respectively, A is the suction parameter, $\bar{\omega}$ is the frequency, T is the temperature, ν is the kinematic viscosity of the fluid, ρ is the fluid density, $\alpha (= k/\rho c_p)$ is the thermal diffusivity of the fluid, k is the thermal conductivity, β is the thermal expansion coefficient, g is the acceleration due to gravity, B_0 is the magnetic field in the \bar{y} -direction, σ is the electrical conductivity. The subscript ∞ refers to the condition at infinity.

The boundary conditions at the plate surface and far into the cold fluid can be written as

$$\bar{u} = L \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right), \quad -k \frac{\partial T}{\partial \bar{y}} = h_f (T_f - T_w) \quad \text{at } \bar{y} = 0 \quad (2.3)$$

$$\bar{u} \rightarrow 0, \quad T \rightarrow T_\infty, \quad \text{as } \bar{y} \rightarrow \infty \quad (2.4)$$

where $T_w = T(0, t)$ and L is some body dimension which is a characteristic dimension in the flow field. Note that the relative importance of effects due to the Rarefaction of a fluid may be indicated by a comparison of the magnitude of the mean free molecular path in the fluid with some significant body dimension which is taken as L here.

We now introduce the non dimensional variables into equations (2.1)-(2.4) as follows:

$$y = \frac{\bar{y}V_o}{v}, \quad t = \frac{\bar{t}V_o^2}{4\nu}, \quad \omega = \frac{4\nu\bar{\omega}}{V_o^2}, \quad u = \frac{\bar{u}}{V_o}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}$$

So that equations (2.1) and (2.2) reduce to

$$\frac{1}{4} \frac{\partial u}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - Mu \quad (2.5)$$

$$\frac{1}{4} \frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \quad (2.6)$$

The boundary conditions (2.3) and (2.4) in the dimensionless form become

$$u = H \frac{\partial u}{\partial y}, \quad \frac{\partial \theta}{\partial y} = -H_f (1 - \theta) \quad \text{at } y = 0, \quad (2.7)$$

$$u = 0, \quad \theta = 0 \quad \text{as } y \rightarrow \infty, \quad (2.8)$$

where $Gr = \frac{g\beta\nu}{V_o^3} (T_f - T_\infty)$ is the Grashof Number, $M = \frac{\sigma\nu B_o^2}{\rho V_o^2}$ is the Magnetic Field Parameter, $H = \frac{LV_o}{v}$ is the Rarefaction Parameter, $H_f = \frac{\nu h_f}{kV_o}$ is the Convective Heat Change Parameter and $Pr = \frac{\mu c_p}{k}$ is the Prandtl number.

3. Solution of the Problem

In order to solve the boundary value problem given by (2.5)-(2.8), system of partial differential equations (2.5) and (2.6) are first reduced to ordinary differential equations using perturbation technique. Assuming small amplitude oscillations $\varepsilon \ll 1$, the velocity u and temperature θ near the plate can be represented as

$$u(y, t) = u_o(y) + \varepsilon e^{i\omega t} u_1(y) + O(\varepsilon^2) \quad (3.1)$$

$$\theta(y, t) = \theta_o(y) + \varepsilon e^{i\omega t} \theta_1(y) + O(\varepsilon^2) \quad (3.2)$$

Substituting equations (3.1) and (3.2) into equations (2.5) and (2.6), and comparing the coefficients of identical powers of ε (neglecting those of $O(\varepsilon^2)$), we obtain

$$u_o'' + u_o' - Mu_o + Gr\theta_o = 0 \quad (3.3)$$

$$u_1'' + u_1' - (M + i\omega/4)u_1 = -Gr\theta_o - Au_o' \quad (3.4)$$

$$\theta_o'' + Pr\theta_o' = 0 \quad (3.5)$$

$$\theta_1'' + Pr\theta_1' - Pr\frac{i\omega}{4}\theta_1 = APr\theta_0' \quad (3.6)$$

where the primes denote differentiation with respect to y .

The corresponding boundary conditions (2.7) and (2.8) reduce to

$$u_o = H\frac{\partial u_o}{\partial y}, u_1 = H\frac{\partial u_1}{\partial y}, \frac{\partial \theta_o}{\partial y} = -H_f(1 - \theta_o), \frac{\partial \theta_1}{\partial y} = H_f\theta_1 \text{ at } y = 0 \quad (3.7)$$

$$u_o = 0, u_1 = 0, \theta_o = 0, \theta_1 = 0, \text{ as } y \rightarrow \infty \quad (3.8)$$

Solving the set of equations (3.3)-(3.6) under boundary conditions (3.7)-(3.8), we get

$$u_o(y) = B_3e^{-m_1y} - B_2e^{-Pr y} \quad (3.9)$$

$$u_1(y) = B_9e^{-m_3y} + B_6e^{-m_2y} + B_7e^{-m_1y} - B_8e^{-Pr y} \quad (3.10)$$

$$\theta_o(y) = B_1e^{-Pr y} \quad (3.11)$$

$$\theta_1(y) = -B_5e^{-m_2y} - B_4e^{-Pr y} \quad (3.12)$$

where

$$m_1 = \frac{1}{2}(1 + \sqrt{1 + 4M}), m_2 = \frac{Pr}{2}(1 + \sqrt{1 + i\omega/Pr}), m_3 = \frac{1}{2}(1 + \sqrt{1 + 4M + i\omega}),$$

$$B_1 = \frac{H_f}{Pr + H_f}, B_2 = \frac{B_1 Gr}{Pr^2 - Pr - M}, B_3 = \frac{Gr(1 + HPr)B_1}{(1 + Hm_1)(Pr^2 - Pr - M)}, B_4 = \frac{4APrB_1}{\omega}i,$$

$$B_5 = \frac{(Pr + H_f)B_4}{m_2 + H_f}, B_6 = \frac{GrB_5}{m_2^2 - m_2 - (M + i\omega/4)}, B_7 = \frac{AB_3m_1}{m_1^2 - m_1 - (M + i\omega/4)},$$

$$B_8 = \frac{GrB_4 + APrB_2}{Pr^2 - Pr - (M + i\omega/4)}, B_9 = \frac{-(1 + Hm_2)B_6 - (1 + Hm_1)B_7 + (1 + HPr)B_8}{1 + Hm_3}.$$

Substituting equations (3.9)-(3.12) in equations (3.1) and (3.2) we get the expressions for the velocity and temperature profiles. We can further express the velocity and temperature in terms of the small fluctuating parts as

$$u(y, t) = u_o(y) + \varepsilon(u_r \cos \omega t - u_i \sin \omega t) \quad (3.13)$$

$$\theta(y, t) = \theta_o(y) + \varepsilon(\theta_r \cos \omega t - \theta_i \sin \omega t) \quad (3.14)$$

where u_r and u_i are respectively the real and imaginary parts of $u_1(y)$ and θ_r and θ_i are respectively the real and imaginary parts of $\theta_1(y)$.

For $\omega t = \pi/2$ we can write the expressions for the transient velocity and temperature profiles from equations (3.13) and (3.14) as

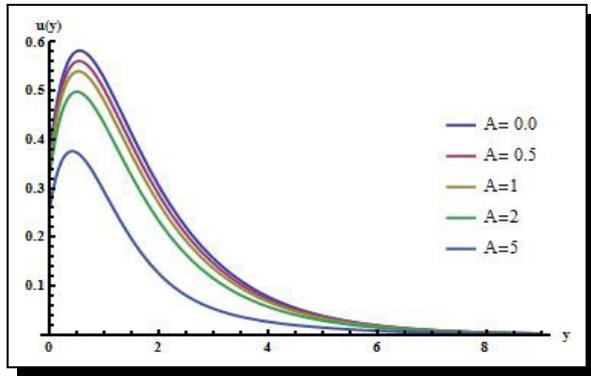
$$u(y, \pi/2\omega) = u_o(y) - \varepsilon u_i(y) \quad (3.15)$$

$$\theta(y, \pi/2\omega) = \theta_o(y) - \varepsilon \theta_i(y) \quad (3.16)$$

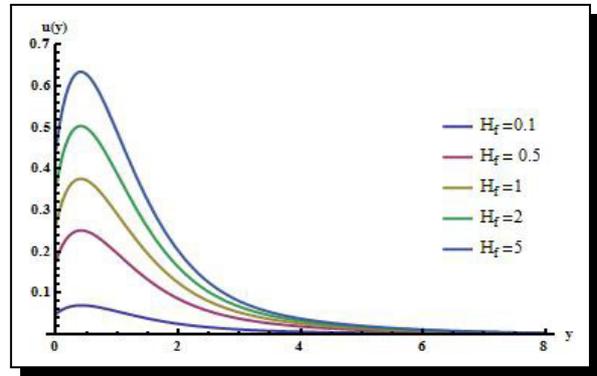
4. Results and Discussion

The momentum and energy equations of the flow problem considered in this paper are solved by using perturbation technique and consequently the velocity and temperature in terms of various parameters are given by (3.15) and (3.16), respectively. The graphs of transient velocity

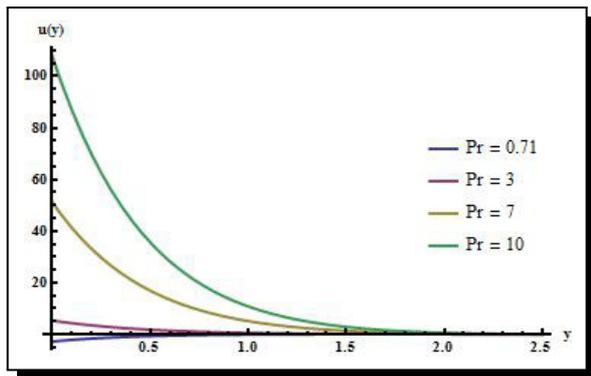
profiles are presented in Figures 2(a)-2(f) and the graphs of transient temperature profiles are presented in Figures 3(a)-3(d).



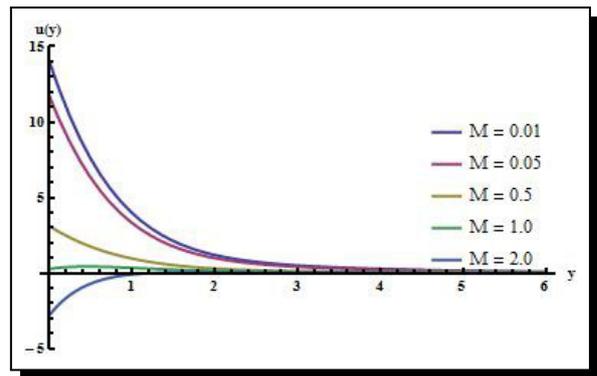
(a) Effect of suction parameter A on velocity profiles, $M = 2$, $Gr = 5$, $w = 5$, $A = 5$, $Pr = 0.71$, $H = 0.4$, $\epsilon = 0.2$



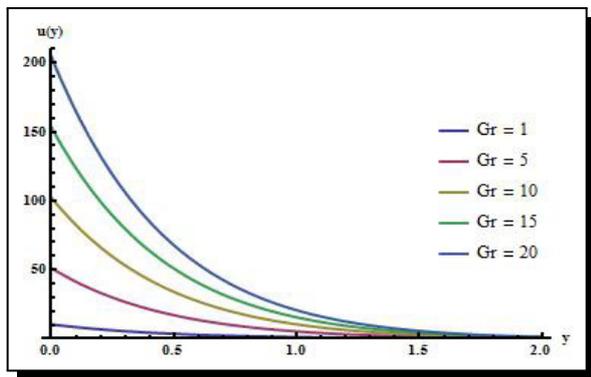
(b) Effect of H_f on velocity profiles, $M = 2$, $Gr = 5$, $w = 5$, $H_f = 1$, $Pr = 0.71$, $H = 0.4$, $\epsilon = 0.2$



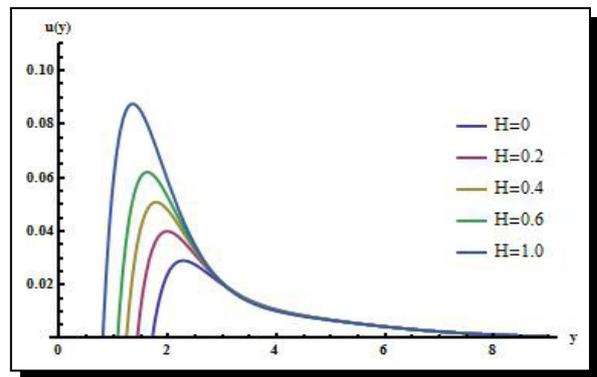
(c) Effect of Prandtl number Pr on velocity profiles, $M = 2$, $H = 0.4$, $w = 5$, $A = 5$, $H_f = 1$, $Gr = 5$, $\epsilon = 0.2$



(d) Effect of M on velocity profiles, $w = 5$, $H = 0.4$, $Pr = 0.71$, $A = 5$, $H_f = 1$, $Gr = 5$, $\epsilon = 0.2$

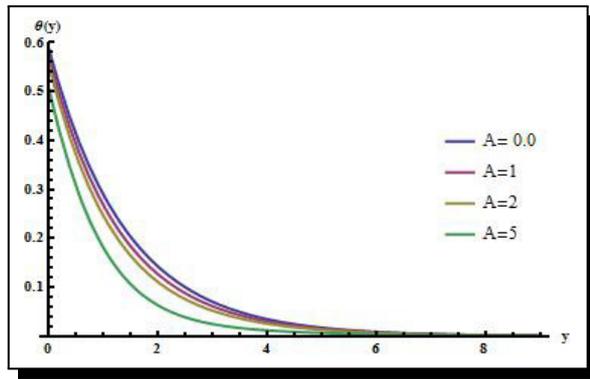


(e) Effect of Gr on velocity profiles, $M = 2$, $A = 5$, $w = 5$, $A = 5$, $H_f = 1$, $Pr = 0.71$, $H = 0.4$, $\epsilon = 0.2$

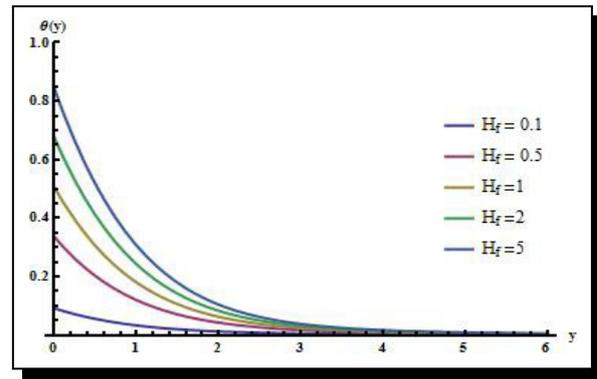


(f) Effect of Rarefaction parameter H on velocity profiles, $M = 2$, $w = 5$, $A = 5$, $H_f = 1$, $Pr = 0.71$, $Gr = 5$, $\epsilon = 0.2$

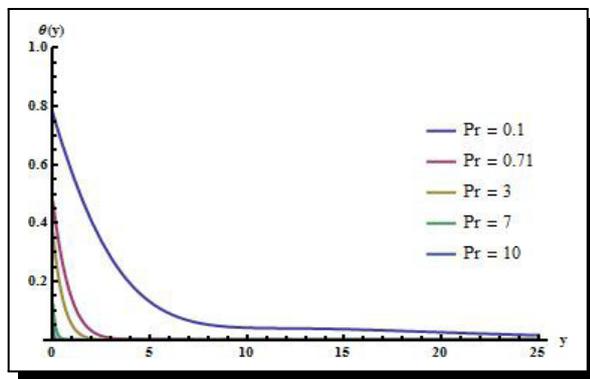
Figure 2



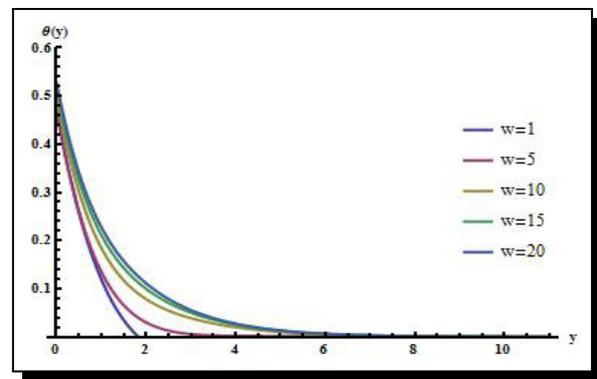
(a) Effect of A on temperature profiles, $M = 2$, $Gr = 5$, $w = 5$, $H_f = 1$, $Pr = 0.71$, $H = 0.4$, $\varepsilon = 0.2$



(b) Effect of H_f on temperature profiles, $M = 2$, $Gr = 5$, $w = 5$, $A = 5$, $Pr = 0.71$, $H = 0.4$, $\varepsilon = 0.2$



(c) Effect of Pr on temperature profiles, $M = 2$, $H = 0.4$, $w = 5$, $A = 5$, $H_f = 1$, $Gr = 5$, $\varepsilon = 0.2$



(d) Effect of frequency w on temperature profiles, $M = 2$, $H = 0.4$, $Pr = 0.71$, $A = 5$, $H_f = 1$, $Gr = 5$, $\varepsilon = 0.2$

Figure 3

It is observed from Figures 2(a)-2(f) that transient velocity at the plate decreases with the increase in the suction parameter A as well as with the increase in the Magnetic field parameter M . However reverse effect is observed for other parameters i.e. the velocity profile increases with increasing Convective heat change parameter H_f , Prandtl number Pr and the Grashof number Gr . Furthermore, the velocity which is more in the vicinity of the plate gradually decreases to the free stream velocity at far away from the plate. From Figures 3(a)-3(d), it may be observed that transient temperature decreases with increase in Pr and A while it increases with the increase in w and H_f . The dimensionless temperature at far away from the plate also decreases and approaches zero.

Skin-Friction

By Newton's law of viscosity, shearing stress is given by

$$\bar{\tau}_x = \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0} = \mu \left(\frac{V_o^2}{\nu} \frac{\partial u}{\partial y} \right)_{y=0}.$$

The dimensionless shearing stress on the surface of the body, due to fluid motion, is known as skin-friction. It is given by:

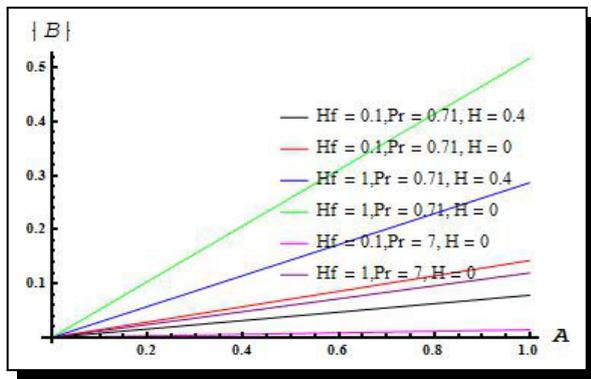
$$\tau_x = \frac{\bar{\tau}_x}{\rho V_o^2} = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \tau_m + \varepsilon e^{i\omega t} B = \tau_m + \varepsilon |B| \cos(\omega t + \alpha) \tag{4.1}$$

where

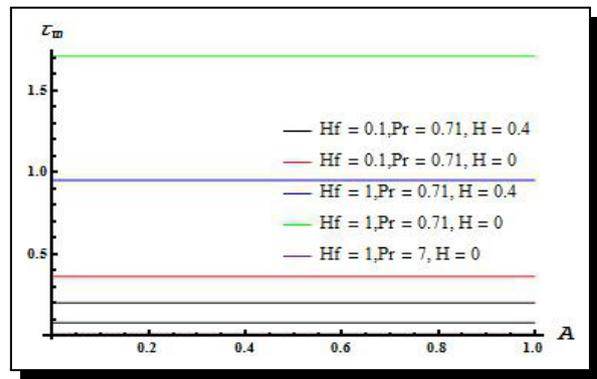
$$\tau_m \text{ (Mean-friction)} = -m_1 B_3 + Pr B_2 \quad \text{and} \quad B = -m_3 B_9 - m_2 B_6 - m_1 B_7 + Pr B_8,$$

$$B = B_r + i B_i = |B|(\cos \alpha + i \sin \alpha), \quad |B| = \sqrt{B_r^2 + B_i^2}, \quad \tan \alpha = B_i/B_r,$$

The effect of suction on amplitude of skin-friction and mean skin-friction are shown in Figures 4(a) and 4(b), respectively. It can be seen that for prescribed values of various parameters mean skin-friction remains constant as suction parameter A changes.



(a) Amplitude of skin-friction coefficient for $M = 2, Gr = 5, w = 5, \varepsilon = 0.2$



(b) Mean skin-friction coefficient for $M = 2, Gr = 5, w = 5, \varepsilon = 0.2$

Figure 4

Coefficient of Heat Transfer:

The main interest in any fluid dynamics problem is to predict the heat transfer rate between the fluid and differentially heated solid body. Since at the surface boundary, the heat exchanged is only due to conduction, therefore by Fourier’s Law it is given by

$$\bar{q}_x = -k \left(\frac{\partial T}{\partial y} \right)_{\bar{y}=0} = -\frac{k V_o}{\nu} (T_f - T_\infty) \left. \frac{\partial \theta}{\partial y} \right|_{y=0}$$

Nusselt Number (Nu), the dimensionless coefficient of heat transfer is:

$$Nu = \frac{\bar{q}_x}{\rho C_p V (T_f - T_\infty)_o} = -\frac{1}{Pr} \left(\frac{\partial \theta}{\partial y} \right)_{y=0} = B_1 + \varepsilon |Q| \cos(\omega t + \delta) \tag{4.2}$$

where $|Q| = \sqrt{Q_r^2 + Q_i^2}, \delta = \tan^{-1}(Q_i/Q_r), Q = Q_r + i Q_i = B_4 - \frac{m_2}{Pr} B_5$

The magnitude $|Q|$ and phase $\tan \delta$ of rate of heat transfer has been shown in Figure 5(a) and Figure 5(b), respectively. As can be seen from Figure 5(b), phase remains constant for all values of suction parameter A .

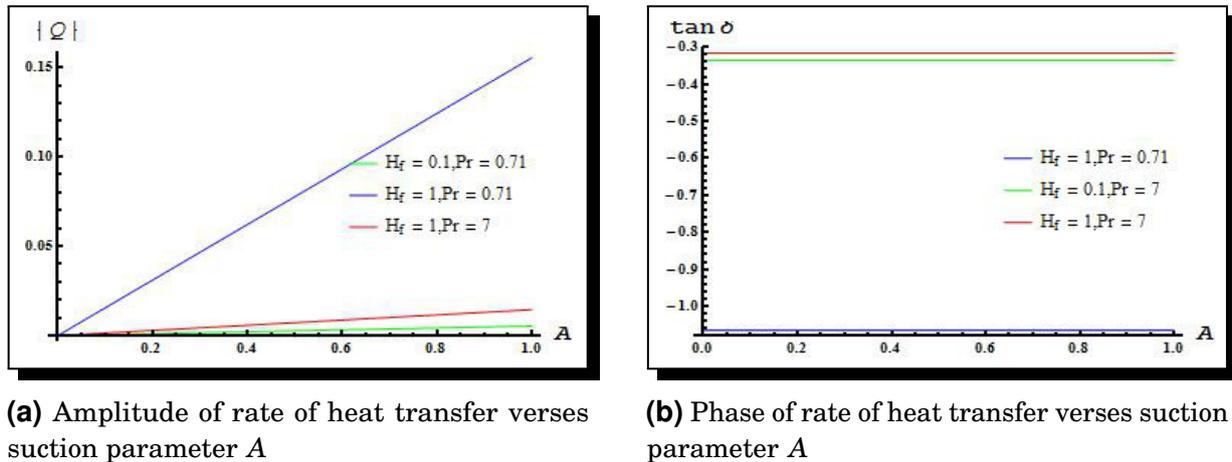


Figure 5

5. Conclusions

- (i) The transient velocity at the plate decreases with the increase in the suction parameter A as well as with the increase in the Magnetic field parameter M . However reverse effect is observed for other parameters i.e. the velocity profile increases with increasing Convective heat change parameter H_f , Prandtl number Pr and the Grasshof number Gr .
- (ii) The transient temperature decreases with increase in Pr and A while it increases with the increase in w and H_f .
- (iii) The amplitude of the skin-friction coefficient increases as suction parameter A increases while the mean skin-friction coefficient remains constant for all values A .
- (iv) The amplitude of the heat transfer increases as suction parameter A increases while the phase of heat transfer remains same as A changes.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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