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Research Article

Radio Geometric Mean Labeling of Some Star Like Graphs

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Abstract. A radio Geometric Mean Labeling of a connected graph G is a one to one map f from the vertex set $V(G)$ to the set of natural numbers N such that for two distinct vertices u and v of G, $d(u,v)+\left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1+\text{diam}(G)$. The radio geometric mean number of f, $r_{gmn}(f)$ is the maximum number assigned to any vertex of G. The radio geometric mean number of G, $r_{gmn}(G)$ is the minimum value of $r_{gmn}(f)$ taken over all radio geometric mean labeling f of G. In this paper, we find the radio geometric mean number of some star like graphs.

Keywords. Radio Geometric Mean labeling; Star; Bistar; Diameter; Lotus inside a circle; k-ary tree

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1. Introduction

We consider finite, simple, undirected graphs only. Let $V(G)$ and $E(G)$ respectively denote vertex set and edge set of G. Chartand *et al*. [1] defined the concept radio labeling of G in 2001. Radio labeling of graphs is applied in channel assignment problem [1]. Radio number of several graphs determined [2, 5, 7, 9]. In this sequence Ponraj *et al*. [8] introduced the radio mean labeling in G. Here we introduce a new type of labeling, a radio geometric mean labeling is a one to one mapping f from $V(G)$ to N satisfying the condition

$$d(u,v)+\left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1+\operatorname{diam}(G)$$

for every $u,v \in V(G)$.

The span of a labeling f is the maximum integer that f maps to a vertex of graph G. The radio geometric mean number of G, $r_{gmn}(G)$ is the lowest span taken over all radio geometric mean labeling of the graph G. In this paper we determine the radio geometric mean number of some star like graphs. Let x be any real number. Then $\lceil x\rceil$ stands for smallest integer greater than or equal to x. Terms and definitions not defined here are followed from Harary [12] and Gallian [13].

Definition 1.1. A **Star** is the complete bipartite graph $K_{1,n}$.

Definition 1.2. The graph **Bistar** $B_{n,n}$ obtained by joining the center vertices of two copies of $K_{1,n}$ with an edge.

Definition 1.3. The graph $T_{k,3}$ is the complete k-ary tree with 3 levels.

Definition 1.4. The graph lotus inside a circle LC_n is obtained from the cycle $C_n : w_1w_2 \dots w_nw_1$ and a star $K_{1,n}$with central vertex u and the end vertices $u_1,u_2,\dots,u_n$ by joining each u_i to w_i and $w_{i+1(\bmod n)}$.

2. Main Results

Theorem 2.1. *Radio Geometric Mean number of star,* $r_{gmn}(K_{1,n}) = n+1$.

Proof. Denote the central vertex u, and pendant vertices by u_i, $(1 \leq i \leq n)$.

The diameter of $K_{1,n}$, $n > 1$ is 2 and that of $K_{1,1}$ is 1.

Case (i): $n = 1$

$$d(u,v)+\left\lceil\sqrt{f(u)f(v)}\right\rceil \geq 1+2=3$$

It is easy to verify that, the given labeling satisfies the radio geometric mean condition and hence $r_{gmn}(K_{1,1}) = 2$.

Case (ii): $n > 1$, $\operatorname{diam}(K_{1,n}) = 2$.

We describe a radio geometric mean labeling f as follows. Assign the labels from $\{1,2,3,\dots,n\}$

to the pendant vertices in any order.Assign the label $n+1$ to the central vertex u.

Now, we check the radio geometric mean condition of the labeling f.

Subcase (i): Check the pair (u, u_i)

$$d(u,u_i)+\left\lceil\sqrt{f(u)f(u_i)}\right\rceil \geq 1+\left\lceil\sqrt{(n+1).1}\right\rceil \geq 3$$

Subcase (ii): Verify the pair $\left(u_i, u_j\right)$, $i \neq j$ where $f(u_i)=1$.

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil \geq 2+\left\lceil\sqrt{(1)(2)}\right\rceil \geq 4$$

Subcase (ii): Verify the pair $\left(u_i, u_j\right)$, $i \neq j$ where $f(u_i) \neq 1$.

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil \geq 2+\left\lceil\sqrt{n(n-1)}\right\rceil \geq 4$$

Hence $r_{gmn}(K_{1,n}) = n+1$.

Example 2.1. The radio geometric mean labeling of $K_{1,6}$ is given below in Figure 1.

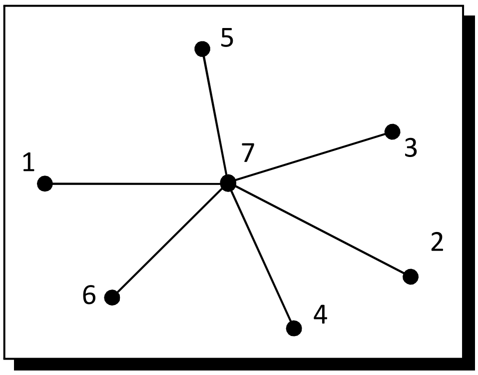

Figure 1

Theorem 2.2. *The radio geometric mean number of the graph Bistar $B_{n,n}$ is 5 if $n=1$ and $2(n+1)$ if $n>1$.*

Proof. Let us consider the central vertices by u and v and the corresponding pendant vertices by u_i, $(1 \leq i \leq n)$ and v_j, $(1 \leq j \leq n)$. The diameter of the graph Bistar $B_{n,n}$is 3.

Case (i): If $n=1$

The vertex labeling of $B_{1,1}$with radio geometric mean number 5 is given by Figure 2.
Hence $r_{gmn}(B_{1,1}) \leq 5$.

Claim. $r_{gmn}(B_{1,1}) > 4$.

The labels 1 and 2 should not be assigned to the adjacent vertices. The labels 1 and 2 assign with the distance atleast 3. Hence without loss of energy we omit the label 1 and assign the label to the remaining vertices in any order.

This gives $r_{gmn}(B_{1,1}) > 4$. Hence $r_{gmn}(B_{1,1}) = 5$.

Case (ii): Assume $n > 1$.

We define the labeling f as follows.

Assign the labels $\{1,2,3,\dots,n\}$ to the vertices u_i in any order and then assign the labels $n+i$ to the vertices v_i, $(1 \le i \le n)$. Assign the label $2n+1$ and $2(n+1)$ to the vertices u and v, respectively.

Now, we check the radio geometric mean condition for any two vertices.

Case 1: Consider the pair (u,v)

$$d(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \ge 1 + \left\lceil \sqrt{(2n+1).2(n+1)} \right\rceil \ge 7$$

Case 2: Consider the pair (u,u_i)

$$d(u,u_i) + \left\lceil \sqrt{f(u)f(u_i)} \right\rceil \ge 1 + \left\lceil \sqrt{(2n+1).1} \right\rceil \ge 4$$

Case 3: Consider the pair (v,v_i)

$$d(v,v_i) + \left\lceil \sqrt{f(v)f(v_i)} \right\rceil \ge 1 + \left\lceil \sqrt{2(n+1).(n+1)} \right\rceil \ge 6$$

Case 4: Consider the pair (u,v_i)

$$d(u,v_i) + \left\lceil \sqrt{f(u)f(v_i)} \right\rceil \ge 2 + \left\lceil \sqrt{(2n+1).(n+1)} \right\rceil \ge 6$$

Case 5: Consider the pair (v,u_i)

$$d(v,u_i) + \left\lceil \sqrt{f(v)f(u_i)} \right\rceil \ge 2 + \left\lceil \sqrt{2(n+1).1} \right\rceil \ge 5$$

Case 6: Consider the pair (u_i,v_j)

Subcase (i): Check the pair (u_i,v_i)w here $f(u_i) = 1$.

It is clear that the pair (u_i,v_i) satisfies the radio geometric mean condition.

Suppose $i \ne j$ then

$$d(u_i,v_j) + \left\lceil \sqrt{f(u_i)f(v_j)} \right\rceil \ge 3 + \left\lceil \sqrt{1.(n+1)} \right\rceil \ge 5$$

Subcase (ii): Check the pair (u_i,v_j) where $f(u_i) \ne 1$.

Clearly, the pair (u_i,v_i) satisfies the radio geometric mean condition.

Let $i \ne j$ then we get,

$$d(u_i,v_j) + \left\lceil \sqrt{f(u_i)f(v_j)} \right\rceil \ge 3 + \left\lceil \sqrt{2.(n+1)} \right\rceil \ge 6$$

Hence $r_{gmn}(B_{n,n}) = 2(n+1)$, $n \ne 1$. □

Example 2.2. The radio geometric mean labeling of $B_{1,1}$ and $B_{4,4}$ are given below.

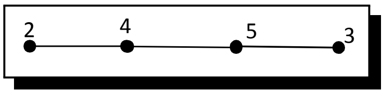

Figure 2

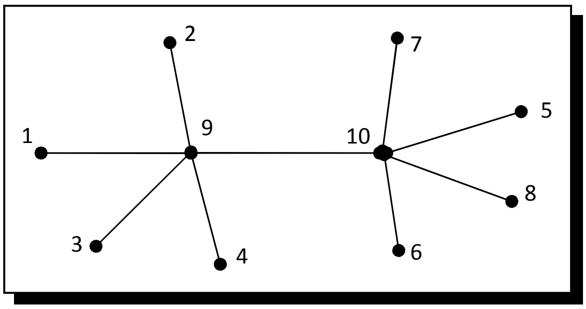

Figure 3

Theorem 2.3. *For $k \geq 1$, $r_{gmn}(T_{k,3}) = k^2 + k + 1$.*

Proof. Consider the central vertex by u and let adjacent with the vertex u by u_i, $(1 \leq i \leq k)$ and also denote the pendant vertices by v_j^i, $(1 \leq i, j \leq k)$.

The diameter of $T_{k,3}$ is 4 for $k \neq 1$ and the diameter of $T_{1,3}$ is 2.

Case (i): If $k = 1$, it is easy to verify that the labeling satisfies the radio geometric mean condition and hence $r_{gmn}\left(T_{1,3}\right) = 3$ is given in Figure 4.

Case (ii): $k = 2$

Assign the labels 1 and 2 (or) 3 (or) 4 at a distance atleast 3. Also assign the label 5 with a distance two. Assign the central vertex by $k^2 + k + 1$. Without loss of generality we assign label 1 adjacent with label 6.

Clearly, the labels are satisfies the radio geometric mean condition.

Case (iii): $k = 3$

Similarly, the same way of case (ii), we get $r_{gmn}(T_{k,3}) = k^2 + k + 1$.

Assume $k \geq 4$. We describe a radio geometric mean labeling f as follows.

Assign the labels $i + k(j-1)$, $(1 \leq i \leq k)$ $(1 \leq j \leq k)$ to the pendant vertices v_j^i. Assign the labels $k^2 + i$ to the support vertices u_i, $(1 \leq i \leq k)$ and also assign the label $k^2 + k + 1$ to the central vertex u.

Now, we check the radio geometric mean condition for any two vertices.

Case (iv): Check the pair (u, u_i)

$$d(u, u_i) + \left\lceil \sqrt{f(u)f(u_i)} \right\rceil \geq 1 + \left\lceil \sqrt{(k^2 + k + 1).(k^2 + i)} \right\rceil \geq 20$$

Case (v): Check the pair (u, v_j^i)

$$d(u, v_j^i) + \left\lceil \sqrt{f(u)f(v_j^i)} \right\rceil \geq 2 + \left\lceil \sqrt{(k^2 + k + 1).1} \right\rceil \geq 6$$

Case (vi): Check the pair (u_i, v_j^i)

Subcase 1: verify the pair (u_i, v_j^i) where $f(v_j^i) = 1$.

Clearly, the pair (u_i, v_i^i)satisfies the radio geometric mean condition.

Suppose $i \neq j$, then

$$d(u, v_j^i) + \left\lceil \sqrt{f(u)f(v_j^i)} \right\rceil \geq 1 + \left\lceil \sqrt{(k^2+i).1} \right\rceil \geq 5$$

Subcase 2: Verify the pair (u_i, v_j^i) where $f(v_j^i) \neq 1$.

It is easy to verify that the pair (u_i, v_i^i)satisfies the radio geometric mean condition.

Let $i \neq j$, then we get

$$d(u, v_j^i) + \left\lceil \sqrt{f(u)f(v_j^i)} \right\rceil \geq 1 + \left\lceil \sqrt{(k^2+i).2} \right\rceil \geq 7$$

Case (vii): Check the pair (u_i, u_j)

$$d(u_i, u_j) + \left\lceil \sqrt{f(u_i)f(u_j)} \right\rceil \geq 2 + \left\lceil \sqrt{(k^2+1).(k^2+2)} \right\rceil \geq 20$$

Case (viii): Check the pair (v_j^i, v_r^s)

Subcase 1: Suppose $i = j$ and $s = r$, $(i \neq s, j \neq r)$. We get

$$d(v_j^i, v_r^s) + \left\lceil \sqrt{f(v_j^i)f(v_r^s)} \right\rceil \geq 4 + \left\lceil \sqrt{1.2} \right\rceil \geq 6$$

Subcase 2: Suppose $i \neq j$ and $s = r$. We get

$$d(v_j^i, v_r^s) + \left\lceil \sqrt{f(v_j^i)f(v_r^s)} \right\rceil \geq 4 + \left\lceil \sqrt{(k+1).2} \right\rceil \geq 7$$

Subcase 3: Suppose $i = j$ and $s \neq r$. We get

$$d(v_j^i, v_r^s) + \left\lceil \sqrt{f(v_j^i)f(v_r^s)} \right\rceil \geq 4 + \left\lceil \sqrt{1.(k+1)} \right\rceil \geq 6$$

Subcase 4: Suppose $i \neq s$, $j = r$.

$$d(v_j^i, v_r^s) + \left\lceil \sqrt{f(v_j^i)f(v_r^s)} \right\rceil \geq 4 + \left\lceil \sqrt{1.(k+1)} \right\rceil \geq 6$$

Subcase 5: Suppose $i = s$, $j \neq r$.

$$d(v_j^i, v_r^s) + \left\lceil \sqrt{f(v_j^i)f(v_r^s)} \right\rceil \geq 2 + \left\lceil \sqrt{1.(k+1)} \right\rceil \geq 5$$

Hence $r_{gmn}(T_{k,3}) = k^2 + k + 1$, $\forall$ k. □

Example 2.3. The radio geometric mean number of $T_{1,3}$ and $T_{4,3}$ is given below.

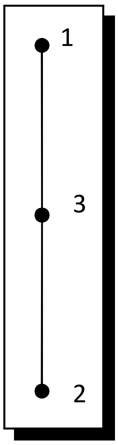

Figure 4

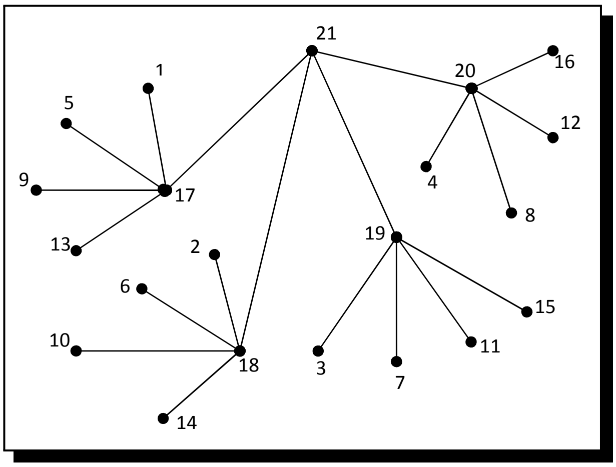

Figure 5

Theorem 2.4. $r_{gmn}(LC_n) = 2n+1$, $n \geq 3$.

Proof. Let us denote the central vertex by u and end vertices of star by u_i, $(1 \leq i \leq n)$ and also denote the vertices of cycle by w_i, $(1 \leq i \leq n)$.

$$\text{The diameter } (LC_n) = \begin{cases} 2, & \text{if } n = 3,4 \\ 3, & \text{if } n = 5,6,7 \\ 4, & \text{if } n \geq 8. \end{cases}$$

Case 1: $n = 3,4$

Assign the consecutive labels for cycle and star. Also, assign label for the central vertex is 7 and 9, respectively. It is enough to prove that

$$d(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 3$$

for every pair of vertices (u,v) where $u \neq v$.

Case 2: $n = 5,6,7$

Consider the label 1 and assign the labels 2, 3, 4 at a distance of atleast 2 with label 1. Further, assign the remaining labels for all other vertices in any order.

Clearly, any two vertices satisfies radio geometric mean condition

$$d(u,v) + \left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 4$$

Case 3: $n \geq 8$

Consider the vertex labels 2, 3, 4 at a distance of atleast 3 with a vertex label 1 and assign the labels $5 \leq i \leq n$ for other vertices in any order.

Clearly, $\left\lceil \sqrt{f(u)f(v)} \right\rceil \geq 2$.

Subcase (i): Compare the pair (u, u_i)

$$d(u,u_i) + \left\lceil \sqrt{f(u)f(u_i)} \right\rceil \geq 2 + \left\lceil \sqrt{(2n+1).1} \right\rceil \geq 7$$

Subcase (ii): Compare the pair (u, w_i)

$$d(u,w_i)+\left\lceil\sqrt{f(u)f(w_i)}\right\rceil \geq 1+\left\lceil\sqrt{(2n+1).1}\right\rceil \geq 6$$

Subcase (iii): Compare the pair (u_i, u_j)

$$d(u_i,u_j)+\left\lceil\sqrt{f(u_i)f(u_j)}\right\rceil \geq 2+\left\lceil\sqrt{2.3}\right\rceil \geq 5$$

Subcase (iv): Compare the pair (u_i, w_j)

$$d(u_i,w_j)+\left\lceil\sqrt{f(u_i)f(w_j)}\right\rceil \geq 3+\left\lceil\sqrt{1.2}\right\rceil \geq 5$$

Subcase (v): Compare the pair (w_i, w_j)

$$d(w_i,w_j)+\left\lceil\sqrt{f(w_i)f(w_j)}\right\rceil \geq 2+\left\lceil\sqrt{2.3}\right\rceil \geq 5$$

Hence the radio geometric mean number for lotus inside a circle graph is $r_{gmn}(LC_n) = 2n+1$, $n \geq 3$. □

Example 2.4. Figure 6 shows the radio geometric mean number of LC_8.

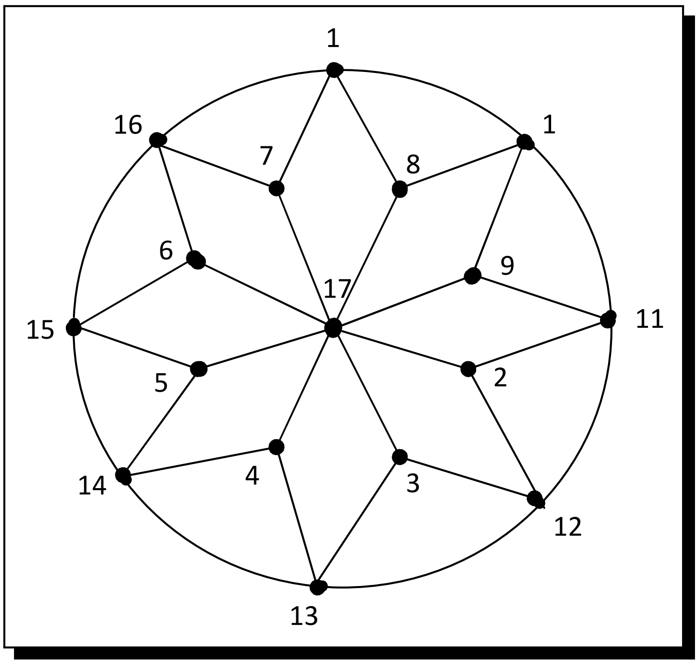

Figure 6

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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