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Research Article

Fuzzy Idealization on *gb*-Continuity

V. Chitra, V. Indhumathi and C. Selvapriya*

Department of Mathematics, NGM College, Pollachi 642001, India
***Corresponding author:** priyamath1729@gmaila.com

Abstract. In this work, we introduce the notions of *fgb*-I-closed sets, *fgb*-I-continuous, *fgb*-I-irresolute, *fgb*-I-open functions and *fgb**-I-open functions in fuzzy ideal topological spaces and investigate some of their characterizations and properties.

Keywords. *fgb*-I-closed sets; *fgb*-I-neighbourhood; *fgbq*-I-neighbourhood; *fgb*-I-continuous functions; *fgb*-I-irresolute functions; *fgb*-I-closed functions; *fgb**-I-open functions; fuzzy gb-I-$T_{1/2}$-space

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1. Introduction

Fuzziness is one of the important and useful concepts in the modern scientific studies. Zadeh [16] introduced the notion of fuzzy sets. Throughout, the development of fuzzy set theory, many interesting phenomena have been observed. Fuzzy topology was introduced by Chang [4]. An alternate definition of fuzzy topology was given by Lowen [7]. Vaidyanathaswamy [14] studied the topic of ideal topological spaces intensively. Mahmoud [9] and Sarkar [13] introduced the fuzzy ideal topology. The concept of *fgb*-closed sets and *fgb*-continuous are introduced by Benchalli and Karnel [3]. In this paper, we introduced *fgb*-I-closed sets, *fgb*-I-continuous, *fgb*-I-irresolute, *fgb*-I-closed functions, *fgb*-I-open functions and *fgb**-I-open functions in fuzzy ideal topological spaces.

2. Preliminaries

Let X be a non-empty set. A family τ of fuzzy sets of X is called a fuzzy topology [4] on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to τ and τ is closed with respect to any union and finite intersection. If τ is a fuzzy topology on X, then the pair (X,τ) is called a fuzzy topological space. A fuzzy point in X with support $x \in X$ and value α $(0 \le \alpha \le 1)$ is denoted by x_α. For a fuzzy set A in X, $Cl(A)$, $Int(A)$ and $1-A$ denote the closure, interior and complement of A. A fuzzy set A in (X,τ) is said to be quasi-coincident [12] with a fuzzy set B, denoted by $A\,\mathrm{q}B$, if there exists a point $x \in X$ such that $A(x)+B(x) > 1$ [5]. A fuzzy set B in (X,τ) is called a q-neighbourhood [12] of a fuzzy point x_β if there exists a fuzzy open set U of X such that $x\beta\,\mathrm{q}U \le V$ [5]. A nonempty collection of fuzzy sets I of a set X is called a fuzzy ideal [6, 11], if

(1) $A \in I$ and $A \le B$, then $A \vee B \in I$.

(2) $A \in I$ and $B \in I$, then $A \vee B \in I$.

The triplet (X,τ,I) is called fuzzy ideal topological space with a fuzzy ideal I and a fuzzy topology τ.

For (X,τ,I), the fuzzy local function of $A \le X$ with respect to τ and I denoted by $A^*(\tau,I)$ (briefly A^*) and is defined $A^* = \vee\{x \in X : A \wedge U \not\in I$ for every $U \in \tau\}$. While A^* is the union of the fuzzy points such that if $U \in \tau$ and $E \in I$, then there is atleast one $y \in X$ for which $U(y)+A(y)-1 > E(y)$ [8]. Fuzzy closure operator of a fuzzy set A in (X,τ,I) is defined as $Cl^*(A) = A \vee A^*$. A subset A in (X,τ,I) is called fuzzy ideal open [10] if $A \le Int(A^*)$.

Definition 2.1 ([2])**.** A fuzzy set A in a fuzzy topological space X is called

(i) fuzzy b-open set if $A \le (IntCl(A) \vee ClInt(A))$.

(ii) fuzzy b-closed set if $(IntCl(A) \vee \wedge Int(A)) \le A$.

Definition 2.2 ([2])**.** Fuzzy b-closure and fuzzy b-interior of a fuzzy set A is given by

(i) $bCl(A) = \wedge\{B : B$ is a fuzzy b-closed set of X and $B \ge A\}$.

(ii) $bInt(A) = \vee\{C : C$ is a fuzzy b-open set of X and $A \ge C\}$.

Definition 2.3. A fuzzy set A in a fuzzy topological space (X,τ) is called

(i) a generalized closed(briefly g-closed) fuzzy set [1] if $Cl(A) \le B$ whenever $A \le B$ and B is fuzzy open set in (X,τ).

(ii) a fuzzy generalized b-closed(briefly *fgb*-closed) set [2] if $bCl(A) \le B$ whenever $A \le B$ and B is fuzzy open in (X,τ).

Definition 2.4 ([3]). A fuzzy topological space (X,τ) is called fuzzy gb-$T_{1/2}$-space (briefly *fgb*-$T_{1/2}$-space) if every *fgb*-closed set in X is fuzzy b-closed in X.

Definition 2.5 ([1]). Let X, Y be two fuzzy topological spaces. A function $f:(X,\tau)\to(Y,\sigma)$ is called a *fg*-continuous if $f^{-1}(A)$ is g-closed fuzzy set in X, for every closed fuzzy set A of Y.

Definition 2.6 ([3]). A function $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy generalized b-continuous (briefly *fgb*-continuous) if $f^{-1}(A)$ is *fgb*-closed set in X, for every fuzzy closed set A in Y.

Definition 2.7 ([15]). A subset A of a fuzzy ideal topological space (X,τ,I) is said to be fuzzy b-I-open set if $A\le Cl^*(Int(A))\vee Int(Cl^*(A)$.

Definition 2.8 ([11]). A function $f:(X,\tau,I)\to(Y,\sigma)$ is called fuzzy-I-continuous if $f^{-1}(V)$ is fuzzy I-open set for each V in Y is fuzzy open.

Definition 2.9 ([15]). A function $f:(X,\tau,I)\to(Y,\sigma)$ is called fuzzy b-I-continuous if the inverse image of each fuzzy open set in Y is fuzzy b-I-open in (X,τ,I).

Remark 2.1 ([2]). Every fuzzy closed set in a fuzzy topological space (X,τ) is fuzzy b-closed.

Theorem 2.1 ([15]). *In a fuzzy ideal topological space (X,τ,I), the following statements are holds:*

(a) *every fuzzy closed set is fb-I-closed set.*

(b) *every fuzzy I-closed set is fb-I-closed set.*

(c) *every fuzzy b-I-closed set is fb-closed set.*

3. Fuzzy *gb*-I-Closed Sets

Definition 3.1. A subset A of a fuzzy ideal topological space (X,τ,I) is said to be fuzzy generalized b-I-closed (briefly *fgb*-I-closed) if $bCl(A)\le B$, whenever $A\le B$ and B is a fuzzy I-open in (X,τ,I).

Remark 3.1. A subset A of a fuzzy ideal topological space (X,τ,I) is called fuzzy generalized b-I-open (*fgb*-I-open) if its complement $1-A$ is *fgb*-I-closed.

Theorem 3.1. *Every fuzzy closed set in (X,τ,I) is fgb-I-closed.*

Proof. Assume A is a f-closed set in X. Let $A\le B$, B is fuzzy I-open set in X. By Remark 2.1, A is *fb*-closed, $Cl(A)=bCl(A)=A\le B$. Thus $bCl(A)\le B$. Hence A is fgb-I-closed. □

Theorem 3.2. *Every fuzzy I-closed set in (X,τ,I) is fgb-I-closed.*

Proof. Assume A is a fuzzy I-closed set in X. Let $A\le B$, B is a fuzzy I-open set in X. By Theorem 2.1 every fuzzy I-closed is *fb*-I-closed. Thus A is *fb*-I-closed. Again by Theorem 2.1, every *fb*-I-closed set is *fb*-closed, $Cl(A) = bCl(A) = A \leq B$. Thus $bCl(A) \leq B$. Hence A is *fgb*-I-closed. □

Theorem 3.3. *Every fb-closed set in fuzzy ideal topological space* (X,τ,I) *is fgb-*I*-closed.*

Proof. Assume A is *fb*-closed set in X. Let $A \leq B$, B is fuzzy I-open set in X. Since A is *fb*-closed, we have $bCl(A) = A \leq B$. Therefore $bCl(A) \leq B$. Hence A is *fgb*-closed. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.1. Let $X = \{a,b\}$ and A, B, C be the fuzzy sets defined as follows:

$A(a) = 0.3,\ A(b) = 0.4,$

$B(a) = 0.4,\ B(b) = 0.5,$

$C(a) = 0.3,\ C(b) = 0.7.$

Let $\tau = \{0,1,A\}$ be the topology on X and $I = \{0\}$ be the fuzzy ideal on X. Then C is *fgb*-I-closed but not *fb*-closed in X.

Theorem 3.4. *A subset* A *of a fuzzy ideal topological space* (X,τ,I) *is called fgb-*I*-open if and only if* $B \leq bInt(A)$*, whenever* B *is fuzzy* I*-closed set and* $B \leq A$.

Proof. Suppose A is a *fgb*-I-open set in X. Then $1-A$ is *fgb*-I-closed in X. Let B be a fuzzy I-closed set in X such that $B \leq A$. Then $1-A \leq 1-B$, $1-B$ is fuzzy I-open set in X.
Since $1-A$ is *fgb*-I-closed, we have $bCl(1-A) \leq 1-B$, implies $1-bInt(A) \leq 1-B$. Therefore, $B \leq bInt(A)$.
Conversely, assume that $B \leq bInt(A)$, whenever $B \leq A$ and B is fuzzy I-closed set in X. Then $1-bInt(A) \leq 1-B = C$, C is fuzzy I-open set in (X,τ,I). Thus $bCl(1-A) \leq C$, implies $1-A$ is *fgb*-I-closed. Hence A is *fgb*-I-open. □

Theorem 3.5. *If* A *is a fgb-*I*-closed set in* (X,τ,I) *and* $A \leq B \leq bCl(A)$*, then* B *is a fgb-*I*-closed set in* (X,τ,I).

Proof. Let C be fuzzy I-open in X such that $B \leq C$. Suppose $A \leq B$, then $A \leq C$. Since A is a *fgb*-I-closed set in X, it follows that $bCl(A) \leq C$. Now, $B \leq bCl(A)$ implies $bCl(B) \leq bCl(bCl(A)) = bCl(A)$. Thus $bCl(B) \leq C$. Hence B is a *fgb*-I-closed set in X. □

Theorem 3.6. *If* A *is a fgb-*I*-open set in fuzzy ideal topological space* (X,τ,I) *and* $bInt(A) \leq B \leq A$ *then* B *is fgb-*I*-open in* (X,τ,I).

Proof. Let A be a *fgb*-open set and B be any fuzzy set in X, such that $blnt(A) \leq B \leq A$.
Then $1-A$ is *fgb*-I-closed in X and $1-A \leq 1-B \leq bCl(1-A)$. Then from Theorem 3.5, $1-B$ is a *fgb*-I-closed set in X. Hence B is a *fgb*-I-open set in X. □

3.1 Fuzzy Generalized *b-I*-Neighbourhood and Fuzzy Generalized *bq-I*-Neighbourhood

Definition 3.2. Let A be a subset of a fuzzy ideal topological space (X,τ,I) and x_p be a fuzzy point in (X,τ,I), then A is called fuzzy generalized b-I-neighbourhood (briefly *fgb*-I-neighbourhood) of x_p if and only if there exists a *fgb*-I-open set B in (X,τ,I) such that $x_p \in B \leq A$.

Definition 3.3. Let A be a subset of a fuzzy ideal topological space (X,τ,I) and x_p be a fuzzy point in (X,τ,I), then A is called fuzzy generalized bq-I-neighbourhood (briefly *fgbq*-I-neighbourhood) of x_p if and only if there exists a *fgb*-I-open set B such that $x_p \, \mathrm{q} B \leq A$.

Theorem 3.7. *A is fgb-I-open set in (X,τ,I) if and only if for each fuzzy point $x_p \in A$, A is a fgb-I-neighbourhood of x_p.*

Proof. Let A be a *fgb*-I-open set in X. For each $x_p \in A$, $A \leq A$. Therefore A is a *fgb*-I-neighbourhood of x_p. Conversely, let A be a *fgb*-I-neighbourhood of x_p. Then there exists a *fgb*-I-open set B in $X,\tau,I)$ such that $x_p \in B \leq A$. Hence A is *fgb*-I-open set in (X,τ,I). □

Theorem 3.8. *If A and B are fgb-I-neighbourhoods of x_p then $A \wedge B$ is also a fgb-I-neighbourhood of x_p.*

Proof. Let A and B be *fgb*-I-neighbourhoods of x_p. Then there exist *fgb*-I-open sets L and M such that $x_p \in L \wedge M \leq A \wedge B$. Thus $A \wedge B$ is also a *fgb*-I-neighbourhood of x_p. □

4. Fuzzy *gb-I*-Continuous Functions

Definition 4.1. A function $f:(X,\tau,I) \to (Y,\sigma)$ is said to be fuzzy generalized b-I-continuous (briefly *fgb*-I-continuous), if $f^{-1}(A)$ is *fgb*-I-closed set in (X,τ,I), for every fuzzy closed set A in Y.

Theorem 4.1. *$f:(X,\tau,I) \to (Y,\tau)$ is fgb-I-continuous if and only if the inverse image of each fuzzy open set of (Y,σ) is fgb-I-open set of (X,τ,I).*

Proof. Suppose B is a f-open set of Y. Then $1-B$ is f-closed in Y. Since $f:X \to Y$ is *fgb*-I-continuous $f^{-1}(1-B) = 1-f^{-1}(B)$ is *fgb*-I-closed set of X. Hence $f^{-1}(B)$ is a *fgb*-I-open set of X.

Conversely, let A be a fuzzy closed set in Y. Then $1-A$ is f-open in Y. By hypothesis, inverse image of $1-A$ in Y is a *fgb*-I-open set in X. Thus $f^{-1}(1-A) = 1-f^{-1}(A)$ is a *fgb*-I-open set in X. Therefore $f^{-1}(A)$ is a *fgb*-I-closed set in X. Hence f is *fgb*-I-continuous. □

Theorem 4.2. *If $f:(X,\tau,I) \to (Y,\sigma)$ is fgb-I-continuous then*

(a) *for each fuzzy point x_p of X and each $A \in Y$ such that $f(x_p) \mathrm{q} A$, there exists a fgb-I-open set A of X such that $x_p \in B$ and $f(B) \leq A$.*

(b) *for each fuzzy point x_p of X and each $A \in Y$ such that $f(x_p)\mathrm{q}A$, there exists a fgb-I-open set B of X such that $x_p \mathrm{q} B$ and $f(B) \le A$.*

Proof. (a) Let x_p be a fuzzy point of X. Then $f(x_p)$ be a fuzzy point in Y. Also, let $A \in Y$ be a *fgb*-open set such that $f(x_p)qA$. Take $B = f^{-1}(A)$. Since $f : X \to Y$ is *fgb-I*-continuous, B is a *fgb-I*-open set of X and $x_p \in B$, it follows that $f(B) = f(f^{-1}(A)) \le A$.

(b) Let x_p be a fuzzy point of X and $A \in Y$ be such that $f(x_p)\mathrm{q}A$. Take $B = f^{-1}(A)$. Since $f : X \to Y$ is *fgb-I*-continuous and there exists a *fgb-I*-open set B of X such that $x_p \mathrm{q} B$ and $f(B) = f(f^{-1}(A)) \le A$. □

Theorem 4.3. *Every fuzzy-I-continuous function is fgb-I-continuous function.*

Proof. Suppose $f : (X,\tau,I) \to (Y,\sigma)$ is a fuzzy-I-continuous function. Assume A is a fuzzy open set in Y. Since f is fuzzy-I-continuous, $f^{-1}(A)$ is fuzzy I-open in X. Therefore, by Theorem 3.2, $f^{-1}(A)$ is a *fgb-I*-open set in X. Thus f is *fgb-I*-continuous. □

The converse of the above theorem need not be true as seen from the following example.

Example 4.1. Let $X = Y = \{a,b\}$ and the fuzzy sets A and B be defined as follows:

$$A(a) = 0.9,\ A(b) = 0.8, \quad B(a) = 0.3,\ B(b) = 0.4.$$

Let $\tau = \{0,1,A\}$ and $\sigma = \{0,1,B\}$ be the topologies on X and Y respectively and $I = \{0\}$ be the fuzzy ideal on X.

Let $f : X \to Y$ be the identity function. Then f is *fgb-I*-continuous but not fuzzy-I-continuous.

Definition 4.2. A function $f : (X,\tau,I) \to (Y,\sigma,I)$ is said to be fuzzy *gb-I*-irresolute (briefly *fgb-I*-irresolute), if $f^{-1}(A)$ is *fgb-I*-closed set in (X,τ,I), for every *fgb-I*-closed set A in (Y,σ,I).

Theorem 4.4. *A function $f : (X,\tau,I) \to (Y,\sigma,I)$ is fgb-I-irresolute if and only if the inverse image of every fgb-I-open set in Y is fgb-I-open set in X.*

Proof. Let A be a *fgb-I*-open set in Y. Then $1 - A$ is *fgb-I*-closed in Y. Since $f : X \to Y$ is *fgb-I*-irresolute, $f^{-1}(1 - A) = 1 - f^{-1}(A)$ is a *fgb-I*-closed set in X. Thus $f^{-1}(A)$ is a *fgb-I*-open set in X.

Conversely, assume that A is a *fgb-I*-closed set in Y. Then $1-A$ is *fgb-I*-open in Y. By hypothesis, $f^{-1}(1 - A) = 1 - f^{-1}(A)$ is a *fgb-I*-open set in X. Thus $f^{-1}(A)$ is a *fgb-I*-closed set in X. Hence f is *fgb-I*-irresolute. □

Theorem 4.5. *Every fgb-I-irresolute function is fgb-I-continuous.*

Proof. Suppose $f : (X,\tau I) \to (Y,\sigma,I)$ is *fgb-I*-irresolute and let F be a fuzzy I-closed set in Y. Then F is a *fgb-I*-closed set in Y, by Theorem 3.2. Since f is *fgb-I*-irresolute, $f^{-1}(F)$ is a *fgb-I*-closed set in X. Thus f is *fgb-I*-continuous. □

The converse of the above theorem need not be true as seen from the following example.

Example 4.2. Let $X = Y = \{a,b\}$, and the fuzzy sets A, B, C, D and E be defined as follows:

$$A(a) = 0.9,\ A(b) = 0.9,\quad B(a) = 0.8,\ B(b) = 0.5,\quad C(a) = 0.6,\ C(b) = 0.5,$$
$$D(a) = 0.5,\ D(b) = 0.2,\quad E(a) = 0.5,\ E(b) = 0.6.$$

Let $\tau = \{0,1,A,B,C,D\}$ and $\sigma = \{0,1,C\}$ be the topologies on X and Y respectively and $I = \{0\}$ be the fuzzy ideal on X and Y respectively. Define $f : X \to Y$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is *fgb-I*-continuous but not *fgb-I*-irresolute.

Theorem 4.6. *Let $f : (X,\tau,I) \to (Y,\sigma,I)$ and $g : (Y,\sigma,I) \to (Z,\gamma,I)$ be two functions. Then*

(1) *$g \circ f : (X,\tau,I) \to (Z,\gamma,I)$ is fgb-I-continuous, if f is fgb-I-continuous and g is f-I-continuous.*

(2) *$g \circ f : (X,\tau,I) \to (Z,\gamma,I)$ is fgb-I-irresolute, if f is fgb-I-irresolute and g is fgb-I-irresolute function.*

(3) *$g \circ f : (X,\tau,I) \to (Z,\gamma,I)$ is fgb-Icontinuous if, f is fgb-I-irresolute and g is fgb-I-continuous.*

Proof. (1) Suppose B is a fuzzy I-closed set of Z. Since $g : (Y,\sigma,I) \to (Z,\gamma,I)$ is fuzzy-I-continuous, $g^{-1}(B)$ is a fuzzy I-closed set of Y. Also since $f : (X,\tau,I) \to (Y,\sigma I)$ is *fgb-I*-continuous and $g^{-1}(B)$ is a fuzzy I-closed set of Y; then from Definition 4.1, $f^{-1}(g^{-1}(B)) = (g\circ, f)^{-1}(B)$ is a *fgb-I*-closed set in X. Thus $g \circ f : (X,\tau,I) \to (Z,\gamma,I)$ is *fgb-I*-continuous.

(2) Let $g : (Y,\sigma,I) \to (Z,\gamma I)$ be *fgb-I*-irresolute and B be a *fgb-I*-closed subset of Z. Since g is *fgb-I*-irresolute, $g^{-1}(B)$ is a *fgb-I*-closed set of Y. Also since $f : (X,\tau,I) \to (Y,\sigma,I)$ is *fgb-I*-irresolute, we have $f^{-1}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is a *fgb-I*-closed set X. Hence $g \circ f : (X,\tau,I) \to (Z,\gamma,I)$ is *fgb-I*-irresolute.

(3) Let B be fuzzy b-I-closed subset of Z. Since $g : (Y,\sigma,I) \to (Z,\gamma,I)$ is *fgb-I*-continuous, $g^{-1}(B)$ is a *fgb-I*-closed set of Y. Also $f : (X,\tau,I) \to (Y,\sigma I)$ is *fgb-I*-irresolute, $g^{-1}(B)$ is *fgb-I*-closed in X. Therefore $f^{-}(g^{-1}(B)) = (g \circ f)^{-1}(B)$ is a *fgb-I*-closed set of X. Hence $g \circ f : (X,\tau,I) \to (Z,\gamma,I)$ is *fgb-I*-continuous. □

Definition 4.3. A fuzzy ideal topological space (X,τ,I) is fuzzy *gb-I*-$T_{1/2}$-space (briefly *fgb-I*-$T_{1/2}$-space) if every *fgb-I*-closed set in X is fuzzy b-I-closed in X.

Theorem 4.7. *A fuzzy ideal topological space (X,τ,I) is* fgb-I-$T_{1/2}$*-space if and only if every fuzzy set in (X,τ,I) is both fuzzy b-I-open and fgb-I-open.*

Proof. Suppose X is *fgb-I*-$T_{1/2}$-space and A is a *fgb-I*-open set in X. Then $1 - A$ is a *fgb-I*-closed set in X. Since X is *fgb-I*-$T_{1/2}$-space, every *fgb-I*-closed set in X is fuzzy b-I-closed in X.

Therefore $1 - A$ is a *fb-I*-closed set and hence A is a *fb-I*-open set in X.

Conversely, assume that A is a *fgb-I*-closed set in X. Then $1 - A$ is a *fgb-I*-open set. By hypothesis, $1-A$ is fuzzy b-I-open. Therefore A is fuzzy b-I-closed. Thus every *fgb*-I-closed set in X is fuzzy b-I-closed. Hence X is *fgb*-I-$T_{1/2}$-space. □

Definition 4.4. A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be *fb**-I-continuous if $f^{-1}(A)$ is fuzzy b-I-closed set in (X,τ,I), for every fuzzy b-closed set of Y.

Theorem 4.8. *If $f:(X,\tau,I)\to(Y,\sigma)$ is fb*-I-continuous and $g:(Y,\sigma)\to(Z,\gamma)$ is fgb-continuous then $g\circ f:(X,\tau,I)\to(Z,\sigma)$ is fb*-I-continuous if Y is fgb-$T_{1/2}$-space.*

Proof. Let B be a fuzzy b-closed set of Z. Since $g:(Y,\sigma)\to(Z,\gamma)$ is *fgb*-continuous, $g^{-1}(B)$ is a *fgb*-closed set of Y. Since Y is *fgb*-$T_{1/2}$-space, every *fgb*-closed set is fuzzy b-closed in Y. Thus $g^{-1}(B)$ is a fuzzy b-closed set of Y. Also since $f:(X,\tau,I)\to(Y,\sigma)$ is *fb**-I-continuous, $f^{-1}(g^{-1}(B))$ is a fuzzy b-I-closed set in X. Thus $f^{-1}(g^{-1}(B))=(g\circ f)^1(B)$ is a fuzzy b-I-closed set X. Hence $g\circ f:(X,\tau,I)\to(Z,\gamma)$ is *fb**-I-continuous. □

Theorem 4.9. *If (X,τ,I) is fgb-I-$T_{1/2}$-space and $f:(X,\tau,I)\to(Y,\sigma)$ is fgb-I-continuous, then f is fb-I-continuous.*

Proof. Let B be a fuzzy closed set in Y. Since $f:(X,\tau,I)\to(Y,\sigma)$ is *fgb*-I-continuous, $f^{-1}(B)$ is a *fgb*-I-closed set in X. Also X is *fgb*-I-$T_{1/2}$-space, every *fgb*-I-closed set is fuzzy b-I-closed. Thus $f^{-1}(B)$ is a fuzzy b-I-closed set in X. Hence $f:(X,\tau,I)\to(Y,\sigma)$ is *fb*-I-continuous. □

Definition 4.5. A function $f:(X,\tau,I)\to(Y,\sigma)$ is called *fb**-I-closed if $f(A)$ is fuzzy b-closed in Y for every fuzzy b-I-closed set A in X.

Theorem 4.10. *Let $f:(X,\tau,I)\to(Y,\sigma,I)$ be onto, fgb-I-irresolute and fb*-I-closed. If X is fgb-I-$T_{1/2}$-space, then Y is fgb-I-$T_{1/2}$-space.*

Proof. Let A be a *fgb*-I-closed set in Y. Since $f:(X,\tau,I)\to(Y,\sigma,I)$ is *fgb*-I-irresolute, $f^{-1}(A)$ is a *fgb*-I-closed set in X. Also since X is *fgb*-I-$T_{1/2}$-space, $f^{-1}(A)$ is fuzzy b-I-closed set in X. Again $f:(X,\tau,I)\to(Y,\sigma,I)$ is fb*-I-closed, $f(f^{-1}(A))$ is fuzzy b-I-closed in Y. Since f is onto, $f(f^{-1}(A))=A$. Thus A is fuzzy b-I-closed in Y. Hence Y is *fgb*-I-$T_{1/2}$-space. □

Definition 4.6. A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be fuzzy *gb*-I-open (briefly *fgb*-I-open) if the image of every fuzzy I-open set in X, is *fgb*-open set in Y.

Definition 4.7. A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be fuzzy *gb*-I-closed (briefly *fgb*-I-closed) if the image of every fuzzy-I-closed set in X is *fgb*-closed set in Y.

Definition 4.8. A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be fuzzy *gb**-I-open (briefly *fgb**-I-open) if the image of every *fgb*-I-open set in X, is *fgb*-open set in Y.

Definition 4.9. A function $f:(X,\tau,I)\to(Y,\sigma)$ is said to be fuzzy gb^*-I-closed (briefly *fgb**-I-closed) if the image every *fgb*-I-closed set in X is *fgb*-closed set in Y.

Remark 4.1. Every *fgb**-I-open function is *fgb*-I-open.

The converse of the above statement is not true as seen from the following example.

Example 4.3. Let $X=\{a,b\}$, $Y=\{x,y\}$ and the fuzzy sets A, B are defined as follows:

$$A(a)=0.9,\ A(b)=0.7,\quad B(x)=0.5,\ B(y)=0.4.$$

Let $\tau=\{0,1,A\}$, $\sigma=\{0,1,B\}$ be the topologies on X and Y respectively and $I=\{0\}$ be the fuzzy ideal on X. Then the function $f:(X,\tau,I)\to(Y,\sigma)$ defined by $f(a)=x$ and $f(b)=y$ is *fgb*-I-open but not *fgb**-I-open.

Theorem 4.11. *A function $f:(X,\tau,I)\to(Y,\sigma)$ is fgb-I-closed if and only if for each fuzzy set A of Y and for each fuzzy I-open set B such that $f^{-1}(A)\le B$, there is a fgb-open set C of Y such that $A\le C$ and $f^{-1}(C)\le B$.*

Proof. Suppose that f is *fgb*-I-closed. Let A be a fuzzy set of Y, and B be a fuzzy I-open set of X, such that $f^{-1}(A)\le B$. Then $C=1-f(1-B)$ is a *fgb*-open set in Y such that $A\le C$ and $f^{-1}(C)\le B$.
Conversely, suppose that F is a fuzzy I-closed set of X. Then $f^{-1}(1-f(F))\le 1-F$, and $1-F$ is a fuzzy I-open set. By hypothesis, there is a *fgb*-open set C of Y such that $1-f(F)\le C$ and $f^{-1}(C)\le 1-F$. Therefore $F\le 1-f^{-1}(C)$. Hence $1-C\le f(F)\le f(1-f^{-1}(F))\le 1-C$, which implies $f(F)=1-C$. Since $1-C$ is a *fgb*-closed set, $f(F)$ is a *fgb*-closed set and thus f is *fgb*-I-closed. □

Theorem 4.12. *If $f:(X,\tau,I)\to(Y,\sigma)$ is bijective, fuzzy-I-continuous and fgb-I-closed and A is fgb-I-closed set in (X,τ,I), then $f(A)$ is a fgb-closed set in (Y,σ).*

Proof. Let $f(A)\le B$, B is a fuzzy open set in Y. Since f is fuzzy-I-continuous, $f^{-1}(B)$ is an fuzzy I-open set containing A. Also A is *fgb*-I-closed set, we have $bCl(A)\le f^{-1}(B)$. Since f is *fgb*-I-closed, $f(bCl(A))$ is a *fgb*-closed set contained in the fuzzy open set B. It follows that, $bCl(f(bCl(A)))\le B$ and so $bCl(f(A))\le B$. Thus $f(A)$ is a *fgb*-closed set in Y. □

Theorem 4.13. *If $f:(X,\tau,I)\to(Y,\sigma)$ is fgb-I-closed and $g:(Y,\sigma)\to(Z,\gamma)$ is fgb*-closed, then $g\circ f$ is fgb*-I-closed.*

Proof. Let A be a fuzzy I-closed set in X. Then $f(A)$ is *fgb*-closed in Y as f is *fgb*-I-closed. Since $g:(Y,\sigma)\to(Z,\gamma)$ is *fgb**-closed, $g(f(A))$ is *fgb*-closed in Z. Thus $g(f(A))=(g\circ f)(A)$ is *fgb*-closed in Z. Therefore $g\circ f$ is *fgb**-I-closed. □

Theorem 4.14. *Let $f:(X,\tau,I)\to(Y,\sigma,I)$, $g:(Y,\sigma,I)\to(Z,\gamma,I)$ be two functions such that $g\circ f:(X,\tau,I)\to(Z,\gamma,I)$ is* fgb**-I-closed.*

(i) *If f is fgb-I-continuous and surjective, then g is fgb-I-closed.*

(ii) *If g is fgb-I-irresolute and injective, then f is fgb*-I-closed.*

Proof. (i) Let F be a fuzzy I-closed set of Y. Then $f^{-1}(F)$ is *fgb*-I-closed in X as f is *fgb*-I-continuous. Since $g \circ f$ is *fgb**-I-closed, $(g \circ f)(f^{-1}(F)) = g(F)$ is a *fgb*-I-closed set in Z.

Hence $g : (Y, \tau, I) \to (Z, \gamma, I)$ is *fgb*-I-closed.

(ii) Let F be a *fgb*-I-closed set in X. Then $(g \circ f)(F)$ is a *fgb*-I-closed set in Z. Since g is *fgb*-I-irresolute and injective, $g^{-1}(g \circ f)(F) = f(F)$ is *fgb*-I-closed in Y. Hence f is *fgb**-I-closed. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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