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Research Article

New Q-β-Decay Theory Applied to the Calculations of the Rest Mass Energy m_0c^2 of the Electron and of the Q-Value for β^+-Decay Transitions in Mirrors Nuclei $A = 2Z - 1$

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Abstract. A new nuclear approach named Q-β-decay theory (QβT) is presented. This method is applied to express theoretically the β^+-Q-value for mirrors nuclei $A = 2Z - 1$. An important parameter named nuclear charge distribution coefficient (NCDC) noted $\alpha(Z)$ is presented. In the framework of the liquid drop nuclear model where the nuclear charge is uniformly distributed in the nuclear volume, $\alpha(Z) = 3/5$. It this work, it is demonstrated that the protons are not rigorously uniformly distributed within the nucleus. A slight correction is obtained with $\alpha(Z) \approx \alpha_0 = 3/5 + 0.0557$. For ^{37}K and ^{65}As, we find for the electron rest mass energy $m_0c^2 = 0.510\,996$ MeV agreeing excellently with the recommended value 0.510 998 950 MeV (CODATA, 2022). In addition, the Q-value calculated for A ranging between 11 and 99 agree very well with the recent Atomic Mass Evaluation – AME2020 (Wang *et al*. [16]). New accurate Q-values are tabulated for nuclei mirrors with A ranging between 101 and 199. The present Q-β-DT make it possible to understand many nuclear properties and phenomena depending on the Q-value such as comparison between experimental and theoretical predictions of atomic masses, the understanding of the weak force and of the competitive processes between electron capture and β^+-decay.

Keywords. Q-β-decay theory, Liquid drop model, Mirrors nuclei, Q-value, β^+-decay, Nuclear charge distribution coefficient, Electron rest mass energy

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1. Introduction

Beta decay is a nuclear transition, where the atomic number Z of the parent nucleus changes by one unit, while atomic mass A remains the same. Two most fundamental properties of a nuclear β-decay transitions are the lifetime of the parent nuclide and the Q-value of the decay process [11, 13]. Beta nuclear transitions result in three possible modes: β^- (e^--electron emission), β^+ (e^+-position emission) and EC (electron capture). The respective decay transitions between parent, ${}^AX$ (of atomic mass M_X), and daughter, ${}^AY$ (of atomic mass M_Y), isobaric nuclides are the following:

$$\beta^-\text{-decay: } {}^A_ZX \to {}^A_{Z+1}Y + e^- + \overline{\nu}_e. \tag{1.1}$$

$$\beta^+\text{-decay: } {}^A_ZX \to {}^A_{Z-1}Y + e^+ + \nu_e. \tag{1.2}$$

$$EC\text{: } {}^A_ZX + e^- \to {}^A_{Z-1}Y + \nu_e. \tag{1.3}$$

Furthermore, the standard model describing the interaction between elementary particles is based on the four fundamental interactions: strong, electromagnetic, weak and gravitational. In this study, we focus on the weak interaction carried by the vector bosons W^+ and W^- permitting to explain the emission of neutral and uncharged particles called neutrinos and antineutrinos. These bosons have the same mass equal to $(80800 \pm 2700)\,\text{MeV}/c^2$ along with electric charges equal to $+e$ and $-e$, respectively. The neutrino and the antineutrino accompany the β-radioactive emission. Via the weak interaction, a nucleon in the nucleus can transform into another species. To compensate for the change in charge, an electron or positron is then expelled from the nucleus. The emission of the electron is accompanied by an electronic antineutrino $\overline{\nu}_e$ and that of the positron by an electronic neutrino ν_e. The β^- and β^+-decay processes are equivalent to transformations within the nucleus of a neutron into a proton and of a proton into a neutron following the following nuclear processes [13].

$$\begin{cases} n \to p + e^- + \overline{\nu}_e \\ p \to n + e^+ + \nu_e \end{cases} \tag{1.4}$$

In the framework of the quark model, the neutron and the proton have respectively quark structures udd and udu. The electric charges of the u (up) and d (down) quarks are respectively equal to $+2/3e$ and $-1/3e$. By weak interaction, a d quark of the neutron transforms into a u quark by emitting a boson W^-. The boson W^- subsequently decays into an electron and an antineutrino. These two processes of transformation of the d quark and decay of the boson W^- are described below [13],

$$\begin{cases} d \to u + W^- \\ W^- \to e^- + \overline{\nu}_e \end{cases} \tag{1.5}$$

The overall process of transforming a neutron into a proton via the weak interaction process is presented in Figure 1.

Likewise, by weak interaction, a u quark of the proton transforms into a d quark by emitting a boson W^+. The boson W^+ then decays into a positron and a neutrino as shown in Figure 2.

On the other hand, the importance of Q-value measurements is due mainly for the following reasons. In planning the discovery and study of new isotopes, positioned at the neutron-rich or neutron-deficient side of the valley of stability or at its high-Z end, the experimentalist needs an estimate of the properties of their radioactive decay which crucially depend on the Q-value [12].

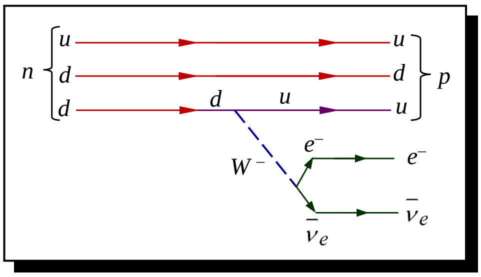

Figure 1. Neutron decay by weak interaction

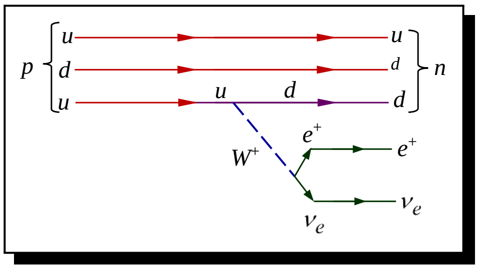

Figure 2. Proton decay by weak interaction

In addition, comparison between experimental and theoretical predictions of atomic masses is generally based on comparison of mass differences depending on the Q-value. Furthermore, the Q-value make it possible to the understand the weak force; the processes describes in Figures 1 and 2 are possible if $Q_{\beta^-} = (m_X - m_Y)c^2 > 0$ (condition of β^--emission) for the β^--decay process and $Q_{\beta^+} = (m_X - m_Y)c^2 - 2m_0c^2 > 0$ that means $(m_X - m_Y)c^2 > 2m_0c^2$ (condition of β^+-emission). Furthermore, if the Q-value is larger than $2m_0c^2$, both EC and β^+-decay leading to the same daughter nucleus are competitive processes. For smaller Q-values only electron capture will occur. Besides, as shown in Figures 1 and 2, β-decay involves neutrinos emission. Postulated in 1930, the existence of the neutrino was not proven experimentally. In 1956, Cowan and Reins carried out the historic experiment that leads to the discovery of the neutrino considered as a non-massless particle [13]. But with the discovery of neutrino oscillation confirmed that neutrinos have mass [12] and motivated physicists to postulate new properties of neutrinos [9], which have created a possible connection between the observed asymmetry of matter over antimatter in our universe [9,10]. This indicates that neutrinos play a key role in understanding the universe [6]. Besides, on the experimental side, the Q-decay values can be measured by studying either the radioactive decay or the inverse nuclear reaction [12] or using Penning trap mass spectrometry [1–3, 5, 11]. On the other hand, the mean experiment performed in the measurement of the electron rest mass energy m_0c^2 is based on the widely used Compton scattering of γ-photon by crystals [7,8]. In this work, we present a new nuclear method for the calculation of the rest mass energy of the electron m_0c^2 in the framework of the Q-β-decay theory (Q-β-T). The Q-β-T is based on the measurement of the Q-value of the β^+ spectrum of the mirrors nuclei $A = 2Z - 1$. Estimation of the electron rest mass energy is aimed intending to compare the present Q-β-T to the widely used Compton scattering of γ-photon by crystals. In addition, we intend also to express theoretically the β^+-Q-value for mirrors nuclei $A = 2Z - 1$ for interpreting the nuclear charge distribution with the nucleus in the framework of the liquid drop nuclear. In Section 2, we give a brief description of the theoretical part of the work. In Section 3 with present and discuss the results obtained and conclude in Section 4.

2. Theory

2.1 Expression of the Maximum Energy of the β^+ Spectrum of the Mirror Nuclei $A = 2Z - 1$

Interpretation of the measurement of maximum energy of β^+ spectrum $e_{\max}$ of mirrors nucleus for which $A = 2Z - 1$ is done in the framework of classical theory [4,14]. This is justified by the simple following argument. Expression of $e_{\max}$ is deduced from the classical Coulomb energy of the nucleus assimilated to a sphere of R radius and of uniform Q charge distribution in the framework of the liquid drop nuclear. Then, theoretical interpretation of $e_{\max}$ can be done in the framework of the present study. In this purpose, we express first $e_{\max}$. The β radioactivity study permits to detect a certain number of nuclei for which the binding energies of the final and initial nucleus are approximately equal. Such nuclei are called "mirrors nucleus". For these nuclei, the mass number A and the charge number Z of the initial mirrors nucleus satisfy the relation $A = 2Z - 1$. In the particular case of β^+ radioactivity, the well know disintegration equation is

$$^{Z}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e^+ + \nu + \gamma. \tag{2.1}$$

Taking into account the ΔW_{coul} Coulombian energy loss between the isobaric pair $({}^{A}_{Z}X, {}^{A}_{Z-1}Y)$, eq. (2.1) becomes

$$^{Z}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e^+ + \nu + \Delta W_{\text{coul}}. \tag{2.2}$$

Using the energy conservation principal, we get

$$M_X c^2 = M_Y c^2 + m_0 c^2 + Ec_b(Y) + Ec(\beta^+) + E(\nu) + \Delta W_{\text{coul}}. \tag{2.3}$$

In this expression, M_X, M_Y, and m_0 represent respectively the masses of the nucleus ${}^{A}_{Z}X$, ${}^{A}_{Z-1}Y$ and of $\beta^+({}^{0}_{1}e^+)$. The constant c denotes the velocity of light, Ec_b the backward movement kinetic energy of ${}^{A}_{Z-1}Y$, $Ec(\beta^+)$ the kinetic energy of β^+ particle and $e(\nu)$ the neutrino energy. By definition, the maximum energy of β^+ decay is

$$E_{\max} = Ec_b(Y) + Ec(\beta^+) + E(\nu). \tag{2.4}$$

Using eq. (2.3), we get

$$E_{\max} = (M_X - M_Y)c^2 - m_0c^2 - \Delta W_{\text{coul}}. \tag{2.5}$$

On the other hand, let us express the difference $(M_X - M_Y)c^2$ in terms of the binding energies of the nucleus ${}^{A}_{Z}X$ and ${}^{A}_{Z-1}Y$.

- for the initial nucleus ${}^{A}_{Z}X$:

$$B(X) = [Zm_p + (A - Z)m_n - M_X]c^2. \tag{2.6a}$$

- for the final nucleus ${}^{A}_{Z-1}Y$:

$$B(Y) = [(Z - 1)m_p + (A - Z + 1)m_n - M_Y]c^2. \tag{2.6b}$$

In eqs. (2.6a) and (2.6b), m_p and m_n are respectively the masses of the proton and the neutron. Using eqs. (2.6a) and (2.6b), the binding energy difference $\Delta B = B(Y) - B(X)$ is equal to

$$\Delta B = [(M_X - M_Y) - (m_n - m_p)]c^2. \tag{2.7}$$

Then, the difference $(M_X - M_Y)c^2$ can be written in the form

$$(M_X - M_Y)c^2 = \Delta B - (m_n - m_p)c^2. \tag{2.8}$$

Using eq. (2.8), eq. (2.5) becomes

$$E_{\max} = \Delta B - (m_n - m_p)c^2 - m_0c^2 - \Delta W_{\text{coul}}. \tag{2.9}$$

Besides, as the nuclear forces are symmetric between protons and neutrons, the binding energy difference $\Delta B = 0$. So, Eq.(2.9) gives

$$E_{\max} = -(m_n - m_p)c^2 - m_0c^2 - \Delta W_{\text{coul}}. \tag{2.10}$$

In the Framework of the liquid drop nuclear $R = r_0 A^{1/3}$, the Coulomb energy of the charge nucleus Ze distributed inside the nuclear volume is given by the relation (see Appendix),

$$W_{\text{coul}} = \frac{3}{5}\frac{e^2}{R}Z^2. \tag{2.11}$$

In relation (2.11), the factor 3/5 justifies the uniform volume distribution of protons in the nucleus. This factor is equal to 1/2 in the case of a surface distribution of the nuclear charge [4, 14]. To formulate our Q-β-decay theory (Q-βT), we consider that during its emission, the positive charge of the β^+-particle is repelled by the nucleus. Coulomb repulsions thus occur near the emitting nucleus. The β^+-particle necessarily disturbing the distribution of protons in the nucleus during its emission, we hypothesize that the loss of Coulomb energy during the isobaric transition is reduced by the mass energy m_0c^2 of the positron β^+. Not knowing the real distribution of the nuclear charge in the nucleus, we introduce a nuclear distribution coefficient of protons within the nucleus which we denote $\alpha(Z)$. The Coulomb energy (2.11) is modified as follows;

$$W^{\alpha}_{\text{coul}} = \alpha(Z)\frac{e^2}{R}Z^2. \tag{2.12}$$

In the case where all the protons are distributed uniformly in the nuclear volume (absence of protons distributed on the surface of the nucleus), $\alpha(Z) = \alpha_0 = 3/5$.

Let us now express the coulomb energy loss ΔW_{coul}. We get:

- For ${}^{A}_{Z}X$ $(q = Ze)$,

$$W^{\alpha}_{\text{coul}}(X) = \alpha(Z)\frac{e^2}{R}Z^2. \tag{2.13a}$$

- For ${}^{A}_{Z-1}Y$ $[q = (Z-1)e]$,

$$W^{\alpha}_{\text{coul}}(Y) = \alpha(Z)\frac{e^2}{R}(Z-1)^2 + m_0c^2. \tag{2.13b}$$

Using (2.13a) and (2.13b), we find

$$\Delta W_{\text{coul}} = W_{\text{coul}}(X) - W_{\text{coul}}(Y),$$

$$\Delta W^{\alpha}_{\text{coul}} = \alpha(Z)\frac{e^2}{R}[Z^2 - (Z-1)^2] - m_0c^2.$$

We get finally

$$\Delta W^{\alpha}_{\text{coul}} = \alpha(Z)\frac{e^2}{R}(2Z-1) - m_0c^2. \tag{2.14}$$

Using eq. (2.14), eq. (2.10) becomes

$$E^{\alpha}_{\max} = \alpha(Z)\frac{e^2}{R}(2Z-1) - (m_n - m_p)c^2 - 2m_0c^2. \tag{2.15}$$

For the considered mirrors nuclei with $A = 2Z - 1$, eq. (2.15) is rewritten as follows taking into account that $R = r_0 A^{1/3}$,

$$E^{\alpha}_{\max} = \alpha(Z)\frac{e^2}{r_0}A^{2/3} - (m_n - m_p)c^2 - 2m_0c^2. \tag{2.16}$$

Let us substitute r_0 in eq. (2.16) by its expression [13],

$$r_0 = \frac{\alpha^2}{2}a_0. \tag{2.17}$$

In (2.17), α is the fine structure constant and a_0 the Bohr's radius given by

$$\alpha = \frac{e^2}{\hbar c}\ ; \quad a_0 = \frac{\hbar^2}{m_0 e^2}\,. \tag{2.18}$$

Substituting r_0 by its expression eq. (2.17) into eq. (2.16) and taking into account the expressions (2.18), we obtain finally

$$E_{\max} = 2\alpha(Z)m_0c^2A^{2/3} - (m_n - m_p)c^2 - 2m_0c^2\,. \tag{2.19}$$

Besides, the Q-value and $e_{\max}$ are linked

$$Q = E_{\max} + 2m_0c^2\,. \tag{2.20}$$

Taking into account (2.19), the Q-value is given by

$$Q = 2\alpha(Z)m_0c^2A^{2/3} - (m_n - m_p)c^2\,. \tag{2.21}$$

From eq. (2.21), we express $\alpha(Z)$ as follows:

$$2\alpha(Z) = \frac{Q + (m_n - m_p)c^2}{m_0c^2A^{2/3}}\,. \tag{2.22}$$

To estimate the value of $\alpha(Z)$, we neglect its dependency with Z. So eq. (2.22) becomes

$$2\alpha_0 = \frac{Q + (m_n - m_p)c^2}{m_0c^2A^{2/3}}\,. \tag{2.23}$$

Eq. (2.23) is an equation with two unknowns α_0 and m_0c^2 (we intend in this work to calculate it directly from q). Let us then establish an equation in terms of the nuclear charge distribution coefficient α_0. For that, we consider the following mass ratios from CODATA (2022)[1]

- Neutron-electron mass ratio: 1838.683 662 00
- Proton-electron mass ratio: 1836.152 673 426

Using these data, with find

$$(m_n - m_p) = 2.530988574m_0 = km_0 \tag{2.24}$$

with $k = 2.530988574$.

Using (2.23), eq. (2.23) is rewritten as follows

$$Q = (2\alpha_0A^{2/3} - k)m_0c^2\,. \tag{2.25}$$

The rest mass energy of the electron is finally expressed as follows

$$m_0c^2 = \frac{Q}{(2\alpha_0A^{2/3} - k)}\,. \tag{2.26}$$

Let us now consider two mirrors nuclei ${}^{x}X_1(A_1 = X)$ and ${}^{y}X_2(A_2 = Y)$. We get from eq. (2.26)

$$\begin{cases} m_0c^2 = \frac{Q(x)}{(2\alpha_0x^{2/3}-k)} \\ m_0c^2 = \frac{Q(y)}{(2\alpha_0y^{2/3}-k)} \end{cases} \Rightarrow \quad \frac{Q(x)}{(2\alpha_0x^{2/3} - k)} = \frac{Q(y)}{(2\alpha_0y^{2/3} - k)}\,. \tag{2.27}$$

Using eq. (2.27), we express finally the nuclear charge distribution coefficient α_0 in terms of the Q-value with the condition $X > Y$,

$$2\alpha_0 = \frac{k[Q(x) - Q(y)]}{[Q(x)y^{2/3} - Q(y)x^{2/3}]} > 0. \tag{2.28}$$

[1] CODATA, https://physics.nist.gov/cgi-bin/cuu/Value?mec2mev, (2022).

3. Result and Discussion

3.1 Value of the Nuclear Charge Distribution Coefficient

For various mirrors nuclei, primary calculations indicated that ^{37}K and ^{65}As have the same nuclear charge distribution coefficient. From Wang *et al*. [16], we extract the Q-value experimental data,

- For ^{37}K: $Q(Y) = 6.1475$ MeV; $Y = 37$;
- For ^{65}As: $Q(X) = 9.5400$ MeV; $X = 65$.

Using these experimental data, we get from eq. (2.28) taking into account the value of $k = 2.530988574$,

$$2\alpha_0 = \frac{2.530988574 \times [9.5400 - 6.1475]}{[9.5400 \times 37^{2/3} - 6.1475 \times 65^{2/3}]} = 1.31142.$$

Remarking that $3/5 = 0.6$ and $\alpha_0 = 0.655710$, we obtain approximately

$$\begin{cases} 2\alpha_0 = \frac{6}{5} + 0.1114 = 1.3114 \\ \alpha_0 = \frac{3}{5} + 0.0557 = 0.6557 \end{cases} \tag{3.1}$$

The nuclear charge distribution coefficient (NCDC) $\alpha(Z)$ is then given by

$$\alpha(Z) = \frac{3}{5} + \varepsilon(Z). \tag{3.2}$$

In eq. (3.2), $\epsilon(Z)$ is a corrective term depending on the mirror nucleus of charge number Z. For the tow mirrors nuclei ^{37}K ($Z = 19$) and ^{65}As ($Z = 33$), $\epsilon(19) = \epsilon(33) = 0.0557$ according to the second relation (3.1).

3.2 Deviation From the Nuclear Charge Uniform Distribution Within the Nucleus

To appreciate the distribution of the nuclear charge within the nucleus, let us use eq. (2.23) to plot the variation of $\alpha(Z)$ with the experimental Q-value taken from Wang *et al*. [16]. Using the exact value of m_0c^2 from MeV CODATA[1], we obtained the results quoted in Table 1. For the uniform nuclear charge distribution model, $2\alpha(Z) = 6/5 = 1.2$. Table 1 show clearly that this model is not appropriated for the considered mirrors nuclei. This is also demonstrated considering the variation of the NCDC $\alpha(Z)$ with the charge number Z. The curve obtained is displayed in Figure 1. As shown in Figure 1, all the value of the NCDC ranges between 0.62 and 0.69 with A ranging between 11 and 99. This result demonstrates that for the mirrors nuclei considered in this work, the protons are not strictly distributed uniformly inside the nuclear volume. A best model is to consider that the nuclear density for the mirrors nuclei $A = 2Z - 1$ decreases from the center to the surface of the nucleus as well known. Besides, form Figure 1, we can obtain an average value α_0 as follows

$$\begin{cases} \overline{\alpha}_0 = \frac{0.62+0.63+0.64+0.65+0.66+0.67+0.68+0.69}{8} = 0.6550 \\ 2\overline{\alpha}_0 = \frac{0.62+0.63+0.64+0.65+0.66+0.67+0.68+0.69}{4} = 1.310 \end{cases} \tag{3.3}$$

The average (3.3) of the NCDC agree very well with the value $\alpha_0 = 0.6557$ (3.1) specific to ^{37}K and ^{65}As. Subsequently, this value can be used to calculate the Q-value for the mirrors nucleus considered in this work.

3.3 Calculations of the Rest Mass Energy of the Electron

The average value of the rest mass energy of the electron is calculated from eq. (2.26) using $\alpha_0 = 0.6557$, $k = 2.530988574$ and $A = 37$ and 65 respectively for ^{37}K ($q = 6.1475$ MeV) and ^{65}As ($q = 9.5400$ MeV). We get

$$\begin{cases} m_0c^2 = \frac{6.1475}{(1.3114\times 37^{2/3} - 2.53098857)} = 0.5109968497347 \\ m_0c^2 = \frac{9.5400}{(1.3114\times 65^{2/3} - 2.53098857)} = 0.5109962526110 \end{cases} \tag{3.4}$$

The average value of the rest mass energy of the electron is then equals to

$$m_0c^2 = 0.510996\,551\,323\,\text{MeV} \approx 0.510997\,\text{MeV}. \tag{3.5}$$

The result (3.5) agrees excellently with the CODATA data at 0.510 999 MeV. The percentage deviation relatively to the COADAT value is at 0.000 2%. Table 2 compares the predictions from the present Q-βT and the experimental data from Compton scattering [8]. It is clearly seen that, the Q-βT provides more accurate results that the Compton scattering of γ photons (γ-CSM) by cristals applied by Hosur and Badiger [8] and by Ganesan *et al*. [4] who obtained in their measurements 524 ± 17 keV associated to a percentage deviation at 2.5%. In Table 3, the present rest mass energy of the electron is also estimated from the Q-value of other mirrors nuclei having approximately the same protons distribution within the nucleus. Using eq. (2.26), we find an average value of 0.511 004 MeV agreeing very well with the 0.510 999 MeV CODATA data. Here, the percentage deviation is at 0.000 2%.

3.4 Calculations of the Q-value

Neglecting in eq. (3.3) the correction term $\epsilon(Z)$ depending on the mirror nucleus of charge number Z, we put $2\alpha(Z) \approx 2\alpha_0$. Eq. (2.21) is then written in the shape

$$Q = 2\alpha_0 m_0 c^2 A^{2/3} - (m_n - m_p)c^2. \tag{3.6}$$

Using our results $2\alpha_0 = 1.3114$ (3.1), $m_0c^2 = 0.510\ 996\ 551$ MeV (3.5) and $(m_n - m_p) = 1.293\ 332\ 51$ MeV (CODATA, 2022), eq. (3.6) is in the shape

$$Q(A) = 0.67012088A^{2/3} - 1.29333251. \tag{3.7}$$

Using eq. (3.7), we obtain the results presented in Table 4. Comparison shows a very good agreement between theory and experiment for the listed mirrors nuclei. The maximum percentage deviation is less than 5%. In addition, we note the same experimental value 10.1800 keV for the two mirror nuclei ^{69}Br and ^{71}Kr for which our predictions give respectively 9.9800 MeV with a precision of 2 % and 10.1968 MeV with a precision of 0.2%. It follows that the experimental value 10.1800 attributed to ^{69}Br is not precise. This value is of the order of our prediction 9.9800 MeV. In addition, the decay energy increases with the increase in the mass number A. Table 2 shows the decrease in this energy for the nuclei with the increase in A for the couples ^{63}Ge (9.6300 MeV) and ^{65}As (9.5400 MeV). For these two nuclei, our predictions give a good evolution of the decay energy which increases respectively from 9.3166 MeV to 9.5400 MeV. The excellent agreement for ^{37}K, 65As and ^{85}Tc is remarkable. We also note the same decrease in energy for ^{87}Ru (11.9600 MeV) and ^{89}Rh (11.7200 MeV). Our predictions provide 11.8639 MeV and 12.0648 MeV respectively. Furthermore, the values recorded in Table 2 generally show that the uniform volume distribution model of the nuclear charge is a fairly crude model. There are protons distributed on the surface of the nucleus. The best model would therefore be to consider protons distributed both inside the nucleus and on the surface. Except for fluorine 17 with a very high prediction of 10.9%, the precision of our calculations is less than 6%. Furthermore, for ^{17}F, our prediction is at 3.1372 MeV in disagreement with the experimental value 3.7605 MeV. This measurement leading to a very high precision of 16.6% is probably not precise. Our prediction would be a good reference value. It should be mentioned some discrepancies between the present calculations and the experimental data from Wang *et al*. [16] for the particular nuclei ^{13}N, ^{19}Ne and ^{21}Na. For these nuclei, our predictions are respectively at 2.4116 MeV, 3.4782 MeV and 3.8074 MeV compared with the experimental data respectively equal to (the percentage deviation is putted into parenthesis) 2.2205 MeV (7.9%), 3.2395 MeV (7.9%) and 3.5469 MeV (6.8%). For these results, great percentage deviations compared with those quoted in Table 4 need to be enlightened by new measurements or by another calculations. New accurate Q-values are tabulated for nuclei mirrors with A ranging between 101 and 199 are listed in Table 5.

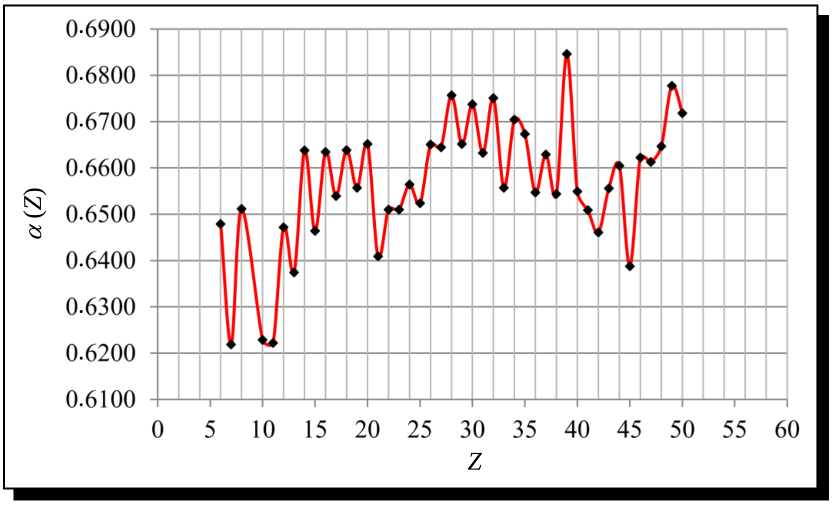

Figure 3. Variation of the nuclear charge distribution coefficient $\alpha(Z)$ (NCDC) with the charge number Z. For the mirrors nuclei considered with A ranging between 11 and 99, all the nuclear charge distribution coefficients are comprised between 0.62 and 0.69

Table 1. Values of the nuclear distribution coefficient $\alpha(Z)$

Element	Z	A	$Q_{\mathrm{exp}}(\beta^+)$ (MeV)^b	$2\alpha(Z)$	$2\alpha(Z)$ (approximate)
Carbon (C)	6	11	1.9817	1.2958	1.30
Nitrogen (N)	7	13	3.2205	1.2437	1.24
Oxygen (O)	8	15	3.754	1.3023	1.30
Fluorine (F)	9	17	3.7605	1.1999	1.20
Neon (Ne)	10	19	3.2395	1.2458	1.25
Sodium (Na)	11	21	3.5469	1.2444	1.24
Magnesium (Mg)	12	23	4.0562	1.2944	1.29
Aluminum (Al)	13	25	4.2768	1.2749	1.27
Silicon (Si)	14	27	4.8124	1.3276	1.33
Phosphorus (P)	15	29	4.9422	1.2928	1.29
Sulfur (S)	16	31	5.3980	1.3269	1.33

Table Contd.

Element	Z	A	$Q_{\exp}(\beta^+)$ (MeV)^b	$2\alpha(Z)$	$2\alpha(Z)$ (approximate)
Chlorine (Cl)	17	33	5.5825	1.3079	1.31
Argon (Ar)	18	35	5.9662	1.3277	1.33
Potassium (K)	19	37	6.1475	1.3114	1.31
Calcium (Ca)	20	39	6.5245	1.3303	1.33
Strontium (Sc)	21	41	6.4955	1.2818	1.28
Titanium (Ti)	22	43	6.8730	1.3020	1.30
Vanadium (V)	23	45	7.1238	1.3020	1.30
Chromium (Cr)	24	47	7.4440	1.3129	1.31
Manganese (Mn)	25	49	7.6345	1.3047	1.30
Iron (Fe)	26	51	8.0540	1.3301	1.33
Cobalt (Co)	27	53	8.2880	1.3289	1.33
Nickel (Ni)	28	55	8.6940	1.3514	1.35
Copper (Cu)	29	57	8.7749	1.3303	1.33
Zinc (Zn)	30	59	9.1428	1.3476	1.35
Gallium (Ga)	31	61	9.2100	1.3264	1.33
Germanium (Ge)	32	63	9.6300	1.3501	1.35
Arsenic (As)	33	65	9.5400	1.3114	1.31
Selenium (Se)	34	67	10.0100	1.3409	1.34
Bromine (Br)	35	69	10.1800	1.3347	1.33
Krypton (Kr)	36	71	10.1800	1.3095	1.31
Rubidium (Rb)	37	73	10.5400	1.3258	1.33
Strontium (Sr)	38	75	10.6000	1.3087	1.31
Yttrium (Y)*	39	77	11.3700	1.3692	1.37
Zirconium (Zr)*	40	79	11.0300	1.3098	1.31
Niobium (Nb)*	41	81	11.1600	1.3018	1.30
Molybdenum (Mo)*	42	83	11.2700	1.2921	1.29
Technetium (Tc)*	43	85	11.660	1.3112	1.31
Ruthenium (Ru)*	44	87	11.9600	1.3210	1.32
Rhodium (Rh)*	45	89	11.7200	1.2775	1.28
Palladium (Pd)*	46	91	13.4000	1.3245	1.32
Silver (Ag)*	47	93	13.5800	1.3226	1.32
Cadmium (Cd)*	48	95	13.8500	1.3294	1.33
Indium (In)*	49	97	13.3300	1.3555	1.36
Tin (Sn)*	50	99	13.4000	1.3436	1.34

^{a}: Present results from eq. (2.22)

^{b}: Experimental values, Wang *et al.* [16]

*: Theoretical values from $Z = 39$

Table 2. Comparison of the present QβT results with the experimental data of Hosur and Badiger [8] from Compton scattering of γ photons (γ-CSM) by crystals

γ-CSM Experiment			QβT Theory		
Nucleus	m_0c^2 (keV)	Accuracy (%)	Nucleus	m_0c^2 (keV)	Accuracy*(%)
^{203}Hg	493.3	3.6	^{37}K	511	< 0.0001
^{137}Cs	524.8	3.7	^{71}Kr	511	< 0.0001
^{54}Mn	529.6	3.6	-	-	-
^{60}Co	513.9	0.4	-	-	-

*: The exact percentage deviation 0.000 24% is evaluated with respect to the CODATA value at 0.510 998 950 690 MeV[1]

Table 3. Comparison of the present QβT results for various mirrors nuclei with the CODATA recommended value. The rest mass electron m_0c^2 is calculated form eq. (2.26). The value of $2\alpha_0$ is obtained from eq. (3.1). The Q-value of a given nucleus is extracted from Wang *et al*. [16]

Nucleus	$2\alpha_0$	A	Q (MeV) [16]	m_0c^2 (MeV)	Accuracy* (%)
^{15}O	1.3023	15	2.754 20	0.510 996 28	0.000 27
^{43}Ti	1.3020	43	6.873 00	0.511 022 85	0.002 39
^{45}V	1.3020	45	7.123 82	0.510 988 56	0.001 04
^{81}Nb	1.3018	81	11.160 00	0.510 996 28	0.000 27
Average value of the rest mass electron				0.510 996 28	-

*: The exact percentage deviation evaluated with respect to the CODATA value at 0.510 998 95 MeV [1]

Table 4. $Q(\beta^+)$-value for various mirrors nuclei. The present calculations are compared with experimental and theoretical data

Element	Z	A	$Q_{\text{exp}}(\beta^+)$ (MeV)^a	$Q_{\text{theo}}(\beta^+)$ (MeV)^b	$\Delta Q/Q$ (%)
Carbon (C)	6	11	1.9817	2.0211	2.0
Oxygen (O)	8	15	3.754	2.7825	1.0
Magnesium (Mg)	12	23	4.0562	4.1263	1.7
Aluminum (Al)	13	25	4.2768	4.4361	3.6
Silicon (Si)	14	27	4.8124	4.7378	1.6
Phosphorus (P)	15	29	4.9422	5.0320	1.8
Sulfur (S)	16	31	5.3980	5.3196	1.5
Chlorine (Cl)	17	33	5.5825	5.6011	0.3
Argon (Ar)	18	35	5.9662	5.8769	1.5
Potassium (K)	19	37	6.1475	6.1475	0.0

Table Contd.

Element	Z	A	Q_{exp} (β^+) (MeV)^a	Q_{theo} (β^+) (MeV)^b	$\Delta Q/Q$ (%)
Calcium (Ca)	20	39	6.5245	6.4133	1.7
Strontium (Sc)	21	41	6.4955	6.6745	2.7
Titanium (Ti)	22	43	6.8730	6.9316	0.8
Vanadium (V)	23	45	7.1238	7.1847	0.8
Chromium (Cr)	24	47	7.4440	7.4341	0.1
Manganese (Mn)	25	49	7.6345	7.6799	0.6
Iron (Fe)	26	51	8.0540	7.9225	1.7
Cobalt (Co)	27	53	8.2880	8.1619	1.5
Nickel (Ni)	28	55	8.6940	8.3983	3.5
Copper (Cu)	29	57	8.7749	8.6318	1.7
Zinc (Zn)	30	59	9.1428	8.8626	3.2
Gallium (Ga)	31	61	9.2100	9.0909	1.3
Germanium (Ge)	32	63	9.6300	9.3166	3.4
Arsenic (As)	33	65	9.5400	9.5400	0.0
Selenium (Se)	34	67	10.0100	9.7611	2.5
Bromine (Br)	35	69	10.1800	9.9800	2.0
Krypton (Kr)	36	71	10.1800	10.1968	0.2
Rubidium (Rb)	37	73	10.5400	10.4116	1.2
Strontium (Sr)	38	75	10.6000	10.6244	0.2
Yttrium (Y)*	39	77	11.3700	10.8354	4.9
Zirconium (Zr)	40	79	11.0300	11.0445	0.1
Niobium (Nb)	41	81	11.1600	11.2518	0.8
Molybdenum (Mo)	42	83	11.2700	11.4575	1.6
Technetium (Tc)	43	85	11.660	11.6615	0.0
Ruthenium (Ru)	44	87	11.9600	11.8639	0.8
Rhodium (Rh)	45	89	11.7200	12.0648	2.9
Palladium (Pd)	46	91	13.4000	12.2642	1.1
Silver (Ag)	47	93	13.5800	12.4621	0.9
Cadmium (Cd)	48	95	13.8500	12.6586	1.5
Indium (In)	49	97	13.3300	12.8538	3.7
Tin (Sn)	50	99	13.4000	13.0476	2.7

^{a}: Present results from eq. (3.7)

^{b}: Experimental values from Wang *et al*. [16]

*: Theoretical values from $Z = 39$ [16]

Table 5. New $Q(\beta^+)$-value for various mirrors nuclei with $A = 101 - 199$

Z	A	$Q(\beta^+)$ (MeV)	Z	A	$Q(\beta^+)$ (MeV)
51	101	13.2401	76	151	17.7089
52	103	13.4313	77	153	17.8763
53	105	13.6213	78	155	18.0430
54	107	13.8101	79	157	18.2090
55	109	13.9977	80	159	18.3743
56	111	14.1842	81	161	18.5388
57	113	14.3696	82	163	18.7027
58	115	14.5538	83	165	18.8660
59	117	14.7370	84	167	19.0286
60	119	14.9192	85	169	19.1905
61	121	15.1004	86	171	19.3518
62	123	15.2805	87	173	19.5124
63	125	15.4597	88	175	19.6725
64	127	15.6379	89	177	19.8319
65	129	15.8152	90	179	19.9908
66	131	15.9916	91	181	20.1490
67	133	16.1671	92	183	20.3067
68	135	16.3417	93	185	20.4638
69	137	16.5154	94	187	20.6203
70	139	16.6883	95	189	20.7763
71	141	16.8604	96	191	20.9317
72	143	17.0317	97	193	21.0866
73	145	17.2021	98	195	21.2409
74	147	17.3718	99	197	21.3947
75	149	17.5407	100	199	21.5480

Appendix: Expression of the Coulomb Energy

The Coulomb energy is evaluated by assuming that the total nuclear charge $Q = Ze$ to be spread uniformly throughout the spherical nuclear volume of radius $R = r_0A^{1/3}$. In the framework of this model. The charge density ρ is

$$\rho = \frac{Q}{4/3 \cdot \pi \cdot R^3}. \tag{A.1}$$

Let us consider then a sphere of r radius and of uniform q charge distribution (Figure A.1).

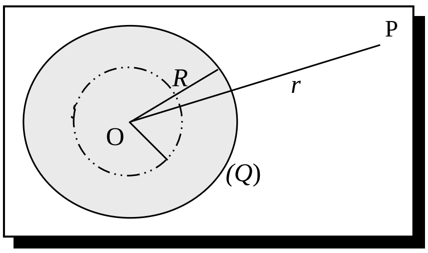

Figure A.1. Sphere of r radius and of uniform q charge distribution

Outside the sphere ($r > R$). The electric field is

$$E_{\text{out}} = k\frac{Q}{r^2}. \tag{A.2}$$

Inside the sphere ($r > R$). The electric field is

$$E_{\text{in}} = k\frac{Q(r)}{r^2}. \tag{A.3}$$

As the volumic charge density ρ is assumed to be constant. The following relations are satisfied

$$Q = \rho V\ ; \quad Q(r) = \rho V(r), \tag{A.4}$$

where

$$V = \frac{4}{3}\pi R^3\ ; \quad V(r) = \frac{4}{3}\pi r^3. \tag{A.5}$$

From eqs. (A.5), we put

$$\frac{Q(r)}{Q} = \frac{V(r)}{V} = \frac{r^3}{R^3}. \tag{A.6}$$

These equations give

$$\frac{Q(r)}{Q} = \frac{V(r)}{V} = \frac{r^3}{R^3}. \tag{A.7}$$

By use of eq. (A.7). We find the expression of the electric charge inside the sphere of r radius and of uniform q charge distribution

$$Q(r) = Q\frac{r^3}{R^3}. \tag{A.8}$$

Considering eq. (A.8). the electric field given by eq. (A.3) is in the form

$$E_{\text{in}} = kQ\frac{r}{R^3}. \tag{A.9}$$

Besides, using the Poynting vector and Maxwell's equations. One can express the electromagnetic energy volumic density in the following form [1]

$$\omega = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}. \tag{A.10}$$

As magnetic phenomena are ignored. The electric energy volumic density is

$$\omega_{el} = \frac{\varepsilon_0 E^2}{2}. \tag{A.11}$$

Then, outside and inside the sphere of r radius and of uniform q charge distribution we find respectively

$$\omega_{\text{out}} = \frac{\varepsilon_0 E_{\text{out}}^2}{2}\ ; \quad \omega_{\text{in}} = \frac{\varepsilon_0 E_{\text{in}}^2}{2}. \tag{A.12}$$

On the other hand, the dW_c elementary volumic electric energy stored inside the elementary volume $d\tau = r^2 dr \sin\theta\, d\theta\, d\varphi$ is

$$dW_c = \omega_{\text{in}} d\tau_{\text{in}} + \omega_{\text{out}} d\tau_{\text{out}} \tag{A.13}$$

That is-to-say using eq. (A.12),

$$dW_c = \frac{\varepsilon_0 E_{\text{in}}^2}{2} d\tau_{\text{in}} + \frac{\varepsilon_0 E_{\text{out}}^2}{2} d\tau_{\text{out}}. \tag{A.14}$$

The integration of eq. (A.14) is taken in the whole space. This involve the following expressions

$$W_c = \iiint \frac{\varepsilon_0 E_{\text{in}}^2}{2} d\tau_{\text{in}} + \iiint \frac{\varepsilon_0 E_{\text{out}}^2}{2} d\tau_{\text{out}}, \quad \text{(A.15)}$$

$$W_c = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\, d\theta \left[\int_0^R \frac{\varepsilon_0 E_{\text{in}}^2}{2} r^2 dr + \int_R^{\infty} \frac{\varepsilon_0 E_{\text{out}}^2}{2} r^2 dr \right]. \quad \text{(A.16)}$$

By use of eqs. (A.2) and (A.9), eq. (A.16) becomes

$$W_c = 4\pi\varepsilon_0 \frac{k^2 Q^2}{2} \left[\frac{1}{R^6} \int_0^R r^4 dr + \int_R^{\infty} \frac{1}{r^2} dr \right]. \quad \text{(A.17)}$$

Knowing that $k = \frac{1}{4\pi\varepsilon_0}$, integration of eq. (A.17) gives after arrangement

$$W_c = k \frac{3}{5} \frac{Q^2}{R}. \quad \text{(A.18)}$$

In uemcgs $k = 1$. So

$$W_c = \frac{3}{5} \frac{Q^2}{R}. \quad \text{(A.19)}$$

4. Conclusion

In this paper have applied our new Q-β-decay theory (QβT) to express theoretically for the first time as far as we know, the β^+-Q-value for mirrors nuclei $A = 2Z - 1$. It is demonstrated that the protons are not rigorously uniformly distributed within the nucleus. In addition, the present formalism gives more accurate electron rest mass energy than the widely used Compton scattering of γ-photon by crystals. Accurate theoretical Q-value for mirrors nuclei with A ranging between 11 and 99 are obtained. New accurate Q-values are tabulated for nuclei mirrors with A ranging between 101 and 199. The present Q-β-DT recently applied the estimation of the velocity of light in vacuum [15], make it possible to understand many nuclear properties and phenomena depending on the Q-value such as comparison between experimental and theoretical predictions of atomic masses, the understanding of the weak force and of the competitive processes between electron capture and β^+-decay.

Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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